AP Calculus AB – The Meaning of the Derivative
Fall 2020 - Unit 3

Date	Topic	Assignment
Tuesday, September 1st	**Keeper 3.1 The Average Rate of Change & the Definition of a Derivative at a Point** *EQ: What is a derivative?*	Rates of Change & the Derivative (Packet p. 1 - 3)
Wednesday, September 2nd	**Keeper 3.2 The Definition of the Derivative** *What is the derivative of a function at a point and how is it related to the tangent line?*	The Derivative as a Function; Differentiability (Packet p. 4 – 6)
Thursday, September 3rd	**Keeper 3.3 Interpreting the Derivative** *How can you interpret derivatives in the real world?*	**Skills Check 3.1 & 3.2** Interpreting the Derivative (Packet p. 7 - 8) Review – Using the Definition of the Derivative (Packet p. 9 - 10)
Friday, September 4th	**Keeper 3.4 Curve Sketching f to f'** *What does the first derivative and the second derivative tell us about the function?*	**Virtual Quiz 3.1-3.3** Curve Sketching – Graphing f' from f (Packet p. 11 - 12)
Tuesday, September 8th	**Keeper 3.5 Curve Sketching f from f'** *What does the first derivative and the second derivative tell us about the function?*	**Skills Check 3.4** Curve Sketching (Packet p. 13 – 14) Curve Sketching – Graphing f from f' (Packet p. 15 – 16)
Wednesday, September 9th	**Review**	Practice Test – The Meaning of Derivatives (Packet p. 17 – 21)
Thursday, September 10th	**Unit 3 Test**	Good Luck!

Rates of Change and The Derivative

Find an equation for the tangent line and the normal line to the graph of each function at the indicated value.

1. $f(x) = x^2 + 2, x = -1$
2. $f(x) = x^3 + 1, x = 1$
3. $f(x) = \frac{2-5x}{1+x}$ at 0
4. $f(x) = \sqrt{x+3}, x = 6$
5. $f(x) = \frac{1}{\sqrt{x}}, x = 4$
6. $f(x) = \frac{1}{x^2}, x = 2$

Find the rate of change of f at the indicated number.

7. $f(x) = 5x - 2, c = 0$

8. $f(x) = x^2 - 1, c = -1$

9. $f(x) = \frac{x^2}{x+3}, c = 0$

10. $f(x) = \frac{x}{x^2-1}, c = 2$

Find the derivative of each function at the given number.

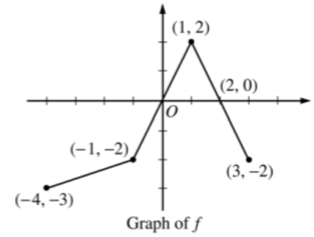
11. $f(x) = 2x + 3$ at 1

12. $f(x) = 3x^2 + x + 5$ at -1

AP Practice Problems

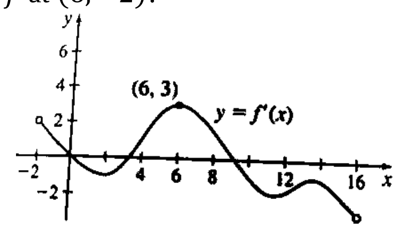
13. The line $x + y = 5$ is tangent to the graph of $y = f(x)$ at the point where $x = 2$. What are the values of $f(2)$ and $f'(2)$?

14. The graph of the function f, given below, consists of three line segments. Find $f'(0)$.



15. What is the instantaneous rate of change of the function $f(x) = 3x^2 + 5$ at $x = 2$?

16. The function f is defined on the closed interval $[-2,16]$. The graph of the derivative of f, $y = f'(x)$, is given below. The point $(6,-2)$ is on the graph of $y = f(x)$. What is the equation of the graph of f at $(6,-2)$?



17. If $x - 3y = 13$ is an equation of the normal line to the graph of f at the point $(2,6)$, then what is the value of $f'(2)$?

18. A tank is filled with 80 liters of water at 7 a.m. $(t = 0)$. Over the next 12 hours the water is continuously used and no water is added to replace it. The table below gives the amount of water $A(t)$ (in liters) remaining in the tank at selected times t, where t measures the number of hours after 7 a.m.

t	0	2	5	7	9	12
$A(t)$	80	71	66	60	54	50

Use the table to approximate $A'(5)$.

The Derivative as a Function; Differentiability

Find the derivative of each function at any real number c.

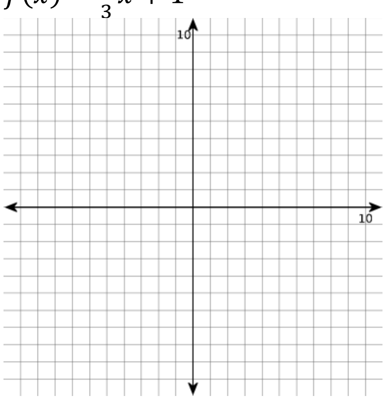
1. $f(x) = 10$
2. $f(x) = 2x + 3$
3. $f(x) = 2 - x^2$

Differentiate each function f and determine the domain of f'.

