Identifying and Naming Polynomial Functions

Polynomial Function:
A function comprised of a monomial, or the sum or difference of monomials.
- Must have REAL NUMBER coefficients
- Must have WHOLE NUMBER exponents

Ex 1: Is it a polynomial?

a) $f(x) = x^3 + 3x$

b) $f(x) = x^4 + 3z - 2x^2 - 5^x$

c) $f(x) = 6x^4 - 2x^{-1} + x$

d) $f(x) = -0.5 + \pi x^2 - \sqrt{2}$

A polynomial is in STANDARD FORM when

The DEGREE is

The LEADING COEFFICIENT is

A CONSTANT is

Ex 2: Write in standard form. Then identify the degree, leading coefficient, and constant.

a) $g(x) = 2x^2 - 4 - 3x^4 + 12x^3$
Standard form:

Degree:

Leading coefficient:

Constant:

b) $h(x) = 3 - x$
Standard form:

Degree:

Leading coefficient:

Constant:

Ex 3: You can classify polynomials by DEGREE and NUMBER OF TERMS.

Polynomial	# of terms	Name by # of terms	DEGREE	Name by Degree
$f(x) = 12$				
$k(x) = 8x$				
$j(x) = 4x^2 + 3$				
$g(x) = 5x^3 + x^2$				
$h(x) = 3x^2 - 4x + 6$				
$s(t) = 7t^4 - 7t + 3$				
$f(x) = x^3 + 2x^2 - 8x - 1$				

Operating with Polynomials

<u>Adding</u>: ① Combine like terms! ② Write the result in standard form.	a) $(2x^3 + 7x - 4) + (2x^2 - 8x + 1)$ b) $(x + 5x^4 + 12x^2) + (x^4 - 10)$
<u>Subtracting</u>: ① Distribute the negative ② Combine like terms. ③ Write the result in standard form.	c) $(7 - 4x^2 - x) - (3x^2 + x + 12)$ d) $(6x^2 - 3) - (2x^3 + 5x^2 - 9x + 8)$
<u>Multiplying</u>: ① Distribute each term from one polynomial to each term in the other. ② Combine like terms. ③ Write the result in standard form.	e) $-2x(5x^2 - 3x + 1)$ f) $(3x - 4)(2x + 1)$ g) $(-3x + 2)(4x^2 + x - 9)$ h) $(2x + 5)^2$ i) $(x + 3)^3$

Practice

Identify the following characteristics:

1. $f(x) = 4x - 1 + 2x^4 + 10x^3$

   Standard form:

   Degree:

   Leading coefficient:

   Constant:

2. $h(x) = 2x - 5x^2 + 4$

   Standard form:

   Degree:

   Leading coefficient:

   Constant:

Classify the following polynomials by <u>DEGREE</u> and <u>TERMS</u>. (You should have 2 words for each.)

3. $-4x^3 + x + 9$
4. $-5x^3 + 3x^2 + 4x - 2$
5. $2x + 1$
6. 3
7. $-3x^2 + 2$
8. $x^4 + 3x - 1$

Algebra 2 Name____________________ ID: 1

Add, Subtract & Multiply Polynomials

Date________________ Period____

Perform the indicated operation. Write your answer in standard form.

1) $(7r^3 + 5r^2 + 5r) + (4r^3 + 8r - 7r^4)$

2) $(2a^2 + a - 2a^3) - (-3 - a - 7a^3)$

3) $(3 + 3x^2 - 5x) - (5x^2 + 1 + 7x)$

4) $(x^3 - 3 + 8x) + (-2x + 3x^3 - 8)$

5) $(-1 + 2a + 5a^2) - (8a^3 + 3a^2 + 3)$

6) $(-5n^3 - 7 - 2n^4) - (-3n^4 - 7n^3 + 1)$

7) $(-x^4 - 8x + 2x^2) + (8x^4 - 8x^2 - 7x^3 - 4x)$

8) $(8 + 3m^3 + 6m) - (-m + 8m^3 - 7 - 3m^2)$

9) $(-2v + 6 + v^2) + (v - v^2) + (-7v^2 - 8v)$

10) $(5v^3 + 4v^2) - (-5v^3 + 2v^4 + 7) + (6v - 5v^4)$

11) $2(5b^2 + 2b - 5)$

12) $8p(2p^2 + 2p - 2)$

13) $(5p + 8)(-3p - 5)$

14) $(k + 7)(6k - 2)$

15) $(5m - 4)(-2m + 7)$

16) $(4p + 4)(p + 8)$

17) $(6k - 2)(2k - 6)$

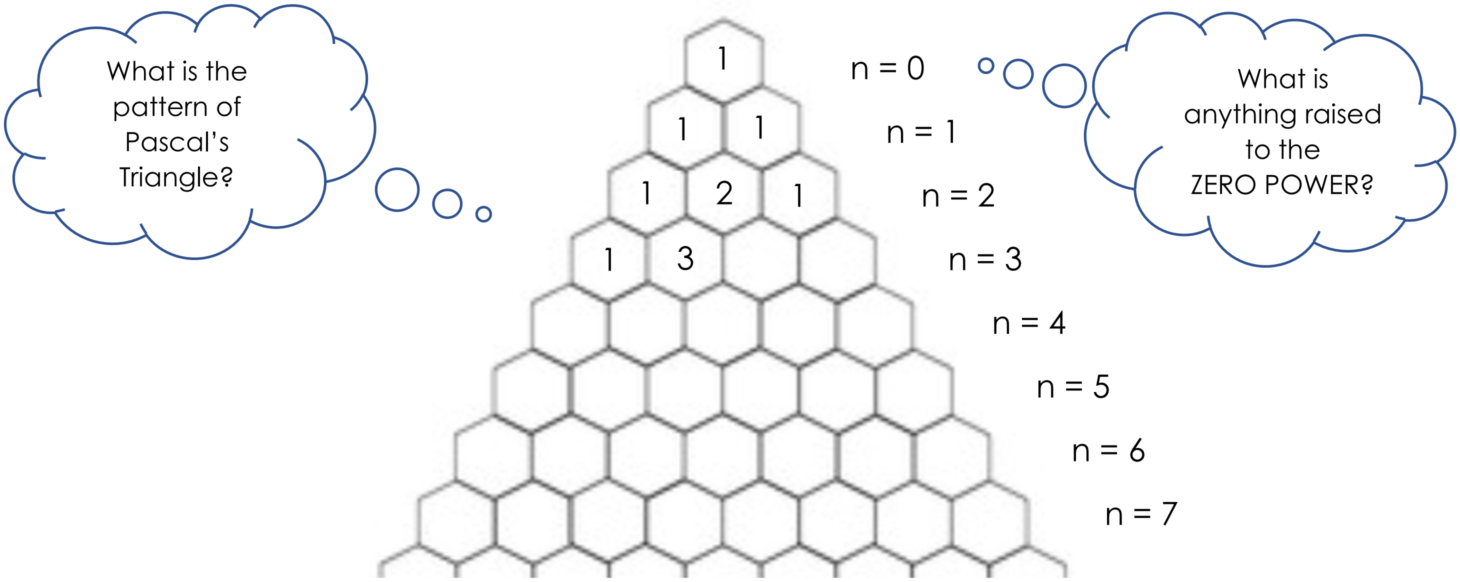
18) $(-8x - 1)(2x + 8)$

19) $(-8n - 7)(2n + 6)$

20) $(x - 4)(-7x - 1)$

21) $(8v + 8)(-6v^2 + 8v + 5)$

22) $(-5n - 7)(6n^2 - 6n + 8)$



We can use Pascal's Triangle to expand binomial expressions. Let's find the pattern…

$(a+b)^0 = 1$

$(a+b)^1 = a+b$

$(a+b)^2 = (a+b)(a+b) =$

$(a+b)^3 = (a+b)(a+b)^2 =$

Do you see the relationship among the exponents, coefficients, and Pascal's Triangle?

