Unit 2 Limits

- Notes and some practice are included
- Homework will be assigned on a daily basis

Topics Covered:

- ❖ Limits from Graphs
- ❖ Graphs from Limits
- ❖ One-Sided Limits & Continuity
- ❖ Creative Factoring
- ❖ Algebraic Limits
- ❖ Intermediate Value Theorem
- ❖ Asymptotes, End Behavior & Infinite Limits

Quiz is _____

Test is _____

Name: _____

Limits from Table of Values

x	-0.3	-0.2	-0.1	0	0.1	0.2	0.3
f(x)	1.971	1.987	1.997	undefined	1.997	1.987	1.971
g(x)	2.018	2.008	2.002	2	2.002	2.008	2.018
h(x)	1	1	1	2	2	2	2

Find the following:

(a) $\lim_{x \to 0} f(x)$

(b) $\lim_{x \to 0} g(x)$

(c) $\lim_{x \to 0} h(x)$

x	2.75	2.9	2.99	2.999	3	3.001	3.01	3.1	3.25
f(x)	5.313	5.710	5.970	5.997	6	6.003	6.030	6.310	6.813
g(x)	1.99499	1.99950	1.99995	1.99999	und	2.00005	2.00050	2.00499	2.01
h(x)	1.99499	1.99950	1.99995	1.99999	2	6.003	6.030	6.310	6.813

Find the following:

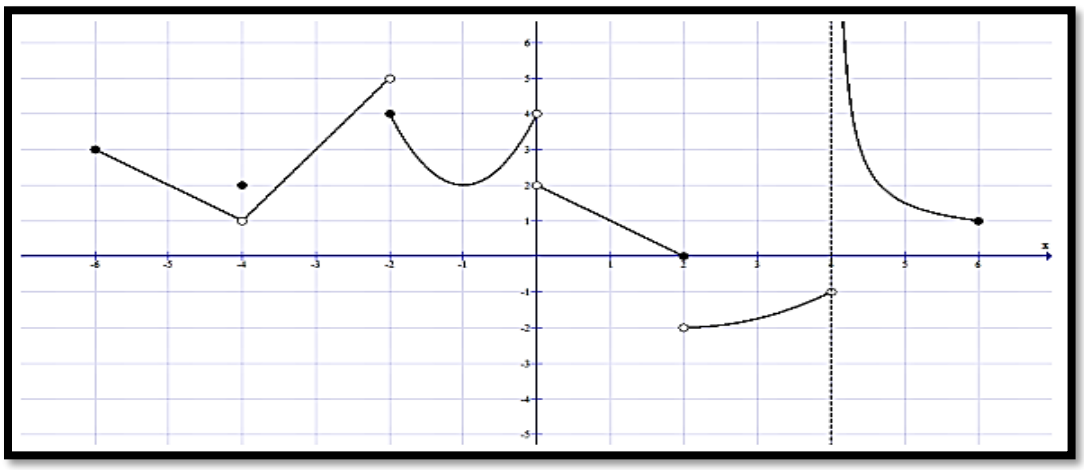
(a) $\lim_{x \to 3} f(x)$

(b) $\lim_{x \to 3} g(x)$

(c) $\lim_{x \to 3} h(x)$

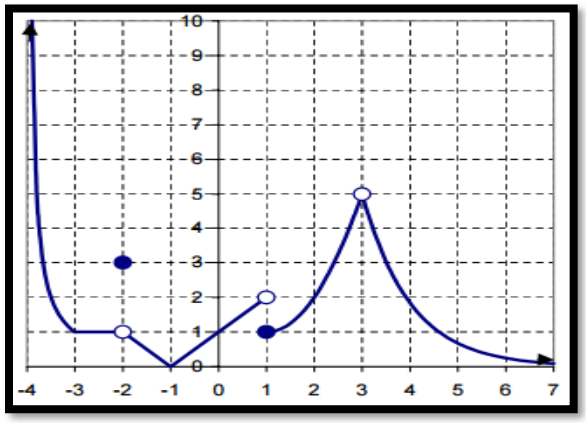
Finding Limits from a Graph

1. Use the graph to evaluate the limits below



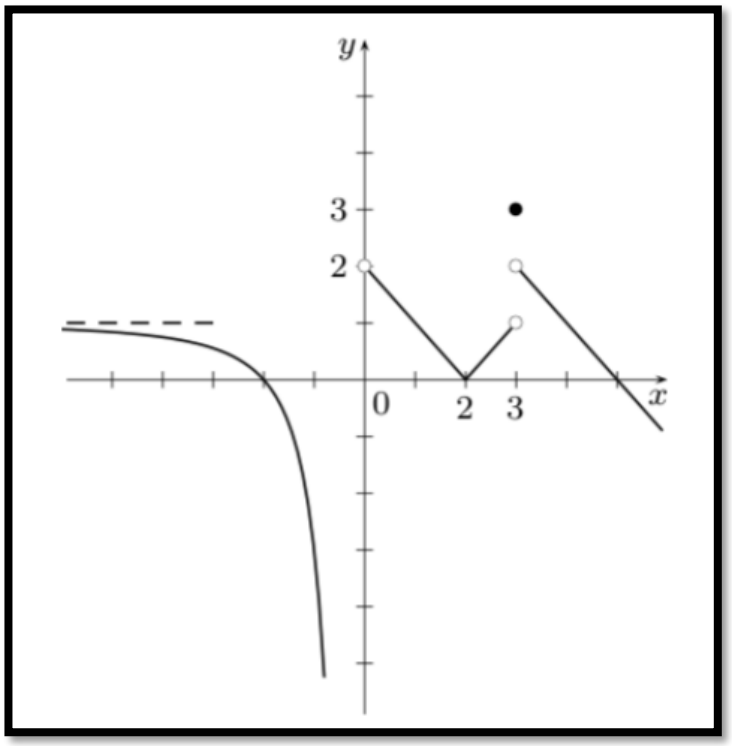
a.	$f(-4)$	b.	$\lim_{x\to -4^-} f(x)$	c.	$\lim_{x\to -4^+} f(x)$	d.	$\lim_{x\to -4} f(x)$
e.	$f(-2)$	f.	$\lim_{x\to -2^-} f(x)$	g.	$\lim_{x\to -2^+} f(x)$	h.	$\lim_{x\to -2} f(x)$
i.	$f(0)$	j.	$\lim_{x\to 0^-} f(x)$	k.	$\lim_{x\to 0^+} f(x)$	l.	$\lim_{x\to 0} f(x)$
m.	$f(2)$	n.	$\lim_{x\to 2^-} f(x)$	o.	$\lim_{x\to 2^+} f(x)$	p.	$\lim_{x\to 2} f(x)$
q.	$f(4)$	r.	$\lim_{x\to 4^-} f(x)$	s.	$\lim_{x\to 4^+} f(x)$	t.	$\lim_{x\to 4} f(x)$