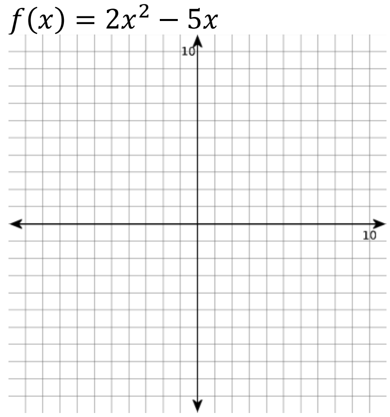
4. $f(x) = 5$
5. $f(x) = 3x^2 + x + 5$
6. $f(x) = 5\sqrt{x - 1}$

Differentiate each function f. Graph $y = f(x)$ and $y = f'(x)$ on the same set of coordinate axes.

7. $f(x) = \frac{1}{3}x + 1$



8. $f(x) = 2x^2 - 5x$



Find the derivative of each function.

9. $f(x) = mx + b$

10. $f(x) = ax^2 + bx + c$

11. $f(x) = \frac{1}{x^2}$

12. $f(x) = \frac{1}{\sqrt{x}}$

Each limit represents the derivative of a function f at some point c. Determine f and c.

13. $\lim_{h \to 0} \frac{(2+h)^2 - 4}{h}$

14. $\lim_{x \to 1} \frac{(x^2-1)}{x-1}$

16. $\lim_{h \to 0} \frac{\sqrt{9+h} - 3}{h}$

17. $\lim_{x \to \frac{\pi}{6}} \frac{\sin x - \frac{1}{2}}{x - \frac{\pi}{6}}$

18. $\lim_{x \to 0} \frac{2(x+2)^2 - (x+2) - 6}{x}$

19. $\lim_{h \to 0} \frac{(3+h)^2 + 2(3+h) - 15}{h}$

For problems 20-24, use the function $f(x) = \frac{x}{x+2}$

20. Find $f'(x)$ by finding $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

21. Find the slope of the normal line drawn to the graph of $f(x)$ at $x = -2$

22. Find the slope of the tangent line drawn to the graph of $f(x)$ at $x = -1$

23. Find the equation of the tangent line drawn to the graph of $f(x)$ at $x = 6$

24. Find $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$, where $a = 1$

Interpreting the Derivative

1. Suppose the cost of drilling x feet for an oil well is $C = f(x)$ dollars
   a. What are the units of $f'(x)$?

   b. In practical terms, what does $f'(x)$ mean in this case?

   c. What can you say about the sign of $f'(x)$?

   d. Estimate the cost of drilling an additional foot, starting at the depth of 300 ft, given that $f'(300) = 1000$

2. A paint manufacturing company estimates that it can sell $g = f(p)$ gallons of paint at a price of p dollars.
   a. What are the units of $\frac{dg}{dp}$

   b. In practical terms, what does $\frac{dg}{dp}$ mean in this case?

   c. What can you say about the sign of $\frac{dg}{dp}$?

   d. Given that $\frac{dg}{dp} = -100$ when $p = 20$, what can you say about the effect of increasing the price from \$20 per gallon to \$21 per gallon?

3. If $q = f(p)$ gives the number of cords of wood sold when the price per cord is p dollars.
   a. What are the units of $f'(p)$?

   b. In practical terms, what does $f'(p)$ mean in this case?

   c. What can you say about the sign of $f'(p)$?

   d. Given that $f'(425) = -\frac{2}{5}$, what can you say about the effect of decreasing the price of a cord of wood from \$425 to \$420?

4. If $q = f(p)$ gives the number of pounds of sugar produced when the price per pound is p dollars, then what are the units and the meaning of the statement $f'(3) = 50$?

The cost of producing x ounces of gold from a new gold mine is $C = f(x)$ dollars. What is the meaning of the derivative $f'(x)$? What are the units? What does the statement $f'(800) = 17$ mean?

5. The number of bacteria after t hours in a controlled laboratory experiment is $n = f(t)$. What is the meaning of $f'(5)$? What are its units?

6. Suppose there is an unlimited amount of space and nutrients for the bacteria. Which do you think is larger $f\,'(5)$ or $f\,'(10)$? If the supply of nutrients is limited, would that affect your conclusion?

7. Let T(t) be the temperature (in °F) in Phoenix t hours after midnight on September 10, 2008. The table shows the values of this function recorded every two hours. What is the meaning of T ' (8)? Estimate its value.

t	0	2	4	6	8	10	12	14
T	82	75	74	75	84	90	93	94

Review - Using the Definition of the Derivative

Each limit represents the derivative of some function f at some number a. State such an f and a in each case.

1. $\lim_{h \to 0} \frac{(1+h)^{10}-1}{h}$

2. $\lim_{h \to 0} \frac{\sqrt[4]{16+h}-2}{h}$

3. $\lim_{x \to 5} \frac{2^x-32}{x-5}$

4. $\lim_{x \to \pi/4} \frac{\tan x - 1}{x-\pi/4}$

5. $\lim_{h \to 0} \frac{\cos(\pi+h)+1}{h}$

6. $\lim_{t \to 1} \frac{t^4+t-2}{t-1}$

Find the derivative of the function using the definition using the definition of derivative. State the domain of the function and the domain of its derivative.