Use the pattern to expand $(a+b)^4$

Ex 1: Simplify $(x+3)^3$

Step 1: Locate the row and determine the number of terms.

Step 2: Write the coefficients for the terms from Pascal's Triangle

Step 3: Write in the powers of a, in descending order, starting with n.

Step 4: Write in the powers of b, in increasing order, starting with 0.

Step 5: Simplify each term.

Ex 2: Simplify $(x-2)^4$

Ex 3: Simplify $(2x+1)^5$

Don't forget PEMDAS and the exponent rules!

Ex 4: Simplify $(3x-2)^3$

You Try
1. $(x-2)^4$
2. $(x+5)^3$
3. $(x+3)^5$
4. $(3x+1)^5$

Characteristics of Polynomial Functions

Interval Notation is a way of writing subsets of real numbers.

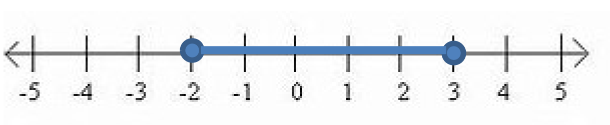
Braces ______ define a set, but that set is limited to the specific elements named within.

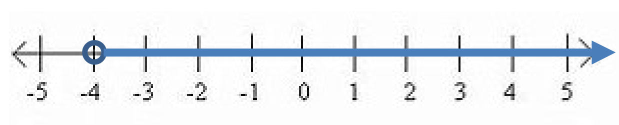
Brackets ______ indicate that all numbers INCLUDING the given values are in the interval.

Parentheses ______ indicate that all numbers BETWEEN the given values are in the interval.

Ex 1: Use interval notation to describe the inequality shown or described.

a. $x < 5$ b. $x \geq -2$ c. $-3 < x \leq 4$

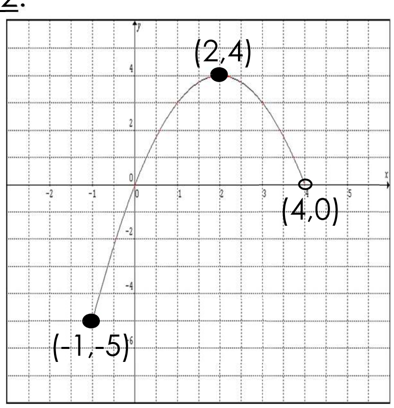
d. 

e. 

We'll use interval notation to describe the domain and range of functions.

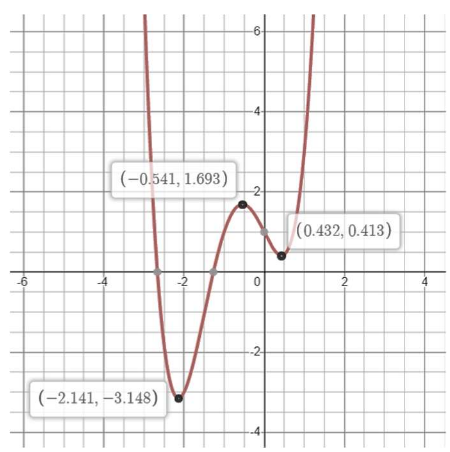
Domain		
Define: All possible values of x	**Think:** How far does the graph go, from **left to right**?	**Write:** [least x-value, greatest x-value] *use () with $-\infty$ and ∞
Range		
Define: All possible values of y	**Think:** How far does the graph go, from **bottom to top**?	**Write:** [least y-value, greatest y-value] *use () with $-\infty$ and ∞

Ex 2:

a. 

Domain:

Range:

b. 

Domain:

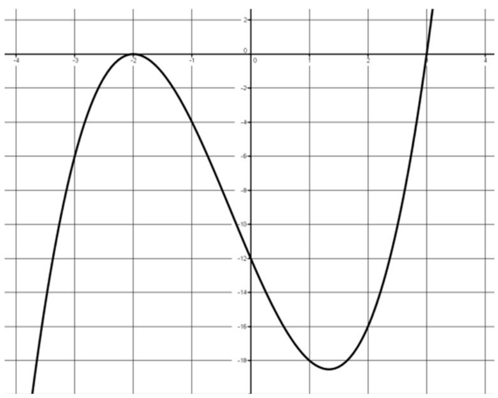
Range:

*For polynomial functions the DOMAIN will ALWAYS be ________________________.

x-intercept	**y-intercept**
• The x-coordinate of the point(s) where the graph crosses the x-axis. • $(x, 0)$ • Also known as ZEROS • Can be calculated by substituting **zero for y**. • A polynomial of degree *n* can have at most *n* real zeros	• The y-coordinate of the point where the graph crosses the y-axis. • $(0, y)$ • Can be calculated by substituting **zero for x**. • Can a function have more than one y-intercept?

Ex 3: Identify the x- and y- intercepts.

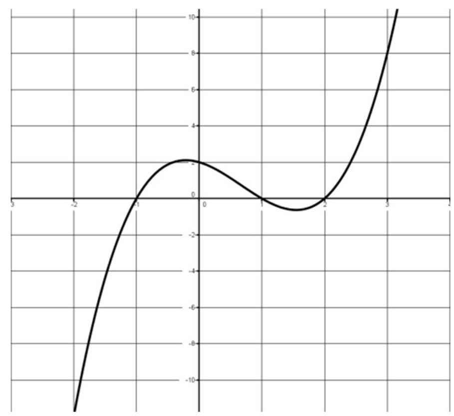
a.



x-intercept(s)____________________

y-intercept ____________________

b.



x-intercept(s)____________________

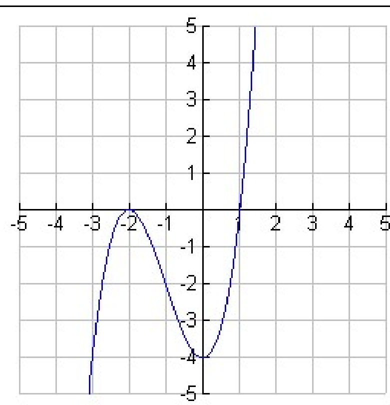
y-intercept ____________________

Ex 4: Find the y-intercepts:

a. $y = 3x^4 + 5x^2 - 1$

b. $f(x) = -2x^2 - 3x + 15$

Putting it all TOGETHER

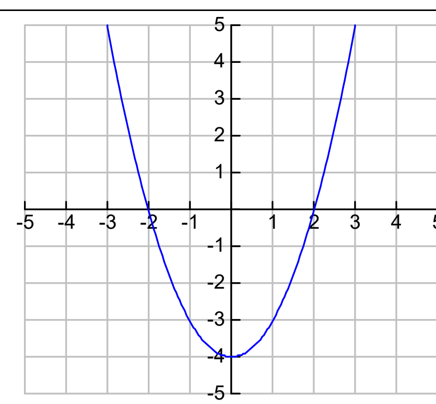


Domain: ____________________

Range: ____________________

X-Intercepts: ____________________

Y-Intercept:____________________



Domain: ____________________

Range: ____________________

X-Intercepts: ____________________

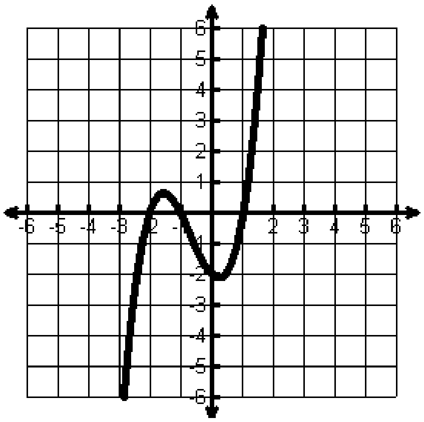
Y-Intercept:____________________

Name______________________________________ Date _____________ Period____

1. $f(x) = x^3 + 2x^2 - x - 2$

Domain:________Range:____________

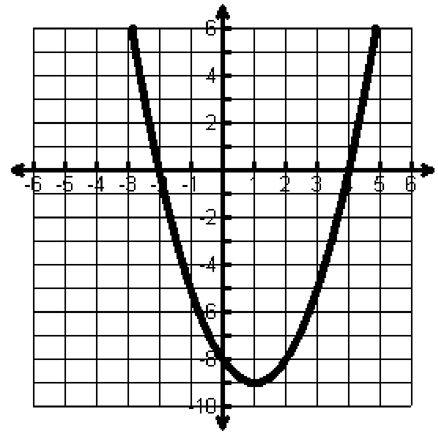
Zeros:__________ Y-Int: _____________



2. $f(x) = x^2 - 2x - 8$

Domain:________Range:____________

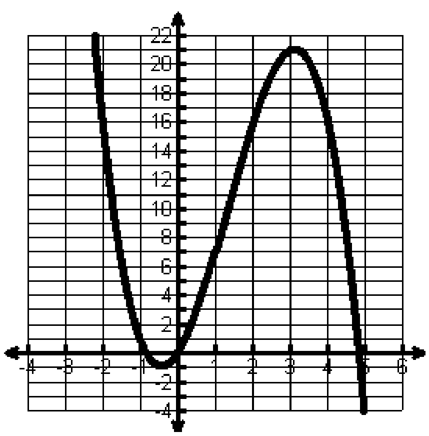
Zeros:__________ Y-Int: _____________



3. $f(x) = -x^3 + 4x^2 + 4x$

Domain:________Range:____________

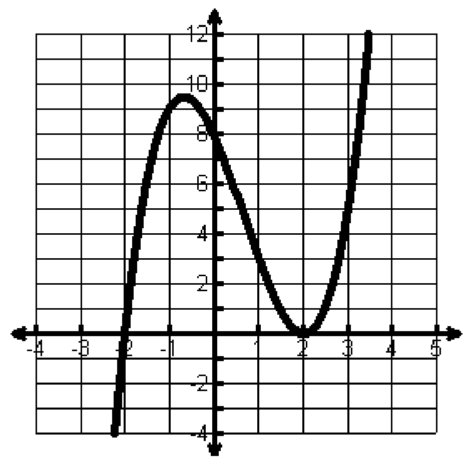
Zeros:__________ Y-Int: _____________



4. $f(x) = x^3 - 2x^2 - 4x + 8$

Domain:________Range:____________

Zeros:__________ Y-Int: _____________



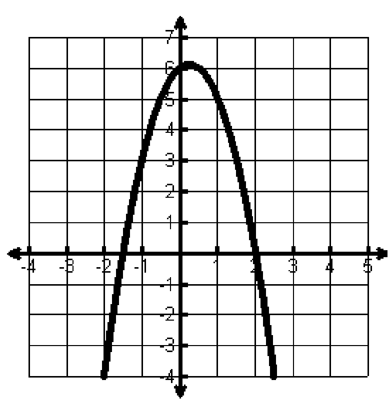
5. $f(x) = -2x^2 + x + 6$

Domain:________

Range:__________

Zeros:____________

Y-Int: _____________



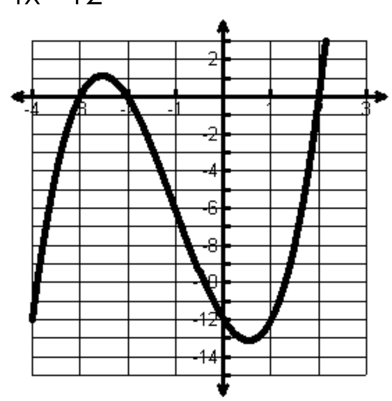
6. $f(x) = x^3 + 3x^2 - 4x - 12$

Domain:________

Range:__________

Zeros:____________

Y-Int: _____________



Degree

The degree, *n*, of a polynomial function, can tell us a lot of helpful information:

- *n* = the maximum number of zeros, or x-intercepts
- *n* = the maximum number of directions in which the graph will travel
- (*n* -1) = the maximum number of turns/extrema
- End Behavior:
    - if the degree is ___________, the ends of the graph will go in _________________ directions
    - if the degree is ___________, the ends of the graph will go in _________________ directions

Maximum and Minimum Values

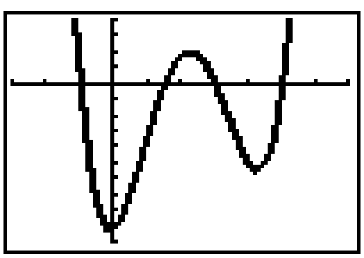
These are the ____ - coordinates of the turning points of the graph.

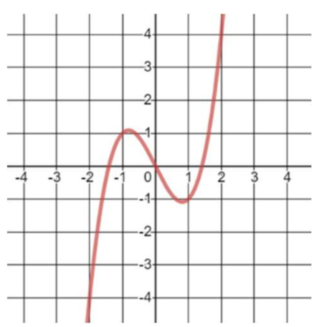
- Absolute maximum is the ___________________ point on the graph
- Absolute minimum is the ____________________ point on the graph
- Relative maximum is found at the ____________ of a peak, and is higher than any point nearby.
- Relative minimum is found at the _____________ of a valley, and is lower than any point nearby.

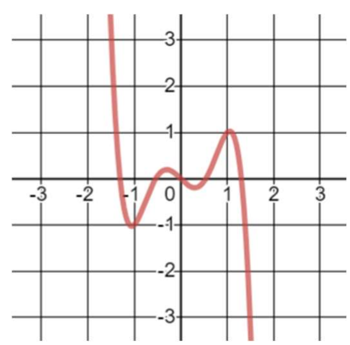
Maximum and minimum values are called __________________________.

To find the least possible degree, count the number of extrema, and ________ 1.

Ex 1: Determine the least possible degree of the function shown.

a.


b.


c.


To find the maximum number of extrema, take the degree and ________________1.

Ex 2: Determine the maximum number of extrema.

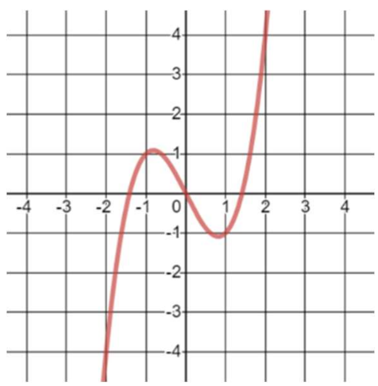
a. $f(x) = 2x^3 - 3x^2 + 5$

b. $y = -3x^4 + 2x^2 - 1$

c. $g(x) = x^5 + 3x^4 - x^3 - 3x^2$

Describes whether the y-values of a function increase or decrease as the x-values approach positive infinity on the right, and as the x-values approach negative infinity on the left.