2. Use the graph to evaluate the expressions below.



a.	$f(-2)$	b.	$\lim_{x\to -2^+} f(x)$	c.	$\lim_{x\to -2} f(x)$
d.	$\lim_{x\to -1^+} f(x)$	e.	$\lim_{x\to -1^-} f(x)$	f.	$\lim_{x\to -1} f(x)$
g.	$\lim_{x\to 1^+} f(x)$	h.	$\lim_{x\to 1^-} f(x)$	i.	$\lim_{x\to 1} f(x)$
j.	$f(3)$	k.	$\lim_{x\to 3^+} f(x)$	l.	$\lim_{x\to 3^-} f(x)$
m.	$\lim_{x\to 3} f(x)$	n.	$\lim_{x\to -4^+} f(x)$	o.	$\lim_{x\to \infty} f(x)$
p.	$f(1)$	q.	$\lim_{x\to -3} f(x)$	r.	$f(-4)$

3. Use the graph of the function $f(x)$ to answer each question. Use $\infty, -\infty, or\ DNE$ where appropriate.



a. $f(0) =$

b. $f(2) =$

c. $f(3) =$

d. $\lim_{x\to 0^-} f(x) =$

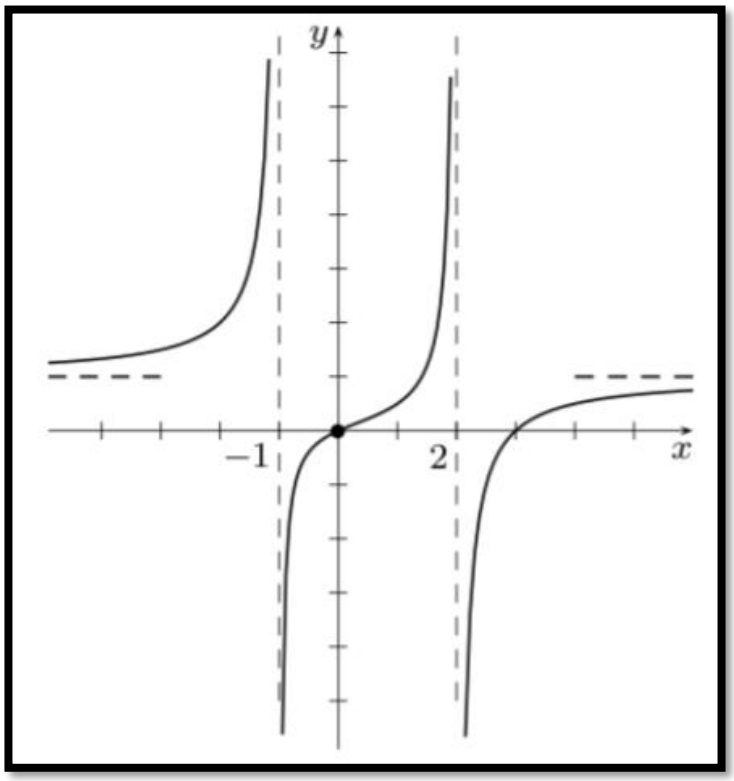
e. $\lim_{x\to 0} f(x) =$

f. $\lim_{x\to 3^+} f(x) =$

g. $\lim_{x\to 3} f(x) =$

h. $\lim_{x\to -\infty} f(x) =$

4. Use the graph of the function $f(x)$ to answer each question. Use $\infty, -\infty, or\ DNE$ where appropriate.



a. $f(0) =$

b. $f(2) =$

c. $f(3) =$

d. $\lim_{x\to -1} f(x) =$

e. $\lim_{x\to 0} f(x) =$

f. $\lim_{x\to 2^+} f(x) =$

g. $\lim_{x\to \infty} f(x) =$

Graphs from Limit Worksheet

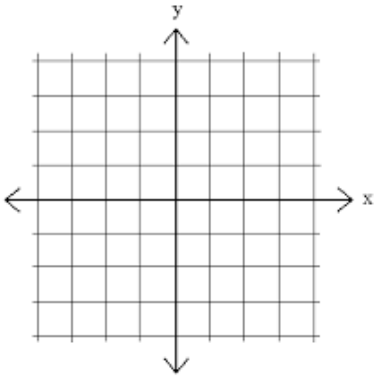
Draw a graph of a function with the give limits.