7. $f(x) = ax^2 + b$

8. $f(x) = x^2 - 2x^3$

9. $G(t) = \frac{1-2t}{3+t}$

10. $f(x) = x^{\frac{3}{2}}$

11. $g(t) = \frac{1}{\sqrt{t}}$

12. $f(x) = x^4$

13. Let $T(t)$ be the average monthly temperature (measured in degrees Fahrenheit) in Atlanta where t is measured in months since January 2000. What is the meaning of each of the following?

a.) $T(4) = 72$

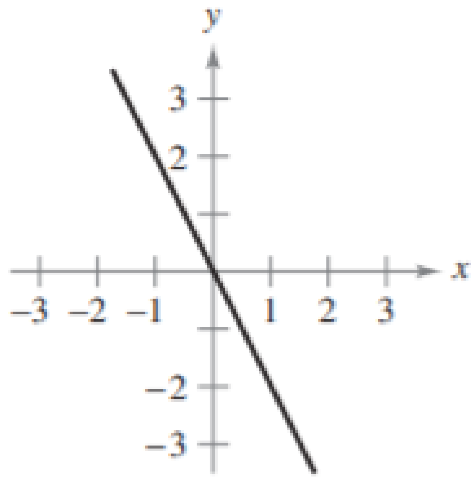
b) $T'(11) = -8$

c) $T'(14) = 12$

Curve Sketching - Graphing f' from f

The graph of f is given below. Sketch a possible graph of f' and f''

1.



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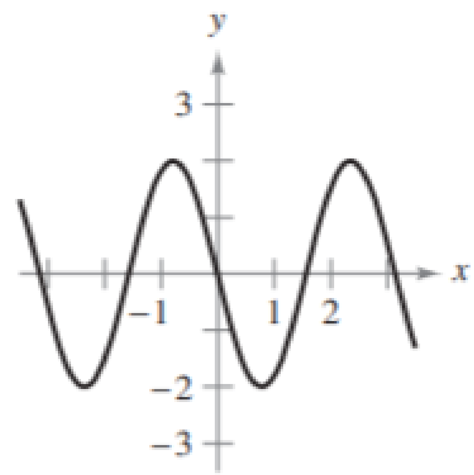

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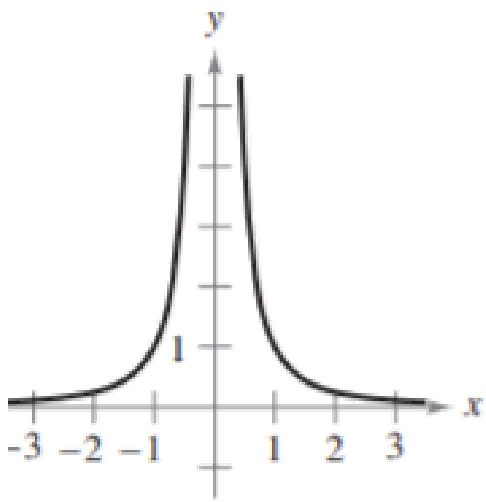

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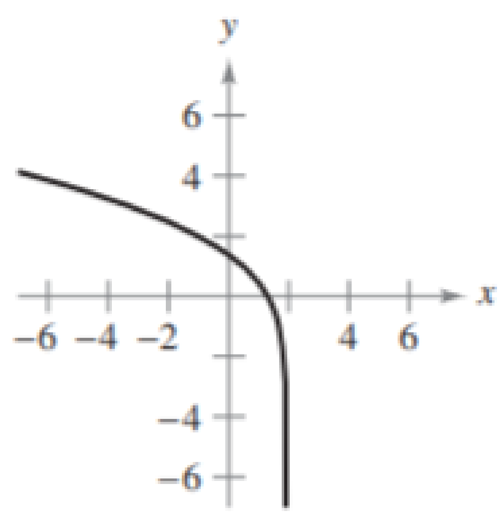

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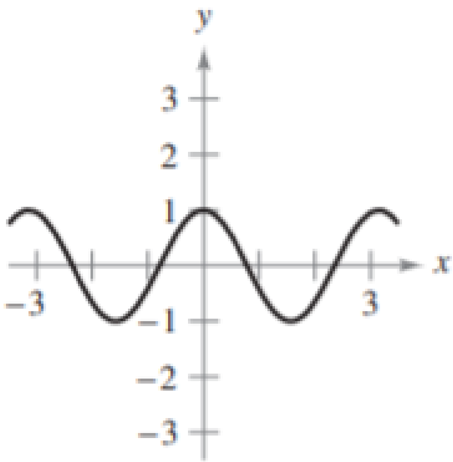

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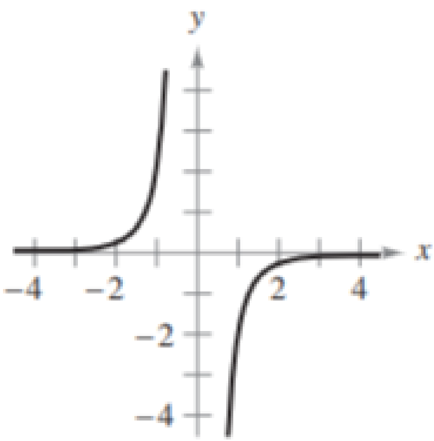

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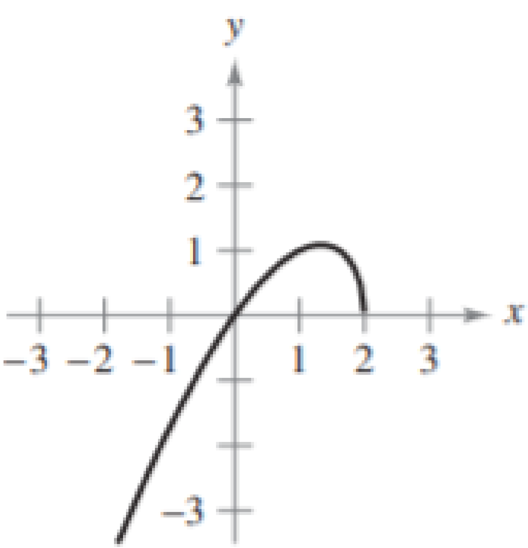