Think: As x goes to the RIGHT (toward positive infinity), does the end of the graph go up or down?	**Say:** "As x approaches infinity, f of x approaches ________"	**Write:** As x → ∞, f(x) → _____
Think: As x goes to the LEFT (toward negative infinity), does the end of the graph go up or down?	**Say:** "As x approaches negative infinity, f of x approaches ________"	**Write:** As x → -∞, f(x) → _____

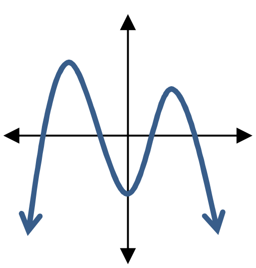


$x \to +\infty \quad f(x) \to$ ________

$x \to -\infty \quad f(x) \to$ ________

Ex 3: Describe the end behavior of each graph.

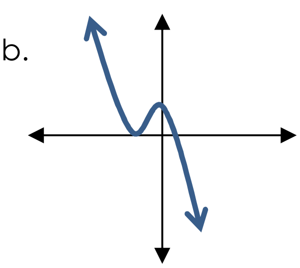
a.



$x \to +\infty \quad f(x) \to$ ________

$x \to -\infty \quad f(x) \to$ ________

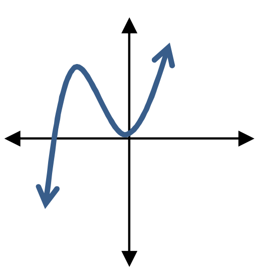
b.



$x \to +\infty \quad f(x) \to$ ________

$x \to -\infty \quad f(x) \to$ ________

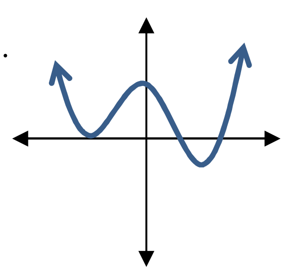
c.



$x \to +\infty \quad f(x) \to$ ________

$x \to -\infty \quad f(x) \to$ ________

d.



$x \to +\infty \quad f(x) \to$ ________

$x \to -\infty \quad f(x) \to$ ________

End behavior can also be determined by looking at the leading coefficient and degree of the function.

Leading Coefficient tells us what happens on the **RIGHT**:

POSITIVE ________

NEGATIVE ________

Degree tells us what happens on the **LEFT**: (same as the right, or opposite?)

EVEN ________

ODD ________

	ODD Degree	EVEN Degree
POSITIVE Leading Coefficient		
NEGATIVE Leading Coefficient		

Ex 4: Determine the end behavior of the function.

a. $f(x) = -2x^3 + x - 4$

$x \to +\infty \quad f(x) \to$ ________

$x \to -\infty \quad f(x) \to$ ________

b. $f(x) = x^4 + 2x^3 - x^2 - 1$

$x \to +\infty \quad f(x) \to$ ________

$x \to -\infty \quad f(x) \to$ ________

c. $f(x) = 6x^5 - 4x^3 - 9$

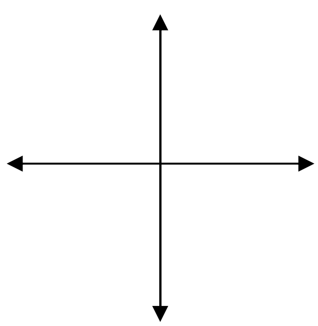
$x \to +\infty \quad f(x) \to$ ________

$x \to -\infty \quad f(x) \to$ ________

Putting it all together!

Ex 5: Given the polynomial and zeros, sketch a graph and determine the characteristics

a. $f(x) = x^2 + 8x - 20$
zeros: -10, 2



# of Zeros: __________

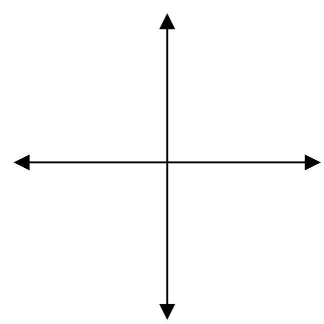
Y-Int: _____________

Max # of extrema: __________

$x \to +\infty \quad f(x) \to$ ________

$x \to -\infty \quad f(x) \to$ ________

b. $f(x) = x^3 + 2x^2 - x - 2$
zeros: -2, -1, 1



# of Zeros: __________

Y-Int: _____________

Max # of extrema: __________

$x \to +\infty \quad f(x) \to$ ________

$x \to -\infty \quad f(x) \to$ ________

Name:______________________________________ Date: ____________________

Complete the following table for each polynomial function:

Function	Degree	End Behavior	Domain
1. $f(x) = x^3 - x^2 - 8x + 12$		As $x \to \infty$ $f(x) \to$ ________ As $x \to -\infty$ $f(x) \to$ ________	
2. $f(x) = 3x^3 - 12x + 4$		As $x \to \infty$ $f(x) \to$ ________ As $x \to -\infty$ $f(x) \to$ ________	
3. $f(x) = -2x^3 + 4x^2 + x - 2$		As $x \to \infty$ $f(x) \to$ ________ As $x \to -\infty$ $f(x) \to$ ________	
4. $f(x) = x^4 + 5x^3 + 5x^2 - x - 6$		As $x \to \infty$ $f(x) \to$ ________ As $x \to -\infty$ $f(x) \to$ ________	
5. $f(x) = x^4 + 2x^3 - 5x^2 - 6x$		As $x \to \infty$ $f(x) \to$ ________ As $x \to -\infty$ $f(x) \to$ ________	

Use the equations to answer the following:

Function	Degree	Max # of Extrema
6. $f(x) = x^3 - x^2 - 8x + 12$		
7. $f(x) = 3x^3 - 12x + 4$		
8. $f(x) = -2x^3 + 4x^2 + x - 2$		
9. $f(x) = x^4 + 5x^3 + 5x^2 - x - 6$		
10. $f(x) = x^4 + 2x^3 - 5x^2 - 6x$		

Determine the end behavior and maximum number of extrema without using a calculator:

$f(x) = -8x^5 - 7x^3 + 3x - 7$
11. $x \to +\infty \quad f(x) \to$ _______ *extrema* _____
$x \to -\infty \quad f(x) \to$ _______

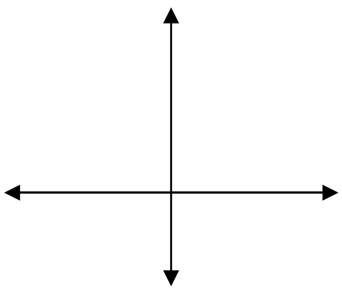
$f(x) = 12 - 3x^3 + 5x^3 - 7x^4$
12. $x \to +\infty \quad f(x) \to$ _______ *extrema* _____
$x \to -\infty \quad f(x) \to$ _______

$f(x) = 1 - 3x - 2x^2 - 5x^3 + 7x^4 - 12x^5$
13. $x \to +\infty \quad f(x) \to$ _______ *extrema* _____
$x \to -\infty \quad f(x) \to$ _______

$f(x) = -7x^3 + 343$
14. $x \to +\infty \quad f(x) \to$ _______ *extrema* _____
$x \to -\infty \quad f(x) \to$ _______

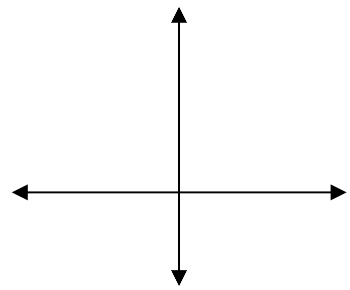
Find the number of zeros, y-int, & end behavior. Sketch the graph:

15. $x^4 - 13x^2 + 36 = 0$
given zeros $: -3, -2, 2, 3$



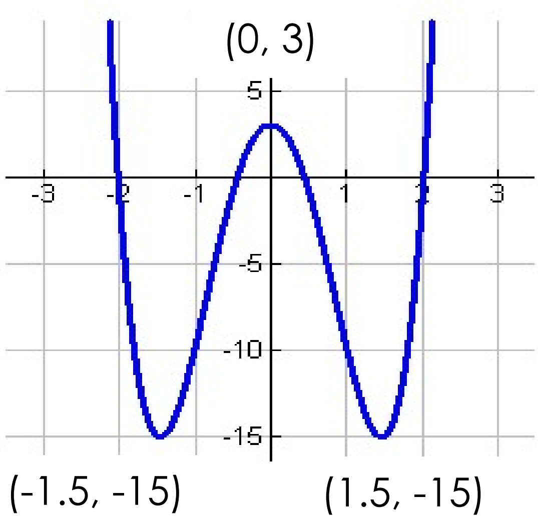
# of Zeros: __________
Y-Int: _____________
$x \to +\infty \quad f(x) \to$ _____
$x \to -\infty \quad f(x) \to$ _____
Max # of extrema _____

16. $x^3 - x^2 - 16x + 16 = 0$
given zeros $: -4, 1, 4$



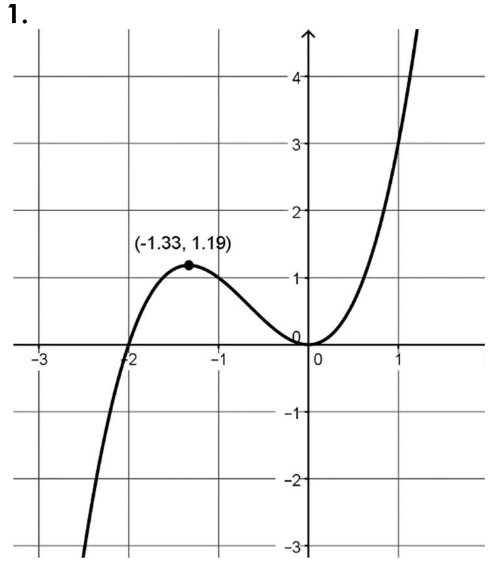
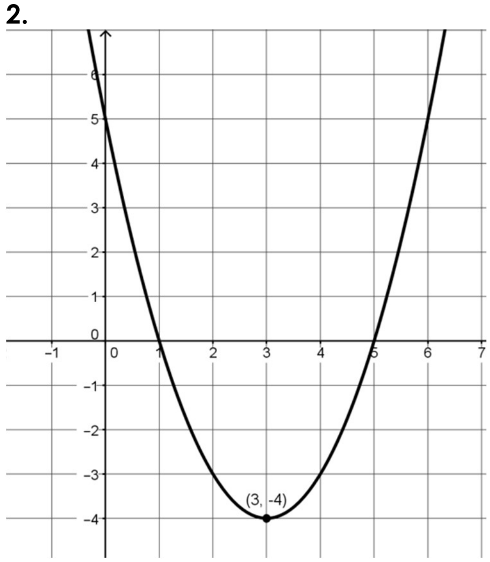
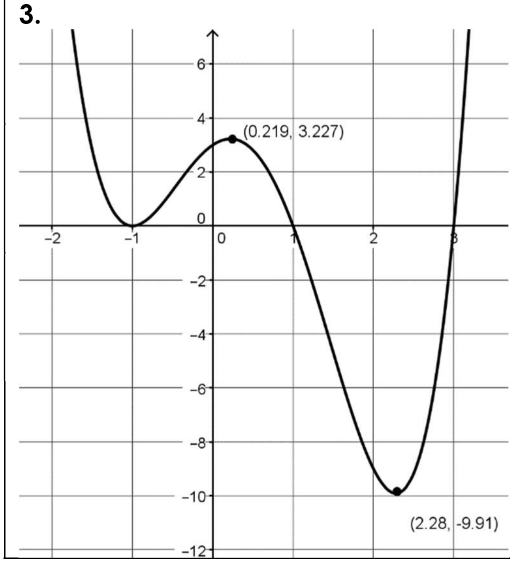
# of Zeros: ________
Y-Int: _____________
$x \to +\infty \quad f(x) \to$ _____
$x \to -\infty \quad f(x) \to$ _____
Max # of extrema _____

17. Determine the following characteristics for the graph shown:

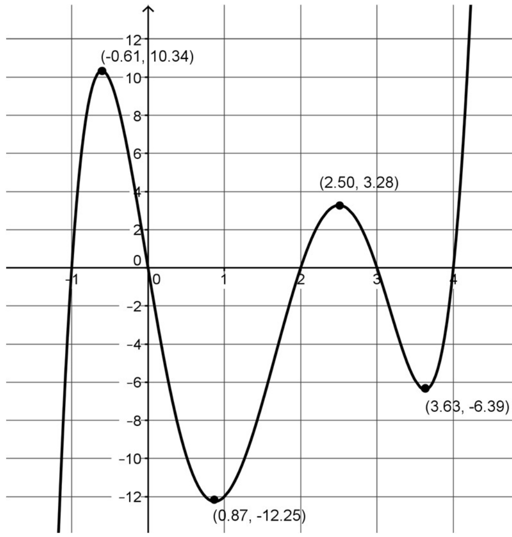


Domain:	Range:
Abs. Max:	Abs. Min:
Rel. Max:	Rel. Min:
Min. degree	Sign of leading Coeff.

Interval of Increase: Using interval notation, name the **x-values** between which the y-values increase
Interval of Decrease: Using interval notation, name the **x-values** between which the y-values decrease

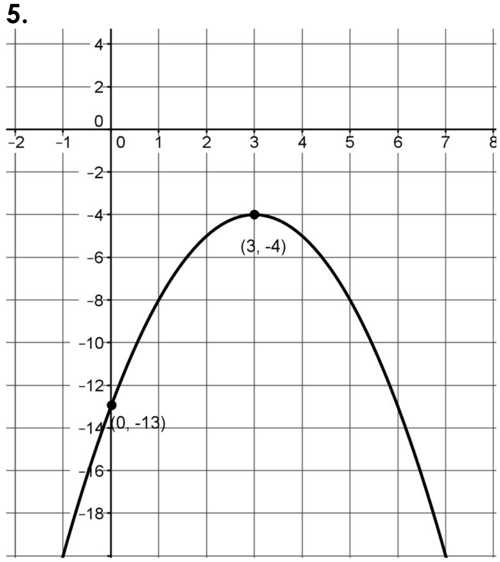
	Increasing, Decreasing, & Constant		**Extrema**	
1. 	**Increasing**		**Absolute Minimum**	
	Decreasing		**Absolute Maximum**	
	Constant		**Relative Minimum(s)**	
			Relative Maximum(s)	
2. 	**Increasing**		**Absolute Minimum**	
	Decreasing		**Absolute Maximum**	
	Constant		**Relative Minimum(s)**	
			Relative Maximum(s)	
3. 	**Increasing**		**Absolute Minimum**	
	Decreasing		**Absolute Maximum**	
	Constant		**Relative Minimum(s)**	
			Relative Maximum(s)	

4.