1. $\lim_{x\to\infty} f(x) = -3$

   $\lim_{x\to 1} f(x) = -2$

   $\lim_{x\to-\infty} f(x) = 2$

   $f(1) = 2$

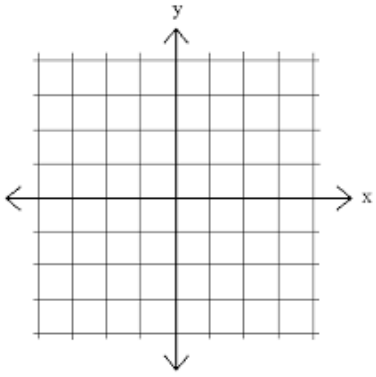


2. $\lim_{x\to\infty} f(x) = 2$

   $\lim_{x\to-2^+} f(x) = 3$

   $\lim_{x\to-2^-} f(x) = \infty$

   $\lim_{x\to-\infty} f(x) = 0$

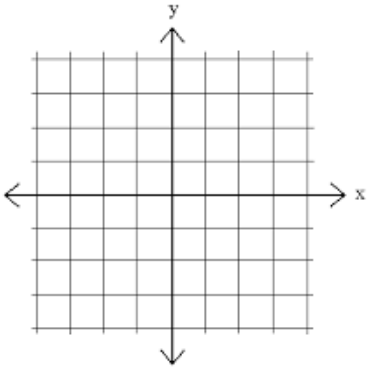


3. $\lim_{x\to\infty} f(x) = \infty$

   $\lim_{x\to 0^+} f(x) = \infty$

   $\lim_{x\to 0^-} f(x) = -\infty$

   $\lim_{x\to-\infty} f(x) = 1$



4. $\lim_{x\to-\infty} f(x) = -\infty$

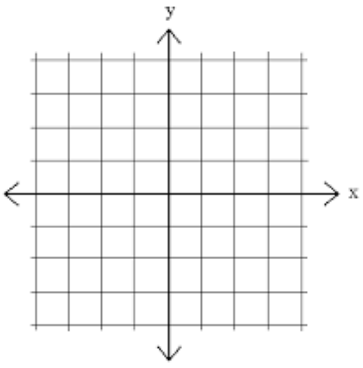
   $\lim_{x\to-1} f(x) = 1$

   $\lim_{x\to 0} f(x) = \infty$

   $\lim_{x\to 2} f(x) = 1$

   $f(2) = 2$

   $\lim_{x\to\infty} f(x) = \infty$



5. $\lim_{x\to-\infty} f(x) = -\infty$

   $\lim_{x\to-2^-} f(x) = \infty$

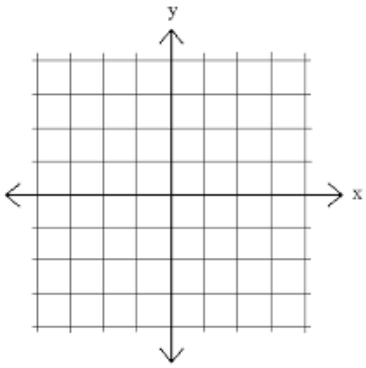
   $\lim_{x\to-2^+} f(x) = -\infty$

   $\lim_{x\to 0} f(x) = 2$

   $\lim_{x\to 2^-} f(x) = -\infty$

   $\lim_{x\to 2^+} f(x) = \infty$

   $\lim_{x\to\infty} f(x) = -\infty$



6. $\lim_{x\to-\infty} f(x) = -2$

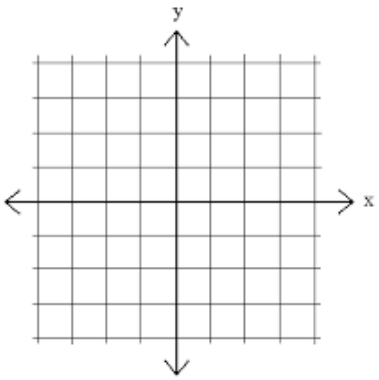
   $\lim_{x\to 0^-} f(x) = 3$

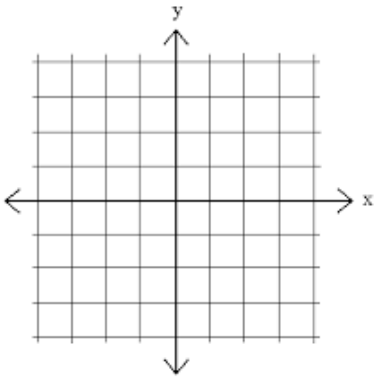
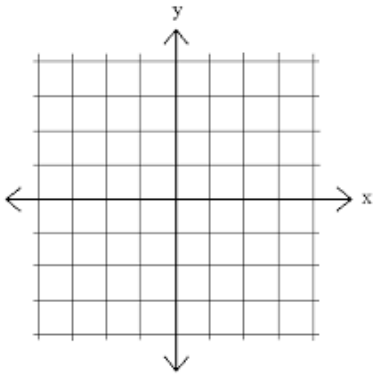
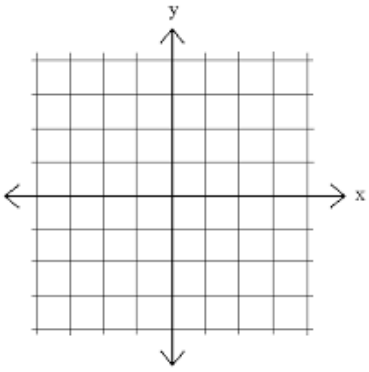
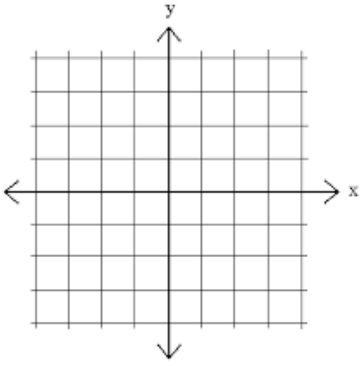
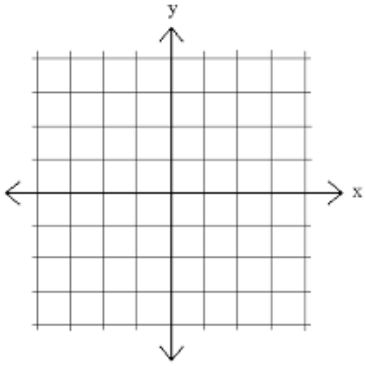
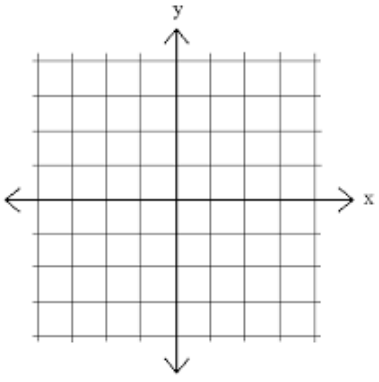
   $\lim_{x\to 0^+} f(x) = -1$

   $\lim_{x\to 2} f(x) = 0$

   $\lim_{x\to\infty} f(x) = 2$

   $f(2) = -1$



7.	$\lim_{x\to\infty} f(x) = 2$ $\lim_{x\to 3^+} f(x) = \infty$ $\lim_{x\to 3^-} f(x) = -\infty$ $\lim_{x\to -\infty} f(x) = -\infty$ 	8.	$\lim_{x\to\infty} f(x) = -\infty$ $\lim_{x\to -\infty} f(x) = \infty$ $\lim_{x\to 0^-} f(x) = -1$ $\lim_{x\to 0^+} f(x) = -1$ 	9.	$\lim_{x\to\infty} f(x) = 0$ $\lim_{x\to 2^+} f(x) = \infty$ $\lim_{x\to 2^-} f(x) = -\infty$ $\lim_{x\to -\infty} f(x) = \infty$ 
10.	$\lim_{x\to\infty} f(x) = \infty$ $\lim_{x\to 3^+} f(x) = 0$ $\lim_{x\to 3^-} f(x) = 0$ $\lim_{x\to -\infty} f(x) = \infty$ $f(3) = 2$ 	11.	$\lim_{x\to\infty} f(x) = 2$ $\lim_{x\to 0^+} f(x) = -\infty$ $\lim_{x\to 0^-} f(x) = \infty$ $\lim_{x\to -\infty} f(x) = -\infty$ 	12.	$\lim_{x\to\infty} f(x) = \infty$ $\lim_{x\to 0^+} f(x) = 3$ $\lim_{x\to 0^-} f(x) = -2$ $\lim_{x\to -\infty} f(x) = -\infty$ 

One-sided Limits Worksheet

Evaluate each limit.

1. $\lim_{x\to 2^+} \frac{x}{x-2}$

2. $\lim_{x\to 3^+} \frac{x+1}{x^2-6x+9}$

3. $\lim_{x\to -3^-} \frac{x+2}{x^2+6x+9}$

4. $\lim_{x\to -2^+} \frac{x-2}{x^2+4x+4}$

5. $\lim_{x\to -3^-} \frac{x^2}{3x+9}$

6. $\lim_{x\to 2^+} \frac{x^2}{2x-4}$

7. $\lim_{x\to -2^+} \frac{1}{x^2-4}$

8. $\lim_{x\to 1^-} -\frac{2}{x^2-1}$

9. $\lim_{x\to 3^-} f(x), f(x) = \begin{cases} -x+4, & x<3 \\ \frac{x}{2}+1, & x\geq 3 \end{cases}$

10. $\lim_{x\to -1^+} f(x), f(x) = \begin{cases} x+3, & x\leq -1 \\ -x-1, & x>-1 \end{cases}$

11. $\lim_{x\to -2^-} f(x), f(x) = \begin{cases} -x^2-8x-17, & x\leq -2 \\ 2x-1, & x>-2 \end{cases}$

12. $\lim_{x\to 1^-} (|x-1|-2)$

13. $\lim_{x\to 0^+} \frac{2x}{|x|}$

14. $\lim_{x\to 1^-} f(x), f(x) = \begin{cases} -\frac{x}{2}-\frac{3}{2}, & x\leq 1 \\ -x^2+4x-5, & x>1 \end{cases}$

15. $\lim_{x\to -3^-} f(x), f(x) = \begin{cases} x+6, & x<-3 \\ 3, & x\geq -3 \end{cases}$

16. $\lim_{x\to 0^-} f(x), f(x) = \begin{cases} -2x+3, & x\leq 0 \\ -\frac{x}{2}+3, & x>0 \end{cases}$

Continuity Worksheet

Determine if each function is continuous. If the function is not continuous, find the x-axis location of and classify each discontinuity.