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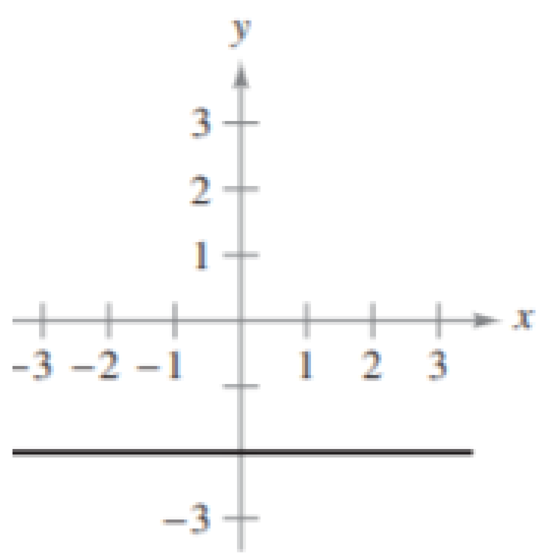

The graphs of f, f', and f'' are shown on the same set of coordinate axes. Which is which?

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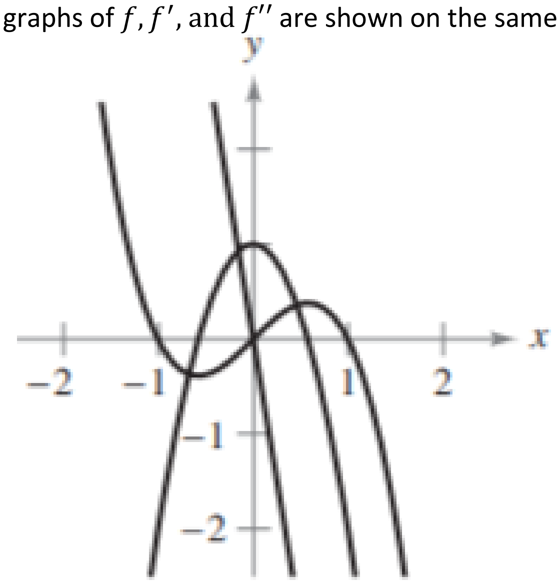

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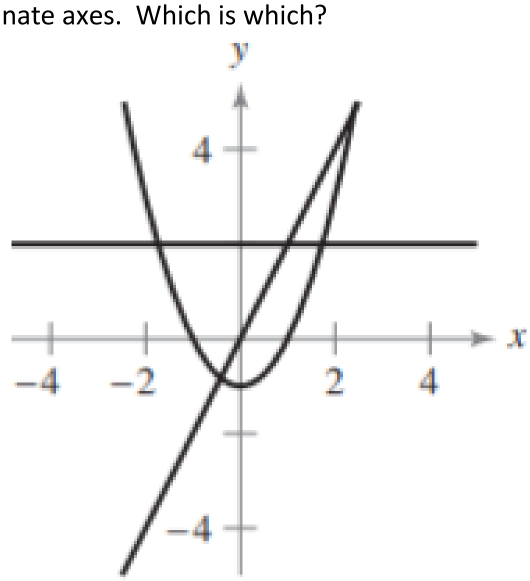


Curve Sketching

Draw a possible graph of $f(x)$ given the information below.

1. **a.** $f(x)$ is continuous

b. $f(3) = 2$

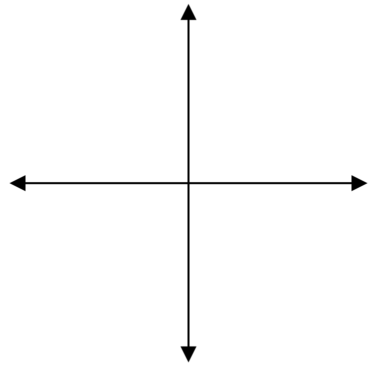
c. $f'(x) > 0$, $(-\infty, 0), (3, \infty)$