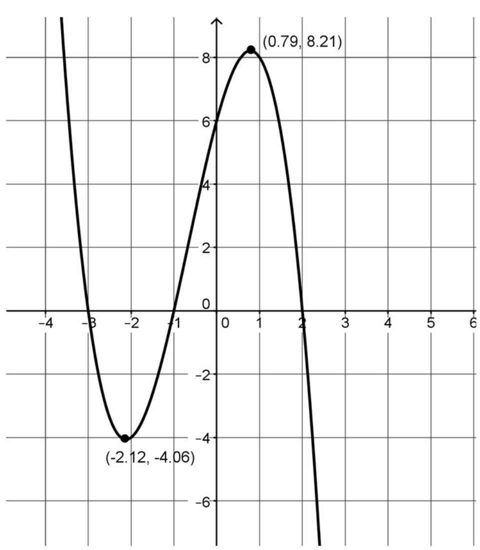
Increasing		**Absolute Minimum**	
Decreasing		**Absolute Maximum**	
Constant		**Relative Minimum(s)**	
		Relative Maximum(s)	

5.



Increasing		**Absolute Minimum**	
Decreasing		**Absolute Maximum**	
Constant		**Relative Minimum(s)**	
		Relative Maximum(s)	

6.



Increasing		**Absolute Minimum**	
Decreasing		**Absolute Maximum**	
Constant		**Relative Minimum(s)**	
		Relative Maximum(s)	

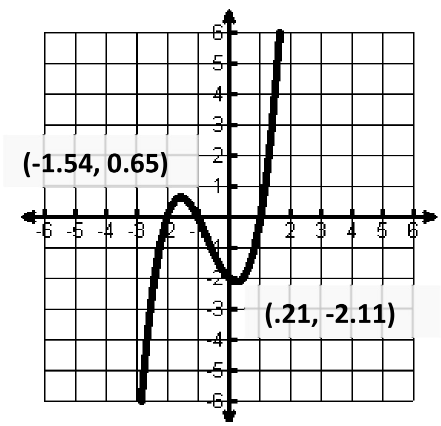
1. $f(x) = x^3 + 2x^2 - x - 2$

Rel. Max: _______ Rel. Min: __________

Abs. Max: ______ Abs. Min: __________

Inc: ___________ Dec: ______________

Domain: _________ Range: __________



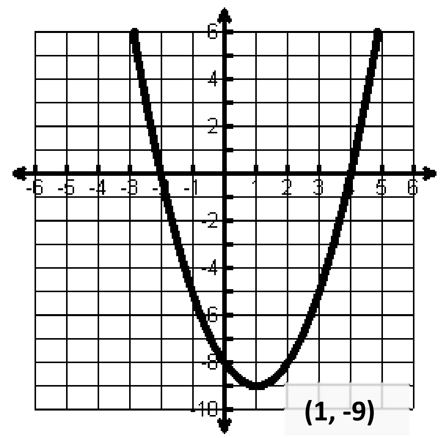
2. $f(x) = x^2 - 2x - 8$

Rel. Max: _______ Rel. Min: __________

Abs. Max: ______ Abs. Min: __________

Inc: ___________ Dec: ______________

Domain: _________ Range: __________



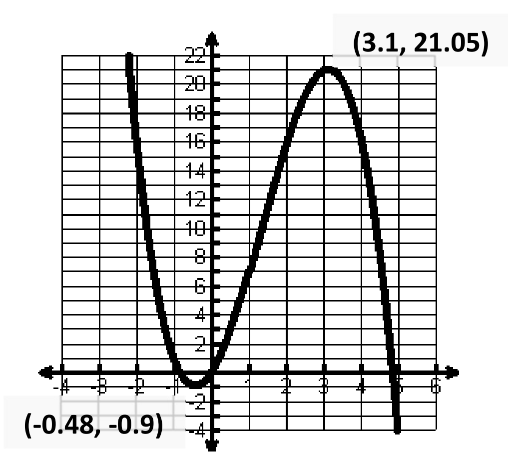
3. $f(x) = -x^3 + 4x^2 + 4x$

Rel. Max: _______ Rel. Min: __________

Abs. Max: ______ Abs. Min: __________

Inc: ___________ Dec: ______________

Domain: _________ Range: __________



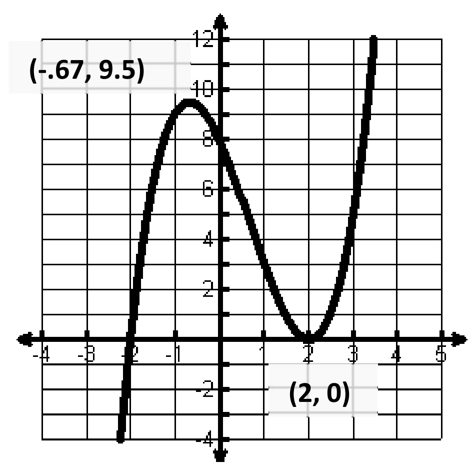
4. $f(x) = x^3 - 2x^2 - 4x + 8$

Rel. Max: _______ Rel. Min: __________

Abs. Max: ______ Abs. Min: __________

Inc: ___________ Dec: ______________

Domain: _________ Range: __________



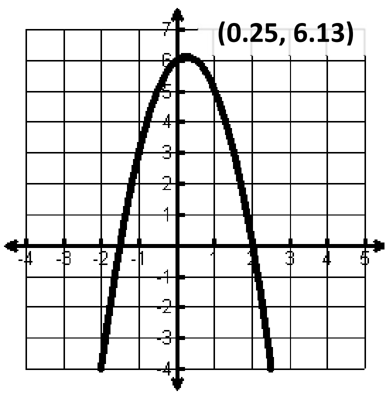
5. $f(x) = -2x^2 + x + 6$

Rel. Max: _______ Rel. Min: __________

Abs. Max: ______ Abs. Min: __________

Inc: ___________ Dec: ______________

Domain: _________ Range: __________



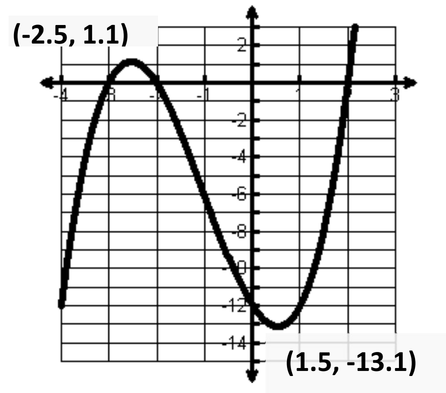
6. $f(x) = x^3 + 3x^2 - 4x - 12$

Rel. Max: _______ Rel. Min: __________

Abs. Max: ______ Abs. Min: __________

Inc: ___________ Dec: ______________

Domain: _________ Range: __________



Identify the **y-intercept** and the **# of zeros**

7. $f(x) = x^3 - 16$

Y-Int: __________ # of Zeros: __________

8. $f(x) = x^2 + x - 1$

Y-Int: ________ # of Zeros: ________

9. $f(x) = 9x^4 + x^3 - 3x - 10$

Y-Int: __________ # of Zeros: __________

10. $f(x) = x^3 - x - 2$

Y-Int: ________ # of Zeros: ________

11. $f(x) = 7x$

Y-Int: __________ # of Zeros: __________

12. $f(x) = -2x^3 + 7$

Y-Int: ________ # of Zeros: ________

Functions and Relations

- Relation: Any set of ____.
- Function: A ____________________ such that every single __________________ has exactly ________________ output.

Ex: {(5, 2), (2, 8), (3, -1), (5, 4)}
Ex: {(1, 2), (2, 3), (3, 4), (4, 5), (5, 3)}

Function Notation:

Function notation replaces ________ with _________ to show that an equation represents a _______________.

- It is pronounced __________________________________.

  Ex: $y = 2x + 4$ is the same as ___________________
- When evaluating functions, it also shows the relationship between the ______________ and _______________ values.

