Unit 1 Pre-Calculus

- Notes and some practice are included
- Homework will be assigned on a daily basis

Topics Covered:

- ❖ Factoring Polynomials
- ❖ Rates of Change & Linear Equations
- ❖ Functions & Their Graphs (Domain, Piecewise Functions, Compositions, & Transformations)
- ❖ Solving Exponential & Logarithmic Functions
- ❖ The Unit Circle & Evaluating Trig Functions
- ❖ Inequalities

Quiz is _____

Test is _____

Name: **Bonanni**

F19

Fun with Factoring

Factoring Trinomials

1. $x^2 + 6x + 8$

$(x+2)(x+4)$

2. $x^2 - x - 6$

$(x-3)(x+2)$

3. $c^2 + 5c + 6$

$(c+3)(c+2)$

4. $y^2 + 3y - 18$

$(y+6)(y-3)$

5. $3y^2 + 14y - 49$

$(3y-7)(y+7)$

6. $3b^2 + 21b - 54$

$3(b^2+7b-18)$
$3(b+9)(b-2)$

7. $6x^2 - 42x + 72$

$6(x^2-7x+12)$
$6(x-3)(x-4)$

8. $7a^2 - 38a - 24$

$(7a+4)(a-6)$

9. $15a^2 + 4a - 4$

$(5a-2)(3a+2)$

10. $8c^2 = 30 + 43c$

$8c^2-43c-30=0$
$(8c+5)(c-6)=0$

11. $x^6 - 14x^3 + 24$

$(x^3-12)(x^3-2)$

12. $15x^3 + 20x^2 - 25x$

$5x(3x^2+4x-5)$

Factoring by GCF

13. $5c^3 - 2c^2$

$c^2(5c-2)$

14. $81r + 48rs$

$3r(27+16s)$

15. $15cd + 30c^2d^2$

$15cd(1+2cd)$

16. $a^2b^2 + a$

$a(ab^2+1)$

Factoring Special Binomials

17. $5a^2 - 20$

$5(a^2-4)$
$5(a+2)(a-2)$

18. $y^3 - 1$

$(y-1)(y^2+y+1)$

19. $8x^3 + 64$

$8(x^3+8)$
$8(x+2)(x^2-2x+4)$

20. $1 - 49c^2$

$(1+7c)(1-7c)$

21. $-27 + p^3$

p^3-27
$(p-3)(p^2+3p+9)$

22. $200r^2 - 18$

$2(100r^2-9)$
$2(10r+3)(10r-3)$

23. $36 - n^4$

$(6+n^2)(6-n^2)$

24. $16 - 81f^8$

$(4+9f^4)(4-9f^4)$
$(4+9f^4)(2+3f^2)(2-3f^2)$

Factoring by Grouping

25. $(6mn - 9m) + (4n + 6)$

$3m(2n-3)-2(2n-3)$
$(3m-2)(2n-3)$

26. $(2x^2y + 6xy) + (x - 3)$

$2xy(x+3)-1(x+3)$
$(2xy-1)(x+3)$

27. $(6x^3 - 3x^2) + (8x - 4)$

$3x^2(2x-1)+4(2x-1)$
$(3x^2+4)(2x-1)$

28. $(4r^3 - 8r^2) + (3r + 6)$

$4r^2(r-2)-3(r-2)$
$(4r^2-3)(r-2)$

①

Average Rate of Change

Use the table of values to find the average rate of change over the given interval.

x	1	2	3	3.5	3.7	6
y	40	25	18	15	18	38

1. [1,3]

$(1,40)$ $(3,18)$

$\frac{18-40}{3-1} = \frac{-22}{2} = -11$

2. [2,6]

$(2,25)$ $(6,38)$

$\frac{38-25}{6-2} = \frac{13}{4}$ or 3.25

3. [2,3.7]

$(2,25)$ $(3.7,18)$

$\frac{18-25}{3.7-2} = \frac{-7}{1.7} = \frac{-70}{17}$

4. [3.5,6]

$(3.5,15)$ $(6,38)$

$\frac{38-15}{6-3.5} = \frac{23}{2.5} = \frac{46}{5}$ or 9.2

Find the average rate of change for each function on the given interval.