d. $f'(x) < 0$, $(0, 3)$

e. $f'(x) = 0$ at $x=0$, $x=3$

$f(x)$ $f(x)$

$f'(x)$



2. **a.** $f(x)$ is continuous

b. $f'(x) < 0$, $(-\infty, 1)$

c. $f'(x) > 0$, $(1, \infty)$

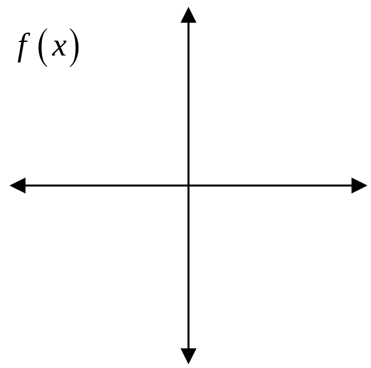
d. $f'(x) =$ undef. at $x=1$

e. $f''(x) < 0$ at $(-\infty, 1) \cup (1, \infty)$

$f(x)$ $f(x)$

$f'(x)$

$f''(x)$



3. **a.** $f(x)$ is continuous

b. $f'(x) < 0; (-\infty, 2), (2, \infty)$

c. $f'(x)$ is undefined at $x = 2$

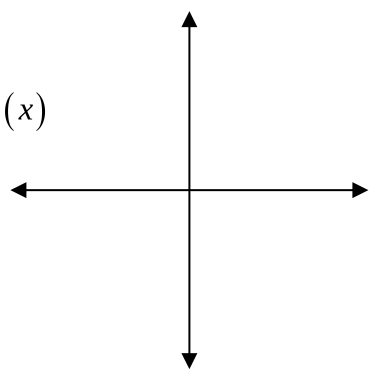
d. $f''(x) < 0$ when $x < 2$

e. $f''(x) > 0$ when $x > 2$

$f(x)$ $f(x)$

$f'(x)$

$f''(x)$



4. a. $f(x)$ has jump discont. at $x = -2$

b. $f'(x) > 0; (-\infty, -2), \cup (-2, \infty)$

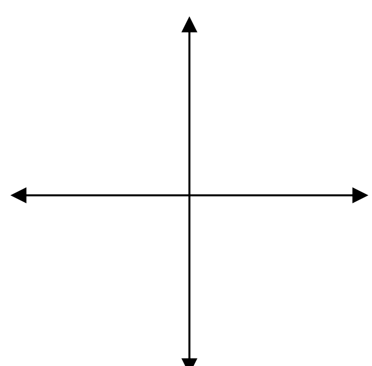
c. $f''(x) < 0; (-\infty, -2)$

d. $f''(x) > 0; (-2, \infty)$

$f(x)$

$f'(x)$

$f''(x)$



5. **a.** $f(x)$ is continuous $\qquad f(x) \qquad f(x)$

b. $f'(x) < 0$ when $x < 1$

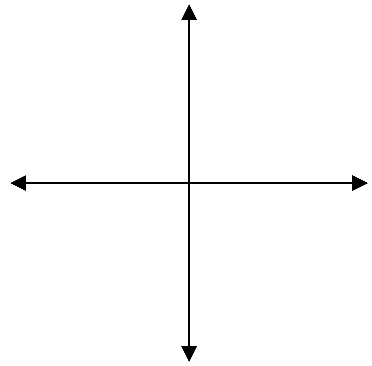
c. $f'(x) > 0$ when $x > 1$ $\qquad f'(x)$

d. $f''(x) > 0$ when $x < 1$

e. $f''(x) < 0$ when $x > 1$ $\qquad f''(x)$

f. $f'(x)$ does not exist at $x = 1$

g. $f''(x)$ does not exist at $x = 1$



6. **a.** $f(x)$ is continuous [-4, 3] $\qquad f(x) \qquad f(x)$

b. $f'(x) < 0$ on $(-4,-2)$

c. $f'(x) > 0$ on $(-2,1)\cup(1,3)$ $\qquad f'(x)$

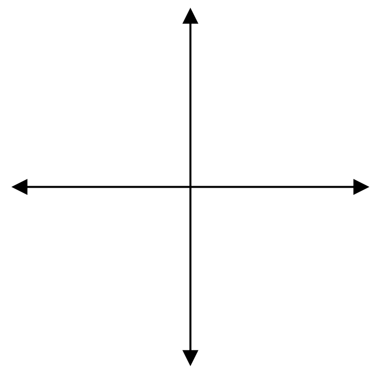
d. $f'(x) = \text{undef. at } x = -2$

e. $f(-2) = -3 \quad f(1) = 3$ $\qquad f''(x)$

f. $f'(x) = 0$ at x=1

g. $f'' < 0$ on $(-4,-2)\cup(-2,1)$

h. $f'' > 0$ on (1, 3)



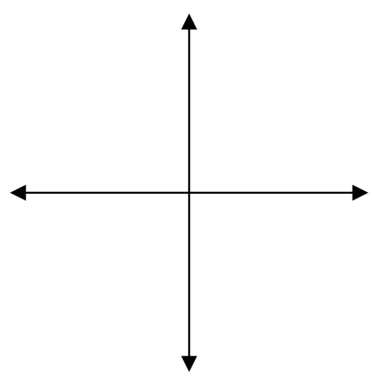
7. **a.** $f(x)$ is continuous $\qquad f(x) \qquad f(x)$

b. $f'(x) > 0$ everywhere

c. $f'(x) = 0$ when $x = -2,\ x = 3$ $\qquad f'(x)$

d. $f''(x) < 0$ on $(-\infty,-2)\cup(1,3)$

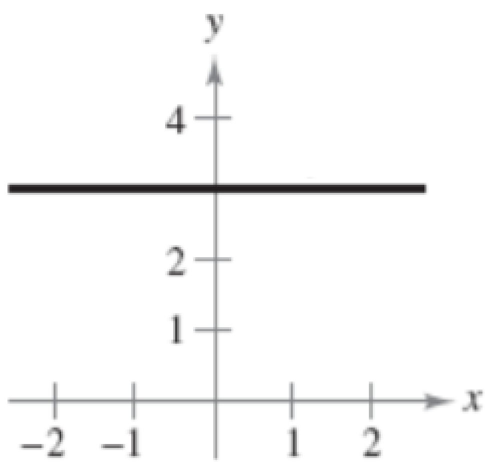
e. $f''(x) > 0$ on $(-2,1)\cup(3,\infty)$ $\qquad f''(x)$



Curve Sketching - Graphing f from f'

The graph of f' is given below. Sketch a possible graph of f

1.