  Ex: Evaluate $f(x) = 2x + 4$ when $x = 2$

Combining Functions

Ex 1: Given the functions $\boxed{f(x) = 6x^2 - 3x + 5}$ and $\boxed{g(x) = 4x^2 + 5x - 8}$

a. Find $f(x) + g(x)$

b. Find $g(x) - f(x)$

Ex 2: Given the functions $\boxed{f(x) = 6x^2 - x + 3}$ and $\boxed{g(x) = x^2 + 3x}$, find $2f(x) + 3g(x)$

Ex 3: Given the functions $\boxed{f(x) = 2x - 4}$ and $\boxed{g(x) = x^2 - 3}$, find $g(x) \bullet f(x)$

Ex 4: Given the functions $\boxed{f(x) = 4x^2 - 2x + 5}$ and $\boxed{g(x) = x^2 + 7x - 8}$

a. Find $f(3) + g(3)$

b. Find $4g(-2) \bullet f(-2)$

Function Compositions

Function composition is applying the rule of one function to another function.

$f(g(x))$ is read as ____.

- $g(x)$ is the input function, because it is ______________ the parentheses.
- Substitute $g(x)$ into the rule of $f(x)$ in the place of each x. Then simplify.
- This is sometimes denoted as $(f \circ g)(x)$. In this case, $g(x)$ is the input function.

Ex: f(x) = 2x +5 & g(x) = x -10

Find f(g(x): f is the _________ function & g is the __________ function. Substitute g(x) inside of f(x).

2(________) + 5 Then simplify to get _________

***Pay attention to the order of the functions to know which one is the inside function.**

Ex 5: Given $f(x) = x^2 + 2$ $g(x) = 3x - 8$ $h(x) = -2x^2 + 1$ and $j(x) = 4x - 3$

a. Find $g(f(x))$

b. Find $g(h(x))$

c. Find $f(g(x))$

d. Find $j(j(x))$

e. Find $j(h(x))$

f. Find $g(j(2))$

g. Find $g(g(-3))$

h. Find $f(j(-4))$

Algebra 2 Name___________________________________

Function Operations & Compositions

Date________________ Period____

Perform the indicated operation.

1) $f(a) = -4a + 4$
$g(a) = a^3 - 2$
Find $f(a) - g(a)$

2) $h(t) = 3t + 3$
$g(t) = -t^2 - 5t$
Find $(h - g)(t)$

3) $f(t) = 4t - 3$
$g(t) = t^2 - 3t$
Find $(f \cdot g)(t)$

4) $f(n) = 4n + 2$
$g(n) = n^2 - 1$
Find $f(n) - g(n)$

5) $f(x) = x^3 - 4x$
$g(x) = 2x + 3$
Find $f(x) + g(x)$

6) $h(a) = 4a - 5$
$g(a) = a^3 - 2a^2$
Find $h(a) \cdot g(a)$

7) $f(t) = 4t + 3$
$g(t) = -2t^2 - 2$
Find $f(t) \cdot g(t)$

8) $h(t) = t^2 - 4t$
$g(t) = 2t - 3$
Find $h(t) \cdot g(t)$

Perform the indicated operation

9) $h(n) = 4n - 1$
$g(n) = n^2 + 3n$
Find $h(g(n))$

10) $g(n) = n^2 - 4n$
$f(n) = 4n + 3$
Find $g(f(n))$

11) $h(n) = n^3 - 4n^2$
$g(n) = 4n + 4$
Find $h(g(n))$

12) $g(x) = x^2 - 1$
$h(x) = 2x - 2$
Find $g(h(x))$

13) $h(n) = 4n - 1$
$g(n) = 3n^3 - 5n^2$
Find $h(g(n))$

14) $g(n) = n^2 - 2$
$h(n) = 4n - 3$
Find $g(h(n))$

Perform the indicated operation.

15) $h(x) = x^2 + 2x$
$g(x) = x - 4$
Find $h(g(-7))$

16) $g(x) = 4x$
$h(x) = x - 5$
Find $g(h(-9))$

17) $g(a) = a^3 + 5a$
$f(a) = a + 3$
Find $g(-3) + f(-3)$

18) $g(n) = 2n + 3$
$f(n) = n^3 - 3$
Find $g(2) \cdot f(2)$

19) $g(x) = 3x + 5$
$h(x) = x + 4$
Find $g(-10) + h(-10)$

20) $h(a) = a^2 - 1$
$g(a) = -a - 5$
Find $h(5) - g(5)$

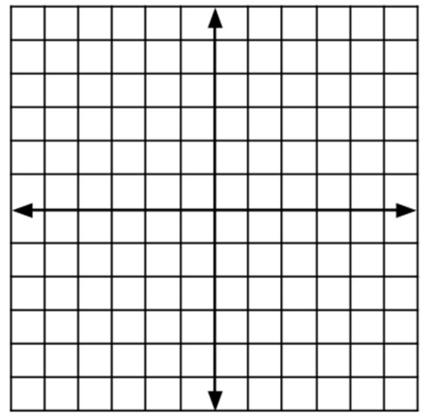
Recall that a **relation** is a set of ________________________________, such as $\{(1,5),(2,6),\ (3,7)\}$.

The ___________________ relation is the set of ordered pairs obtained by _____________________ the x and y coordinates in each of the ordered pairs: {(___,___), (___,___), (___,___)}.

- Notice that the **domain** of the relation becomes the _________________ of its inverse, and the **range** of the relation becomes the ________________ of its inverse.
- Inverse relations are reflected over the line _____________.

Ex 1: The ordered pairs of the relation $\{(-2,0),(-1,5),\ (-3,4)\}$ are the coordinates of the vertices of a triangle.

a. Find the inverse of this relation.
b. Plot the two relations and observe the line of reflection.



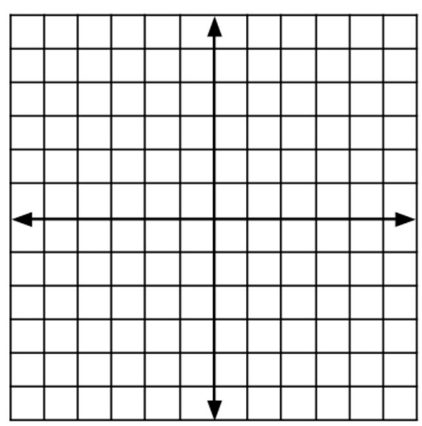
Given the equation of a function, $f(x)$, we can find its inverse, _________.

We read this as

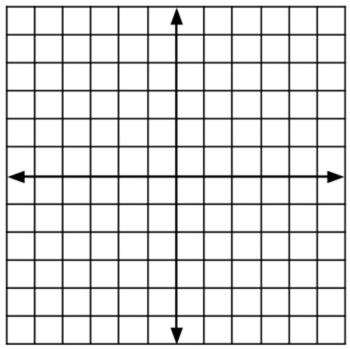
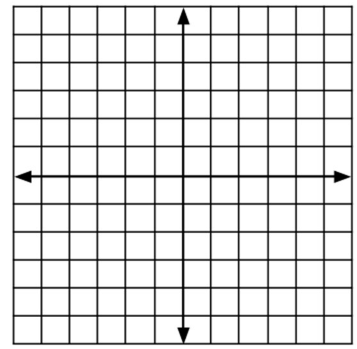
To find the inverse of a function:

1. Replace $f(x)$ with y.
2. Switch x and y.
3. Solve for y.
4. Rewrite using $f^{-1}(x)$ notation.

Ex 2: Given the function $f(x) = x + 4$, find $f'(x)$.
Then sketch both functions and the line of reflection $y = x$.