1. $f(x) = -\frac{x}{2x^2+2x+1}$

2. $f(x) = \frac{x}{x^2+6x+9}$

3. $f(x) = \frac{x^2+4x+3}{x+3}$

4. $f(x) = \frac{x}{x^2-4x}$

5. $f(x) = \begin{cases} x+4, & x \leq -2 \\ -2x-11, & x > -2 \end{cases}$

6. $f(x) = \frac{x+7}{x^2+3x}$

Find the intervals on which each function is continuous.

7. $f(x) = \begin{cases} x, & x \neq 4 \\ 2, & x = 4 \end{cases}$

8. $f(x) = \begin{cases} -2, & x < 3 \\ -2x+6, & x \geq 3 \end{cases}$

9. $f(x) = \frac{(x-1)}{x^2-4x+3}$

10. $f(x) = \frac{x^2}{2} + 4x + 10$

11. $f(x) = -x^2 - 4x + 2$

12. $f(x) = -\frac{x-2}{x^2-3x+2}$

13. $f(x) = -\frac{x-1}{x^2-x}$

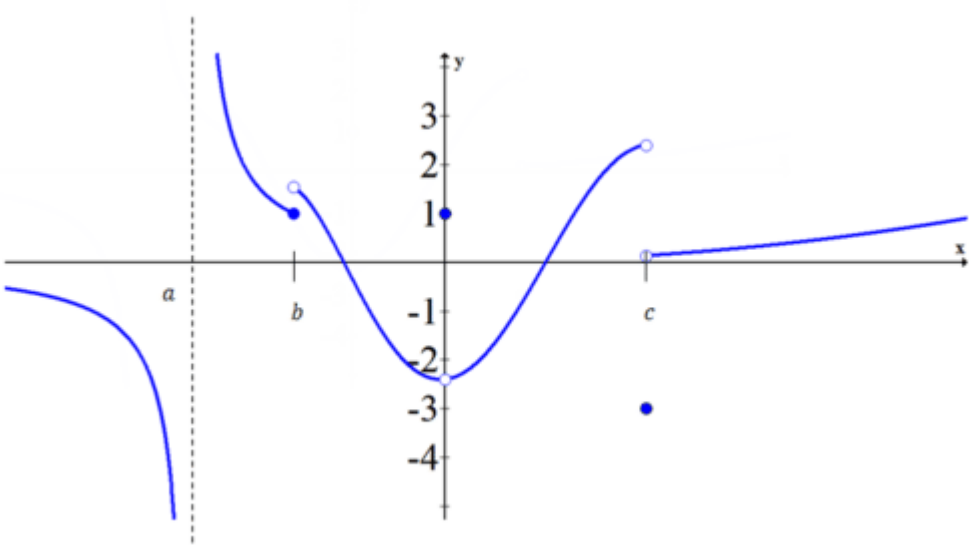
14. $f(x) = \frac{x}{x^2-6x+9}$

15. Critical Thinking: Write a function that has an infinite discontinuity at $x = 100$

16 Critical Thinking: Write a function that is continuous over $(-\infty, 0), (0,1), and\ (1, \infty)$ and discontinuous everywhere else.

Quiz Review

1. Using the graph of $f(x)$ below, find the limits.



a. $\lim_{x\to a^-} f(x)$

b. $\lim_{x\to -\infty} f(x)$

c. $\lim_{x\to a} f(x)$

d. $\lim_{x\to 0} f(x)$

e. $\lim_{x\to b^+} f(x)$

f. $\lim_{x\to b} f(x)$

g. $\lim_{x\to c} f(x)$

h. $\lim_{x\to c^+} f(x)$

2. Using the graph of $f(x)$ above, list any discontinuities and the type of discontinuity.

3. Use the following information to sketch a graph.

$\lim_{x\to -\infty} f(x) = \infty$

$\lim_{x\to -1^-} f(x) = 4$

$\lim_{x\to -1^+} f(x) = -\infty$

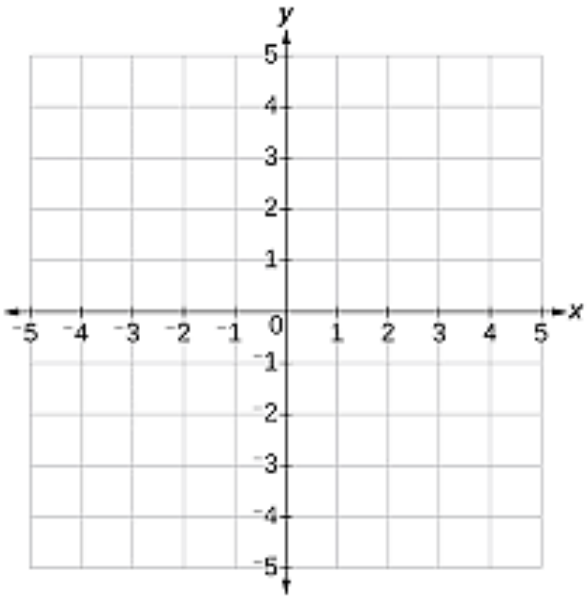
$\lim_{x\to 2} f(x) = 3$

$\lim_{x\to 4^-} f(x) = -\infty$

$\lim_{x\to 4^+} f(x) = \infty$

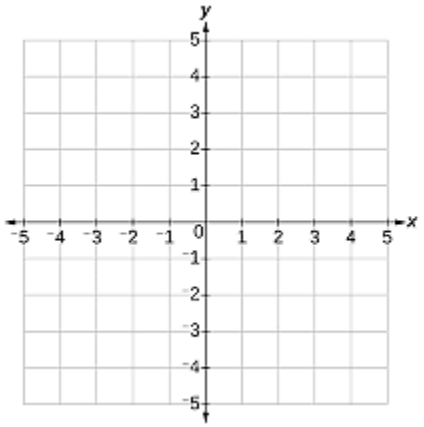
$\lim_{x\to \infty} f(x) = -2$

$f(2) = -1$



Draw a sketch. Find the indicated limit if it exists. If the limit does not exist, explain why.

4. $G(x) = \begin{cases} 3, \text{ if } x > 4 \\ 5, \text{ if } x = 4 \\ -2, \text{ if } x < 4 \end{cases}$



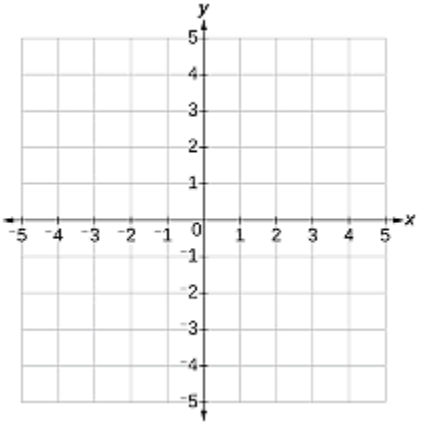
a. $\lim_{x \to 4^+} G(x)$

b. $\lim_{x \to 4^-} G(x)$

c. $\lim_{x \to 4} G(x)$

d. $G(4)$

5. $T(x) = \begin{cases} 3 - 6x, \text{ if } x > 1 \\ -1, \text{ if } x = 1 \\ x^2, \text{ if } x < 1 \end{cases}$