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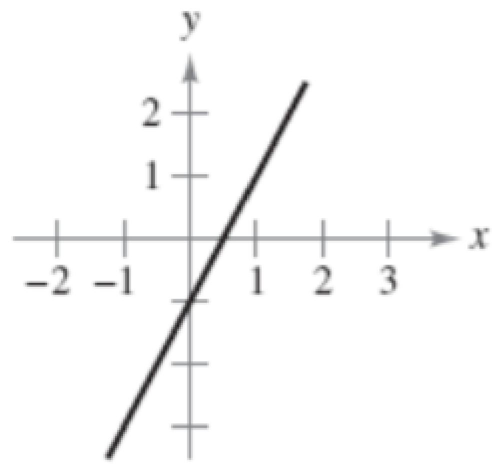

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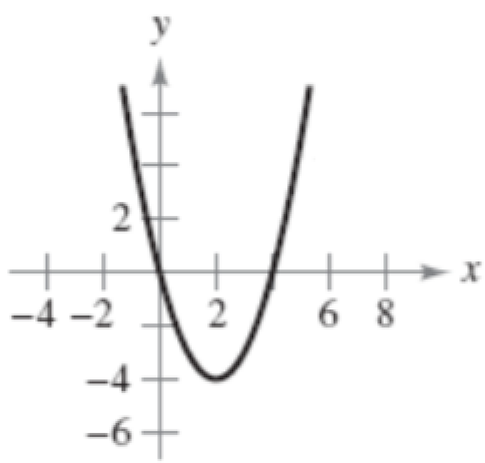

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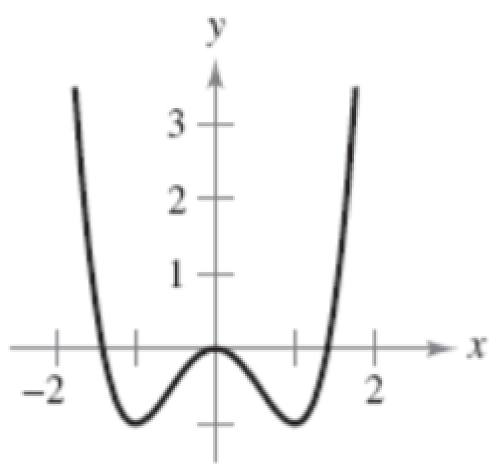

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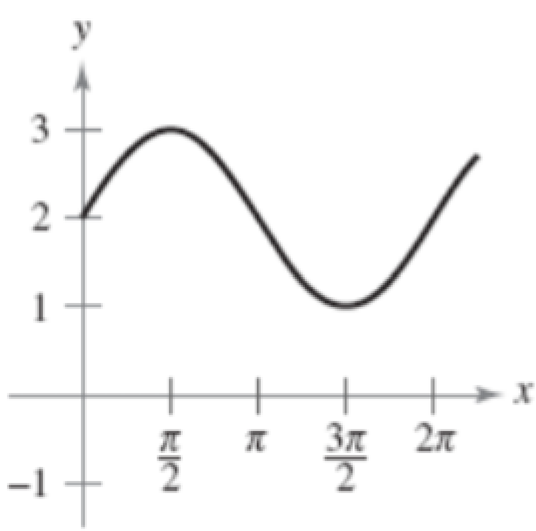

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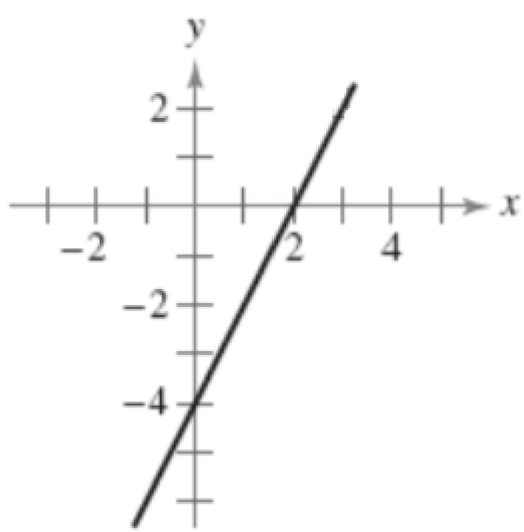

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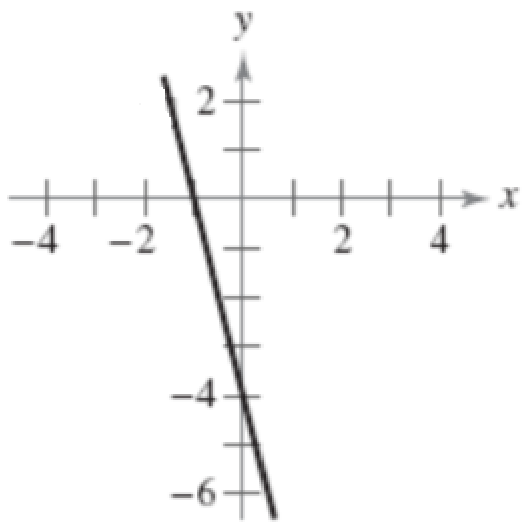