Ex 3: Find the inverse of the function. Then graph the function and its inverse.

a. $g(x) = -\frac{1}{2}x + 1$	b. $h(x) = x^2 + 1$
	

Verifying Inverse Functions

> $f(x)$ and $g(x)$ are inverses if and only if
>
> $f(g(x)) = x$ and $g(f(x)) = x$

Two functions are inverses if and only if both of their compositions are equal to ________.

Ex 4: Determine whether the given functions are inverses. Explain your reasoning.

a. $f(x) = \frac{3}{4}x - 8$
$g(x) = \frac{4}{3}x + 8$

b. $f(x) = \frac{1}{9}(x + 3)^2$
$h(x) = 3\sqrt{x} - 3$

<table>
<tr><th colspan="2">You Try</th></tr>
<tr><td colspan="2">1. Write the inverse of the relation $\{(-4, 1), (0, 2), (7, 1), (5, 3)\}$.</td></tr>
<tr><td>2. Find the inverse of the function $f(x) = 4x - 6$.</td><td>3. Determine whether $g(x) = -6x$ and $h(x) = \frac{1}{6}x$ are inverse functions.</td></tr>
</table>

Algebra 2 Name________________________________

Inverse Functions

Date________________ Period____

Find the inverse of each function.

1) $f(x) = \frac{-3x - 12}{8}$

2) $f(x) = (x + 1)^3 - 1$

3) $g(n) = -3n - 15$

4) $f(x) = \sqrt[3]{x - 1} + 2$

5) $h(x) = \frac{-5x - 10}{4}$

6) $f(n) = n - 2$

7) $h(x) = (x + 1)^3 + 2$

8) $g(x) = \sqrt[3]{x + 3} + 2$

Use composition of functions, for example f(g(x)) and g(f(x)), to determine whether the two functions are inverses or not.

9) $f(n) = -n - 4$
$g(n) = -n - 4$

10) $f(x) = \frac{16 - 3x}{4}$
$g(x) = \frac{3x - 3}{2}$

11) $f(x) = \frac{-9 - x}{3}$
$h(x) = -3x - 9$

12) $g(x) = \frac{1}{3}x - \frac{4}{3}$
$f(x) = 3x + 4$

13) $h(n) = \sqrt[3]{n} + 1$
$f(n) = (n - 1)^3$

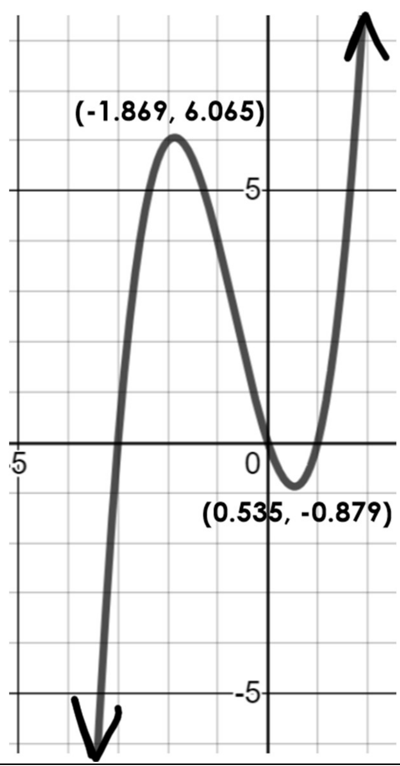
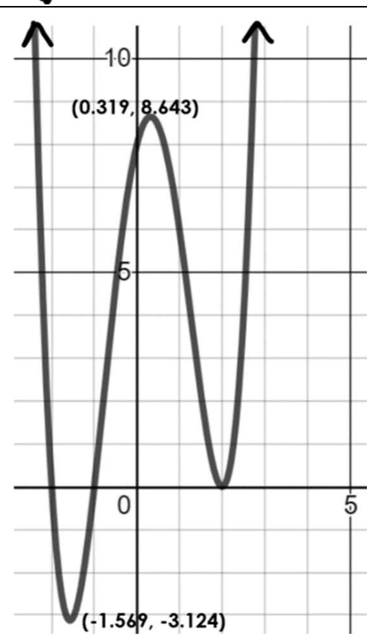
14) $g(x) = \frac{3x - 9}{5}$
$f(x) = \frac{9 + 5x}{3}$

15) $g(x) = 9x + 5$
$f(x) = -x - 5$

16) $f(x) = (x - 2)^3 - 3$
$g(x) = \sqrt[3]{x + 3} + 2$

Name Date

What you need to know & be able to do	Things to remember	Problem	Problem
Classify Polynomials	• Write all answers in **Standard Form** ➢ greatest exp to least • Classify Polynomials based on Degree and # Terms • Leading Coeff - first coeff in standard form • Constant – Term without a variable	1. List all the names for **Degree:** **0 -** ____________ **1 -** ____________ **2 -** ____________ **3 -** ____________ **4 -** ____________ **Number of Terms:** **1 -** ____________ **2 -** ____________ **3 -** ____________ **4 -** ____________	2. $f(x) = x + 2 - x^2 - 4x^4$ Standard form: ____________ Leading coefficient: _______ Constant: _______ Name of Degree: ____________ Name by # terms: ____________ # of zeros: _______ # of turns: _______ End behavior: as $x \to +\infty, f(x) \to$ ____ as $x \to -\infty, f(x) \to$ ____
Adding and Subtracting	Adding: ➢ Combine like terms Subtracting: ➢ Distribute the negative (add the opposite) ➢ Combine like terms	3. $(3x^2 + 7 + x) + (14x^3 + 2 + x^2 - x)$	4. $(1 - x^2) - (3x^2 + 2x - 5)$
Multiply Polynomials	• Distribute each term in the first quantity to each term in the second • Multiply coefficients, add exponents	5. $(3x^2)(2x^2 + 9x - 6)$	6. $(x - y)(x^2 - xy + y^2)$
Combining Functions	Given: $f(x) = 2x^2 + 5x - 3$ $g(x) = -4x + 5$	7. Find $f(x) - 2g(x)$	8. Find $g(x) \cdot f(x)$
		9. Find $f(g(x))$	10. Find $f(-2) + g(3)$

What you need to know & be able to do	Things to remember	Problem	Problem
Inverses	<u>Finding Inverses</u> • Replace $f(x)$ with y • Switch x & y • Solve for y	11. $f(x) = 3x - 7$; find $f^{-1}(x)$	12. $g(x) = \frac{1}{4}x + 3$; find $g^{-1}(x)$
	<u>Proving Inverses</u> • Use compositions • $f(g(x)) = x$ • $g(f(x)) = x$	13. $f(x) = 2x - 5;\ g(x) = -2x + 5$	14. $f(x) = 2x + 8;\ g(x) = \frac{1}{2}x - 4$
Characteristics of Polynomial Graphs	a) Domain: x values b) Range: y values c) Zeros: x-intercepts d) # of extrema – turning points e) Relative max: y-values at peaks f) Relative min: y-values at valleys g) Absolute Max – highest y-value h) Absolute Min – lowest y-value i) Intervals of increase: x-values where graph is rising toward right j) Intervals of decrease: x values where graph is falling toward right k) End behavior: direction that the ends of the graph are going	15. 	Domain: Range: Zeros: # of extrema: Relative max: Relative min: Absolute Max: Absolute Min: Intervals of inc: Intervals of dec: End behavior: As $x \to +\infty, f(x) \to$________ As $x \to -\infty, f(x) \to$________
		16. 	Domain: Range: Zeros: # of extrema: Relative max: Absolute Max: Absolute Min: Intervals of inc: Intervals of dec: End behavior: As $x \to +\infty, f(x) \to$________ As $x \to -\infty, f(x) \to$________