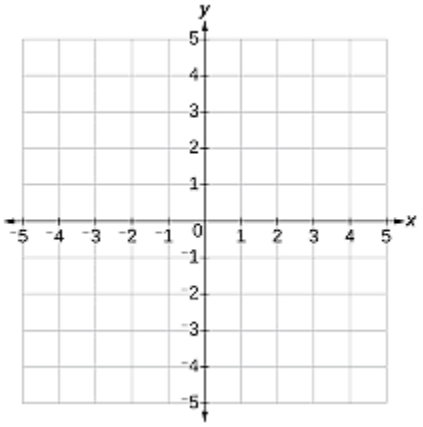
a. $\lim_{x \to 1^-} T(x)$

b. $\lim_{x \to 1^+} T(x)$

c. $\lim_{x \to 1} T(x)$

d. $T(1)$

6. $G(x) = \frac{|3x|}{x}$



a. $\lim_{x \to 0^+} G(x)$

b. $\lim_{x \to 0^-} G(x)$

c. $\lim_{x \to 0} G(x)$

d. $G(0)$

7. Find the limits without sketching the graph:

$T(x) = \begin{cases} x^2 - 16, \text{ if } x < 3 \\ 5, \text{ if } x = 3 \\ 14 - x^2, \text{ if } x > 3 \end{cases}$

a. $\lim_{x \to 3^+} F(x)$

b. $\lim_{x \to 3^-} F(x)$

c. $\lim_{x \to 3} F(x)$

d. $F(3)$

8. $F(x) = \begin{cases} 2x - 5, \text{ if } x > \frac{1}{2} \\ 3kx - 1, \text{ if } x < \frac{1}{2} \end{cases}$

Find the value of k such that $\lim_{x \to \frac{1}{2}} F(x)$ exists.

9. Find the values of *m* and *k* such that $\lim_{x \to 1} G(x)$ and $\lim_{x \to -1} G(x)$ both exist.

$G(x) = \begin{cases} 3x^2 - kx + m, & \text{if } x \geq 1 \\ mx - 2k, & \text{if } -1 < x < 1 \\ -3m + 4x^2k, & \text{if } x \leq -1 \end{cases}$

Find the one-sided limits.

10. $\lim\limits_{x\to 2^-} \frac{3}{x-2}$

11. $\lim\limits_{x\to -3^+} \frac{5}{x+3}$

12. $\lim\limits_{x\to 2} \frac{-7}{2-x}$

13. $\lim\limits_{x\to 2^-} \frac{|x-2|}{x-2}$

14. $\lim\limits_{x\to 5^+} \frac{3x-15}{|4x-20|}$

15. $\lim\limits_{x\to 5^-} \frac{3x-15}{|4x-20|}$

16. $\lim\limits_{x\to 5} \frac{3x-15}{|4x-20|}$

17. $\lim\limits_{x\to 5^+} \frac{|x-4|}{x^2-3x+2}$

At what values are the following functions discontinuous? State the type of discontinuity.

18. $f(x) = \frac{x}{x^2-25}$

19. $f(x) = \frac{x+4}{x^2-16}$

20. $f(x) = \begin{cases} 4x-5, & \text{if } x > 2 \\ 3x-1, & \text{if } x < 2 \end{cases}$

Creative Factoring & Other Interesting Algebra

Difference of Squares

Example: $x-16=\left(\sqrt{x}+4\right)\left(\sqrt{x}-4\right)$

1. $x-9$

2. x^2-5

3. $x^{16}-1$

4. $\left(x+5\right)^2-25$

5. $9y-a^4$

Sums or Differences of Cubes **"SOAP"**

Example: $a^3+b^3=\left(a+b\right)\left(a^2-ab+b^2\right)$

Example: $a^3-b^3=\left(a-b\right)\left(a^2+ab+b^2\right)$

6. $64a^3+125b^3$

7. $64a^3x^3-125$

8. $\left(x+1\right)^3+64$

9. $8c^3-\left(a+b\right)^3$

Factor: x^6-y^6**:**

10. as a difference of **squares**

11. as a difference of **cubes**

Compare #10 & #11. Which way will allow you to factor completely most easily?

Rationalize the Numerator

12. $\dfrac{\sqrt{x+2}-\sqrt{2}}{x}$

13. $\dfrac{\sqrt{x+3}+\sqrt{3}}{x}$

Algebraic Limits Worksheet

1. $\lim_{x\to 3} x^2 + 2x - 7$	2. $\lim_{x\to -1} \dfrac{\frac{1}{x}+1}{x+1}$	3. $\lim_{x\to 1} \dfrac{(4x^4 - 5x^2 + 1)}{x^2 + 2x - 3}$
4. $\lim_{x\to 0} \dfrac{(x+1)^2 - 1}{x}$	5. $\lim_{x\to 1} \dfrac{x^2 - 2x - 15}{x - 5}$	6. $\lim_{x\to -3} \dfrac{2x^2 + 2x - 12}{x^2 + 4x + 3}$
7. $\lim_{x\to 2} \dfrac{(2x+1)^2 - 25}{x - 2}$	8. $\lim_{x\to 2} \dfrac{(3x-2)^2 - (x+2)^2}{x - 2}$	9. $\lim_{x\to 1} \dfrac{\frac{2x}{x+1} - 1}{x - 1}$
10. $\lim_{x\to 2} \dfrac{\frac{2}{x^2} - \frac{1}{2}}{x - 2}$	11. $\lim_{x\to 2} \dfrac{x^4 - 2x^2 - 8}{x^2 - x - 6}$	12. $\lim_{x\to 1} \dfrac{x - 1}{\sqrt{x} - 1}$
13. $\lim_{x\to 0} \dfrac{x^2 + 7x + 6}{x + 3}$	14. $\lim_{x\to -2} \dfrac{\frac{x}{x+4} + 1}{x + 2}$	15. $\lim_{x\to 2} \dfrac{x^3 + x^2 - 4x - 4}{x^2 + x - 6}$
16. $\lim_{x\to 2} \dfrac{x^2 - 2x - 3}{x + 5}$	17. $\lim_{x\to 2} (x^2 - x + 1)$	18. $\lim_{x\to 1} \dfrac{2x + 1}{3x - 2}$
19. $\lim_{x\to 1} \sqrt{10x - 1}$	20. $\lim_{x\to 1} \dfrac{x^2 - x - 2}{x - 2}$	21. $\lim_{x\to 4} \dfrac{\sqrt{x} - 2}{x - 4}$

22. $\lim_{x\to 3}\dfrac{x^2-9}{x+3}$	23. $\lim_{x\to 3}\dfrac{x^2-9}{2x^2+7x+3}$	24. $\lim_{x\to 9}\dfrac{\sqrt{x}-3}{x-9}$
25. $\lim_{h\to 0}\dfrac{(1+h)^2-1^2}{h}$	26. $\lim_{h\to 0}\dfrac{(3+h)^2-3^2}{h}$	27. $\lim_{h\to 0}\dfrac{(x+h)^2-x^2}{h}$
28. $\lim_{x\to 3}(5x^2-6)$	29. $\lim_{x\to -1}\dfrac{x-2}{x^2+4x-3}$	30. $\lim_{x\to 4}\dfrac{x^2-16}{x-4}$
31. $\lim_{x\to 0}\dfrac{6x-9}{x^3-12x+3}$	32. $\lim_{x\to 2}\dfrac{x^2-4x+4}{x^2+x-6}$	33. $\lim_{x\to -2}\dfrac{x^3+8}{x+2}$

Given $\lim_{x\to a} f(x)=-3, \lim_{x\to a} g(x)=0$, *and* $\lim_{x\to a} h(x)=8$, find each limit if it exists.