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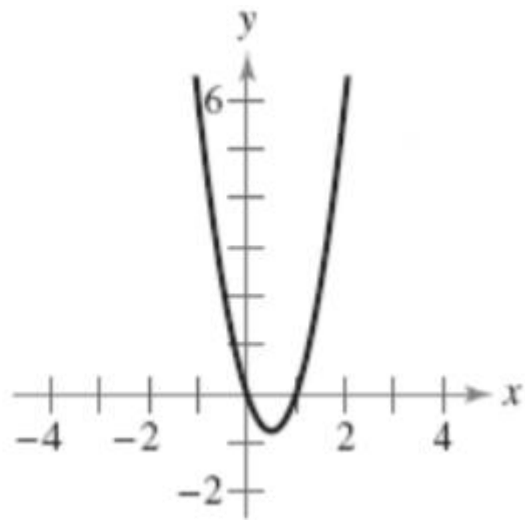

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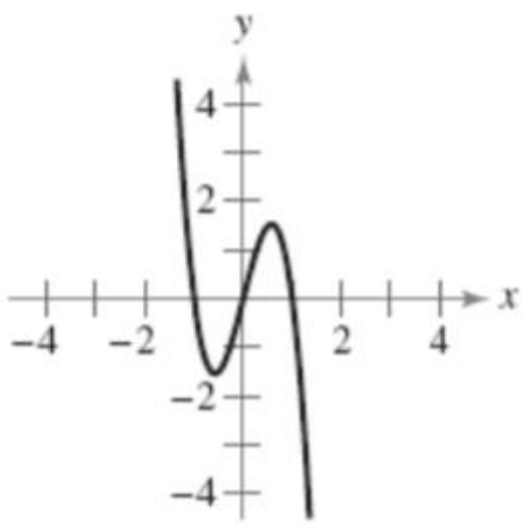

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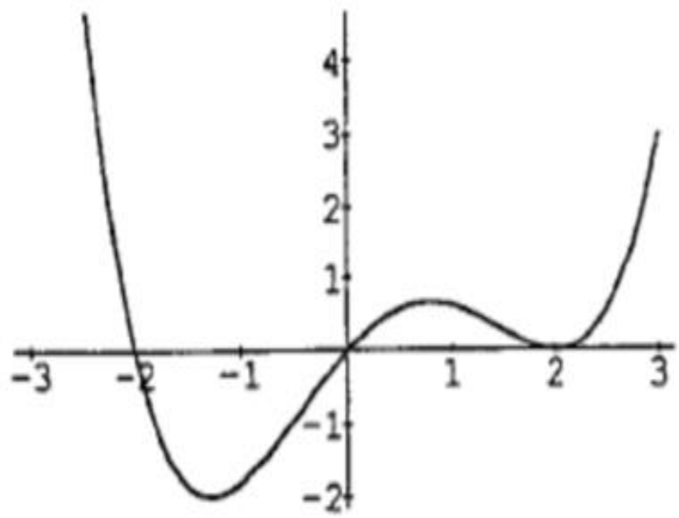


Practice Test – The Meaning of Derivatives

1. Multiple Choice: If f is a differentiable function, the $f'(a)$ is given by which of the following?

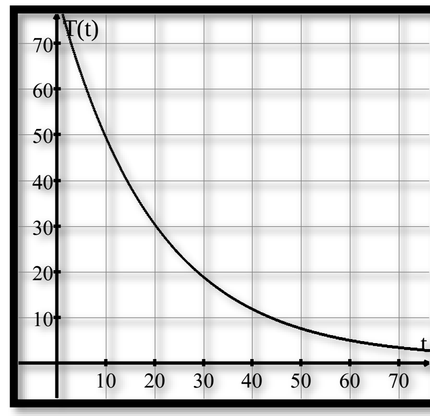
   I. $\lim_{h\to a} \frac{f(a+h)-f(a)}{h}$

   II. $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$

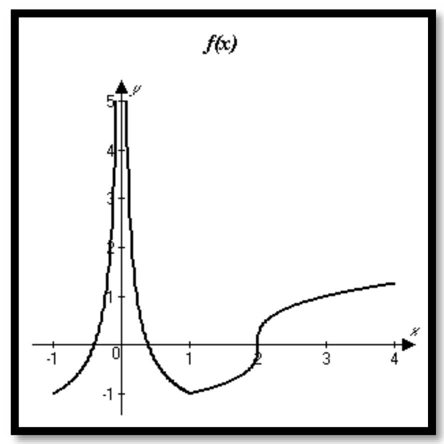
   III. $\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

a. II and III only   b. I and II only   c. III only   d. I and III only   e. I, II, III

2. A pot of hot soup is placed in the freezer to cool down. The temperature, T, of the soup at time t is given by the graph, where T is measured in $°F$ and t is measured in minutes. Estimate $T'(35)$ and interpret its meaning.



3. Use the graph of f to determine all value(s) of x such that g is not differentiable. Give a reason for each answer.



4. a. Find the derivative of $f(x) = \frac{4}{x-5}$ at $x = 7$.

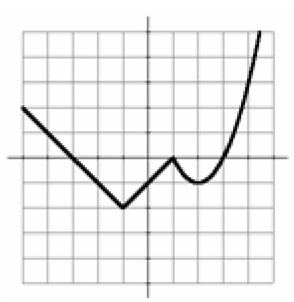
b. Write the equation of the tangent line for part a.

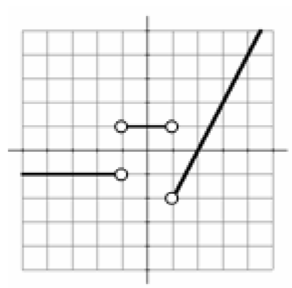
5. a. Use the definition of derivative to find the derivative of $f(x) = x^2 - 3x + 2$.