34. $\lim_{x\to a}[f(x)+h(x)]$	35. $\lim_{x\to a}[f(x)]^2$	36. $\lim_{x\to a}\sqrt[3]{h(x)}$
37. $\lim_{x\to a}\frac{1}{f(x)}$	38. $\lim_{x\to a}\dfrac{g(x)}{h(x)}$	39. $\lim_{x\to a}\dfrac{h(x)}{g(x)}$
40. $\lim_{x\to a}\frac{2f(x)}{h(x)-f(x)}$	41. $\lim_{x\to a}[f(x)h(x)]$	42. $\lim_{x\to a}\left[\dfrac{g(x)+h(x)}{f(x)}\right]$

Intermediate Value Theorem Worksheet

1. Verify the conditions of the IVT and find the guaranteed c value over $[2,6]$ for
$$f(x) = x^2 + 2x - 11$$
when $f(c) = 4$.

2. Verify the conditions of the IVT and find the guaranteed c value over $[-1,3]$ for
$$f(x) = 2x^2 + x - 4$$
when $f(c) = 2$.

3. Use the IVT to show that
$$f(x) = x^3 - 3x^2 - 7x + 1$$
has a root in the interval $(4,5)$

4. Use the IVT to show that
$$f(x) = x^4 + 3x^2 - 6$$
has a root in the interval $(1,2)\ and\ (-2,-1)$

5. Verify the conditions of the IVT and find the guaranteed c value over $[0,5]$ for
$$f(x) = x^2 + x - 1$$
when $f(c) = 11$.

6. Verify the conditions of the IVT and find the guaranteed c value over $[0,3]$ for
$$f(x) = x^2 - 6x + 8$$
when $f(c) = 0$.

7. Verify the conditions of the IVT and find the guaranteed c value over $[0,3]$ for
$$f(x) = x^3 - x^2 + x - 2$$
when $f(c) = 4$.

8. Verify the conditions of the IVT and find the guaranteed c value over $[\frac{5}{2}, 4]$ for
$$f(x) = \frac{x^2 + x}{x - 1}$$
when $f(c) = 6$.

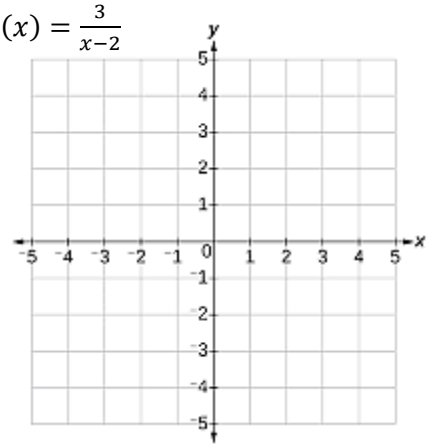
9. Use the IVT to show that
$$f(x) = x^3 + x - 1$$
has a root in the interval $[0,1]$

10. Use the IVT to show that
$$f(x) = x^3 + 3x - 2$$
has a root in the interval $[0,1]$

Vertical and Horizontal Asymptotes Worksheet

State the vertical, horizontal, or slant asymptotes for the following. Sketch the graph and find the end behavior.

1. $f(x) = \frac{3}{x-2}$



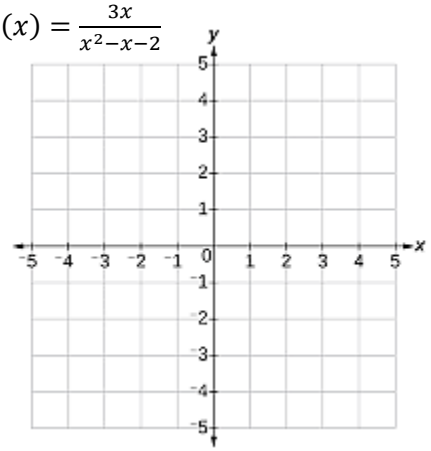
Vertical Asymptote: _____

Horizontal Asymptote: _____

Slant Asymptote: _____

End Behavior: $\lim_{x\to\infty} f(x) =$ _____
$\lim_{x\to-\infty} f(x) =$ _____

2. $f(x) = \frac{3x}{x^2-x-2}$



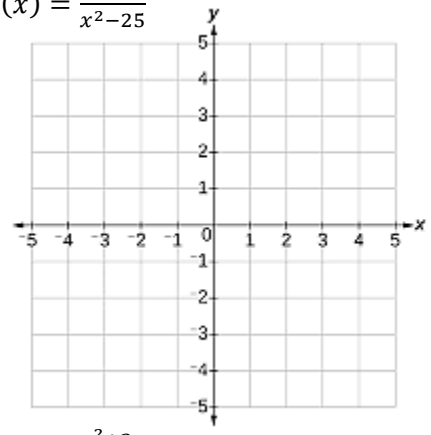
Vertical Asymptote: _____

Horizontal Asymptote: _____

Slant Asymptote: _____

End Behavior: $\lim_{x\to\infty} f(x) =$ _____
$\lim_{x\to-\infty} f(x) =$ _____

3. $f(x) = \frac{x^2-5x}{x^2-25}$



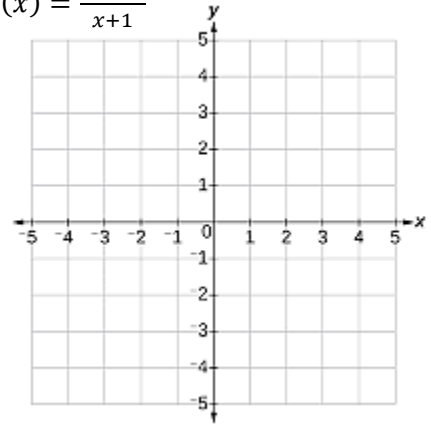
Vertical Asymptote: _____

Horizontal Asymptote: _____

Slant Asymptote: _____

End Behavior: $\lim_{x\to\infty} f(x) =$ _____
$\lim_{x\to-\infty} f(x) =$ _____

4. $f(x) = \frac{x^2+3x}{x+1}$



Vertical Asymptote: _____

Horizontal Asymptote: _____

Slant Asymptote: _____

End Behavior: $\lim_{x\to\infty} f(x) =$ _____
$\lim_{x\to-\infty} f(x) =$ _____

Infinite Limits Worksheet

Find the Limit.