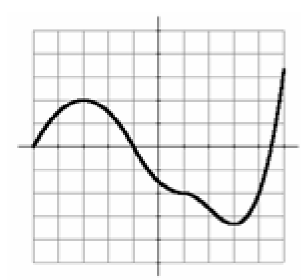
b. Find an equation of the tangent line to the curve at the point where $x = -2$.

6. Find the derivative as a function of x if $f(x) = \sqrt{5x+2}$.

7. Below, in no particular order, are the graphs of $f(x)$, $f'(x)$, and $f''(x)$. Decide which graph goes with each function.







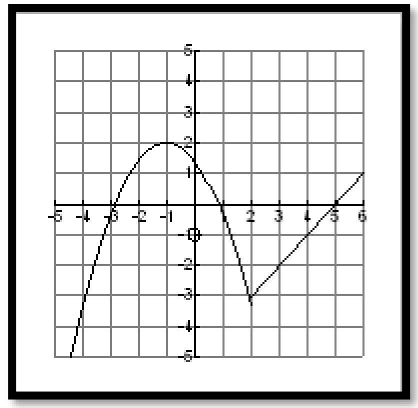
In #8 and 9, Multiple Choice.

8. The graph of a function $y = f(x)$ is shown below. Which of the following are true for the function?

   I. $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x)$

   II. $f'(2)$ is defined

   III. $f'(x) < 0$ for all x in the open interval $(-5,2)$



a. I only   b. II only   c. I and III   d. I and II   e. I, II, and III

9. $\lim_{h\to 0} \frac{\ln(e+h)-1}{h} =$

a. $f'(x)$ where $f(x) = \ln x$ at $x = 0$

b. $f'(x)$ where $f(x) = \frac{\ln x}{x}$ at $x = e$

c. $f'(x)$ where $f(x) = \ln x$ at $x = 1$

d. $f'(x)$ where $f(x) = \ln(x + e)$ at $x = 1$

e. $f'(x)$ where $f(x) = \ln x$ at $x = e$

10. The diameter *D* of a metal shaft, measured in cm, is recorded at various times *t*, measured in minutes, during a particular manufacturing process. Given the table of values below,

t	0	2	4	6	8	10
D	1.112	1.130	1.144	1.139	1.127	1.109

(a) Find the average rate of change over the interval [0, 2]. **Include units**.

(b) Approximate the rate of change of *D* with respect to *t* when $t = 2$ minutes. **Include units.**

11. The cost, C (in dollars) to produce g gallons of ice cream can be expressed as $C = f(g)$. Interpret the following in practical terms, giving units.

a) $f(400) = 580$

b) $f'(100) = 2.9$

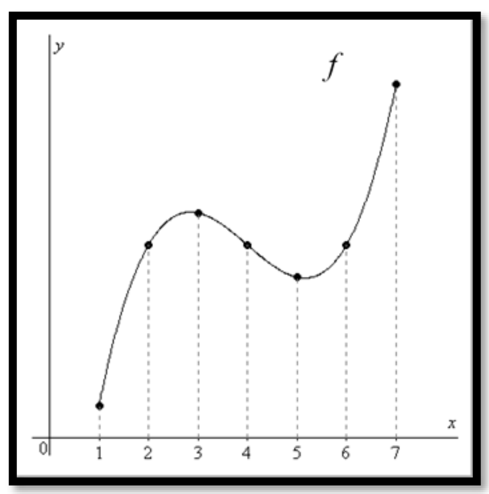
c) $f^{-1}(150) = 65$

12. Suppose that the line tangent to the graph of $y = f(x)$ at $x = 4$ passes through the points $(-2, 6)$ and $(4, 3)$.

a) Find $f(4)$.

b) Find $f'(4)$.

13. Use the graph of f to determine the interval(s) of x that meet the following conditions.



a. $f'(x) > 0$

b. $f'(x) < 0$

c. $f''(x) > 0$

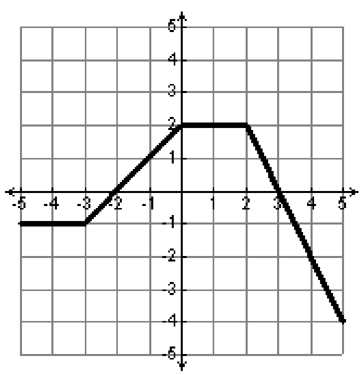
d. $f''(x) < 0$

14. Given $f(4)=8$ and $f'(4)=3$, find the equations for the tangent line to $f(x)$ at $x=4$.

15. Find the average rate of change of $f(x)=e^x+4$ over [0, 3].

Sketch the derivative of the following.

16.



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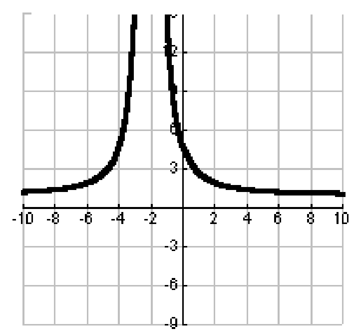

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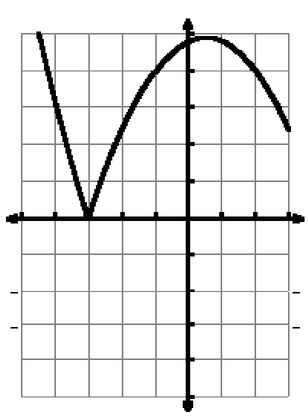

20.