1. $\lim_{x\to\infty} 3$	2. $\lim_{x\to-\infty} 3$	3. $\lim_{x\to-\infty} (-3)$
4. $\lim_{x\to\infty} (-2x)$	5. $\lim_{x\to\infty} (3-x)$	6. $\lim_{x\to\infty} \sqrt{x}$
7. $\lim_{x\to-\infty} (4-x)$	8. $\lim_{x\to\infty} \frac{8}{5-3x}$	9. $\lim_{x\to\infty} \frac{1}{x-12}$
10. $\lim_{x\to-\infty} \frac{3}{x+4}$	11. $\lim_{x\to\infty} (1+2x-3x^5)$	12. $\lim_{x\to\infty} (2x^3-110x+5)$
13. $\lim_{x\to\infty} \frac{(3+2x^2)}{4+5x}$	14. $\lim_{x\to\infty} \frac{x^2+x}{3-x}$	15. $\lim_{x\to\infty} \frac{x+4}{x^2-2x+5}$
16. $\lim_{x\to-\infty} -\frac{x-2}{x^2+2x+1}$	17. $\lim_{x\to\infty} \frac{7-6x^5}{x+3}$	18. $\lim_{x\to\infty} \frac{6-x^3}{7x^3+3}$
19. $\lim_{x\to\infty} \frac{1}{x^2+1}$	20. $\lim_{x\to\infty} \frac{x^4+x^2}{x^4+1}$	21. $\lim_{x\to\infty} \frac{1+x^2}{2-x^2}$
22. $\lim_{x\to\infty} \frac{2x^2}{x^2+1}$	23. $\lim_{x\to-\infty} \frac{x+4}{3x^2-5}$	24. $\lim_{x\to\infty} \frac{3x^3+25x^2-x+1}{4x^3-7x^2+2x+2}$

Limits Review 1

The limit of a constant is a constant.

1. $\lim_{x\to e}\sqrt{7}$

2. $\lim_{x\to\sqrt{5}}\pi$

Direct Substitution – ALWAYS try direct substitution first!

3. $\lim_{x\to 5}(2x^2-x+3)$

4. $\lim_{y\to 2^-}\frac{y^2-3y+2}{y+1}$

5. $\lim_{x\to 4}\frac{|5-3x|}{2x+1}$

6. $\lim_{x\to 4} cos\left(\frac{3\pi}{x}\right)$

If substitution results in $\frac{0}{0}$, try to factor, reduce & substitute again.

7. $\lim_{x\to 1}\frac{x^4-1}{x-1}$

8. $\lim_{x\to 1}\frac{x-1}{x^3-x^2+x-1}$

9. $\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}$

10. $\lim_{x\to -2}\frac{x+2}{x^3+8}$

If substitution results in $\frac{0}{0}$, try to multiply by the conjugate.

11. $\lim_{x\to 2}\frac{\sqrt{5x+6}-4}{x-2}$

12. $\lim_{x\to 4}\frac{3-\sqrt{x+5}}{x-4}$

13. $\lim_{x\to 0}\frac{\sqrt{x+3}-\sqrt{x}}{x}$

If substitution results in $\frac{0}{0}$ with complex fractions, try to clear the "little denominators".

14. $\lim_{h\to 0}\frac{\frac{1}{2+h}-\frac{1}{2}}{h}$

15. $\lim_{x\to 10}\frac{\frac{x}{5}-2}{x-10}$

16. $\lim_{h\to -2}\frac{(h+5)^{-1}-3^{-1}}{h+2}$

Rewrite the absolute value.

17. $\lim_{x\to 5^-} \frac{|2x-10|}{3x+15}$

18. $\lim_{x\to 7^-} \frac{3x-21}{|7-x|}$

19. $\lim_{x\to 2} \frac{|x-2|}{x-2}$

One-sided limits when you get $\frac{\#}{\mathbf{0}}$, do you get ∞ or $-\infty$? Reason it out!

20. $\lim_{x\to 3^-} \frac{5}{x-3}$

21. $\lim_{x\to 3^+} \frac{-4}{x-3}$

22. $\lim_{x\to 6^+} \frac{x+6}{x^2-36}$

Limits to infinity. You can do a **behaves like** only in limits to infinity. You can also divide by the highest power in the denominator, simplify, and then find the limit.

$\lim_{x\to\pm\infty}$ (polynomial) – Use end behavior rules.

23. $\lim_{x\to\infty}(3x^2-4x+2)$

24. $\lim_{x\to-\infty}(5x^3-2x^2+1)$

$$\lim_{x\to\pm\infty} \frac{\text{degree smaller}}{\text{DEGREE LARGER}} = 0$$

25. $\lim_{x\to\infty} \frac{3x-5}{x^2+1}$

26. $\lim_{x\to-\infty} \frac{4x^2-3x}{6x^5-3x+1}$

$$\lim_{x\to\pm\infty} \frac{\text{degree} =}{\text{degree} =} = \text{ratio of the leading coefficients}$$

27. $\lim_{x\to\infty} \frac{5x-11}{x^2+1}$

28. $\lim_{x\to-\infty} \frac{4x^2-5x+2}{3x^2+1}$

29. $\lim_{x\to\infty} \frac{2x^2+1}{(2-x)(2+x)}$

$$\lim_{x\to\pm\infty} \frac{\text{DEGREE LARGER}}{\text{degree smaller}} = \infty \text{ or } -\infty$$

30. $\lim_{x\to\infty} \frac{7-6x^5}{x+3}$

31. $\lim_{x\to-\infty} \frac{7-6x^5}{x+3}$

32. $\lim_{x\to\infty} \frac{5+x^3-3x^4}{2x-1}$

33. $\lim_{x\to-\infty} \frac{5+x^3-3x^4}{2x-1}$

$\lim_{x\to\pm\infty}$ involving square roots: Use the **behaves like** method & remember the $\sqrt{x^2} = |x|$!

34. $\lim_{x\to\infty} \frac{\sqrt{4x^2-2}}{x+1}$ 35. $\lim_{x\to-\infty} \frac{\sqrt{4x^2-2}}{x+1}$ 36. $\lim_{x\to\infty} \frac{2-x}{\sqrt{7+9x^2}}$ 37. $\lim_{x\to-\infty} \frac{2-x}{\sqrt{7+9x^2}}$

Write the equations of the vertical and horizontal asymptotes.

38. $y = \frac{2x^2-5x-3}{x^2-2x-3}$ 39. $y = \frac{3-x}{9-x^2}$

Continuity: Limit from right = limit from left = value of $f(x)$ at the point.

Is $f(x)$ continuous? Why?

40. $f(x) = \begin{cases} -5-x, & x > -1 \\ 6x+2, & x \le -1 \end{cases}$ 41. $f(x) = \frac{|x+2|}{x+2}$

Intermediate Value Theorem.

42. Verify the conditions of the Intermediate Value Theorem, and **find** *c* guaranteed by the theorem when $f(x) = x^2 - 6x + 7$ over the interval $[0,3]$ and *f(c)* = -1.

Finding values that make a function continuous.

43. Find the value of a that would make the function continuous.

$$f(x) = \begin{cases} 3xa+5 & if \quad x \le -1 \\ -2x+5a & if \quad x > -1 \end{cases}$$

44. Find the value of *m* and *n* that would make the function continuous.

$$g(x) = \begin{cases} 3mx-4n & if \quad x \le -1 \\ 4+nx-mx^2 & if \quad -1 < x < 2 \\ x^2-mx+7n & if \quad x \ge 2 \end{cases}$$

Limits Review 2

Evaluate the following limits.

1. $\lim_{x\to 0} \frac{9-4x}{2x^3-4x^2+3}$

2. $\lim_{x\to 2} \frac{2x^2+x-10}{x^2+x-6}$

3. $\lim_{x\to -3} \frac{x^3+27}{x+3}$

4. $\lim_{x\to 5} \frac{x}{x^2-25}$

5. $\lim_{x\to 0} \frac{x^3-8}{x^2-4}$

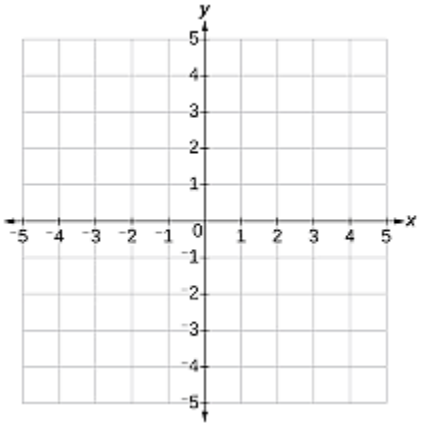
6. $\lim_{x\to 3} \frac{(3-x)^2}{x-3}$

7. $\lim_{\theta\to 0} \frac{1-\cos\theta}{2\sin^2\theta}$

8. $\lim_{x\to -1} \frac{x^4-1}{x+1}$

For problems #9-12, use the function $f(x) = \begin{cases} x-1, & x \le 1 \\ 3x-7, & x > 1 \end{cases}$

9. Graph the function



10. $\lim_{x\to 1^-} f(x)$

11. $\lim_{x\to 1^+} f(x)$

12. $\lim_{x\to 1} f(x)$

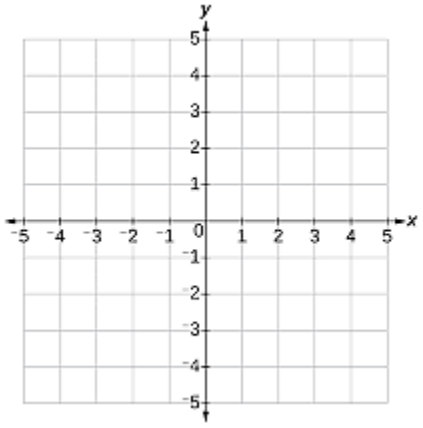
13. Draw a function that meets the following conditions. Is this function continuous? Explain.

$\lim_{x\to\infty} f(x) = 4$

$\lim_{x\to 1} f(x) = 0$

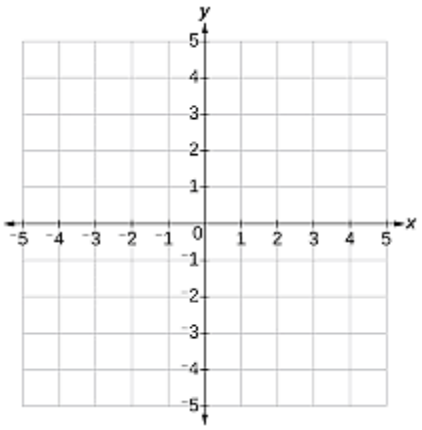
$\lim_{x\to -\infty} f(x) = -4$

$f(-1) = 0$



14. Draw a function that meets the following conditions. Find the indicated limit if it exists. Is this function continuous? Explain.

$$f(x) = \begin{cases} x^2, & x < 1 \\ 2 - x, & x > 1 \\ 2, & x = 1 \end{cases}$$



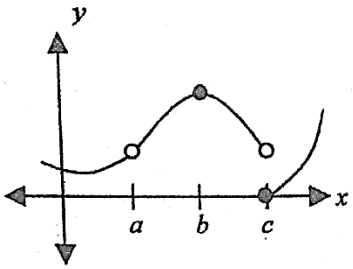
15. $\lim_{x \to 1^+} f(x)$

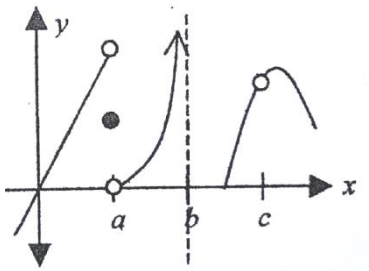
16. $\lim_{x \to 1^-} f(x)$

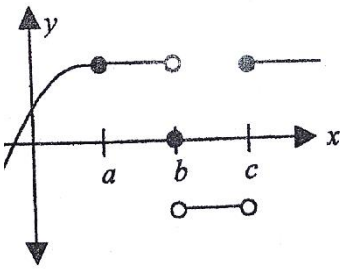
17. $\lim_{x \to 1} f(x)$

18. $f(x)$

Indicate whether the function whose graph is given is continuous at each of the points a, b, and c.

19. 

20. 

21. 

Find a value of k which will cause $f(x)$ to be continuous for all real x values.

22. $f(x) = \begin{cases} kx^2, & x < -3 \\ 5 - 4x, & x \geq -3 \end{cases}$

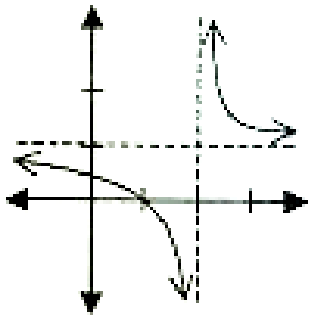
23. $f(x) = \begin{cases} x^3, & x < \frac{1}{2} \\ kx^2, & x \geq \frac{1}{2} \end{cases}$

24. Define $f(3)$so that $f(x) = \frac{x^2-9}{x-3}$ is continuous at x=3.

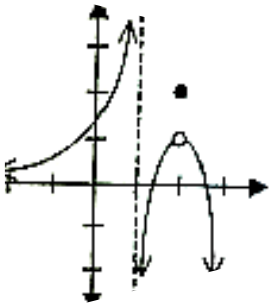
25. Define $f(3)$so that $f(x) = \frac{x^3-1}{x^2-1}$ is continuous at x=1.

Use the graph to determine the intervals for which the function is continuous.

26.



27.



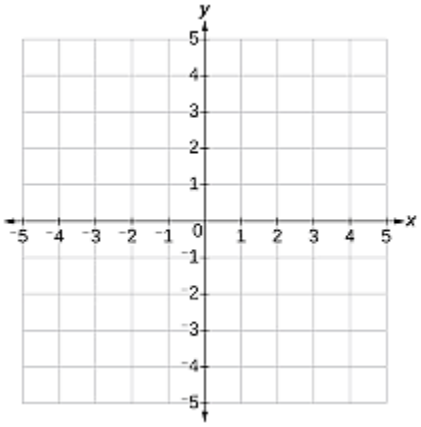
At what values are the following functions discontinuous? Explain the type of discontinuity.

28. $f(x) = \frac{x+3}{x^2-3x-10}$

29. $f(x) = \frac{x+2}{4-x^2}$

For problems #30-42, use the function $\boldsymbol{f(x) = \begin{cases} \frac{|x-2|}{x-2}, & x \neq 2 \\ 0, & x = 2 \end{cases}}$

30. Graph the function.



31. domain:

range:

32. $f(0)$

33. $f(2)$

34. $f(4)$

35. $\lim_{x\to 0^+} f(x)$

36. $\lim_{x\to 0^-} f(x)$

37. $\lim_{x\to 0} f(x)$

38. Is $f(x)$ continuous at x=0?

39. $\lim_{x\to 2^-} f(x)$

40. $\lim_{x\to 2^+} f(x)$

41. $\lim_{x\to 2} f(x)$

42. Is $f(x)$ continuous at x=2?