5. $f(x) = x^2 - 4x - 12$ on [0,6]

$f(0) = -12$ $f(6) = 0$
$(0,-12)$ $(6,0)$

$\frac{0--12}{6-0} = \frac{12}{6} = 2$

6. $f(x) = x^2 - 4x - 12$ on [−1,7]

$f(-1) = -7$ $f(7) = 9$
$(-1,-7)$ $(7,9)$

$\frac{9--7}{7--1} = \frac{16}{8} = 2$

7. $f(x) = 3x^2 - x - 2$ on [−1,4]

$f(-1) = 2$ $f(4) = 42$
$(-1,2)$ $(4,42)$

$\frac{42-2}{4--1} = \frac{40}{5} = 8$

8. $f(x) = 0.02x^2 - 1.6x + 20.5$ on [25,33]

$f(25) = -7$ $f(33) = -10.52$
$(25,-7)$ $(33,-10.52)$

$\frac{-10.52--7}{33-25} = \frac{-3.52}{8} = \frac{-11}{25}$ or $-.44$

9. $f(x) = 3x^5 - 4x^4 + 3x^2 - 4x + 1$ on [−6,1]

$f(-6) = -28,379$ $f(1) = -1$
$(-6,-28,379)$ $(1,-1)$

$\frac{-1+28,379}{1+6} = \frac{28378}{7} = 4,054$

10. $f(x) = \frac{x^2-6}{x+3}$ on [−5,−2]

$f(-5) = -19/2$ $f(-2) = -2$
$(-5,-9.5)$ $(-2,-2)$

$\frac{-2+9.5}{-2+5} = \frac{7.5}{3} = \frac{5}{2}$ or 2.5

(2)

Equations of Lines

11. Write the slope-intercept form of the equation:
$$11x - 8y = -48$$

$$\frac{-8y}{-8} = \frac{-11x - 48}{-8}$$

$$\boxed{y = \tfrac{11}{8}x + 6}$$

12. Write the standard form of the equation of the line through the given point with the given slope:
$$(3,5), m = \frac{5}{3}$$

$$y - 5 = \tfrac{5}{3}(x-3)$$
$$y - 5 = \tfrac{5}{3}x - 5$$
$$y = \tfrac{5}{3}x$$

$$3y = 5x$$
$$0 = 5x - 3y$$
$$\boxed{5x - 3y = 0}$$

13. Write the equation of the line describe: through (4,2), parallel to $y = -5x + 2$

$$m = -5$$

also OK → $\boxed{y - 2 = -5(x-4)}$
$$y - 2 = -5x + 20$$
$$\boxed{y = -5x + 22}$$

14. Write the equation of the line described: through $(-2,4)$, perpendicular to $y = -\frac{5}{2}x + 5$

$$m = 2/5$$

OK → $\boxed{y - 4 = 2/5 (x+2)}$
$$y - 4 = 2/5 x + 4/5$$
$$\boxed{y = 2/5 x + 24/5}$$

15. Write the slope-intercept form of the equation:
$$-4x - 7y + 3 = -48$$

$$-7y = 4x - 51$$

$$\boxed{y = -\tfrac{4}{7}x + \tfrac{51}{7}}$$

16. Write the point-slope form of the equation of the line through the given point with the given slope:
$$(1, -3), m = -\frac{2}{3}$$

$$\boxed{(y+3) = -\tfrac{2}{3}(x-1)}$$

17. Write the equation of the line describe: through $(5, -3)$, parallel to $4y = -3x + 6$

$$m = -\tfrac{3}{4}$$
$$y = -\tfrac{3}{4}x + \tfrac{3}{2}$$

$$\boxed{y + 3 = -\tfrac{3}{4}(x-5)}$$
or
$$\boxed{y = -\tfrac{3}{4}x + \tfrac{3}{4}}$$

18. Write the equation of the line described: through $(5,7)$, perpendicular to $3x - 4y = -10$

$$m = -4/3$$
$$-4y = -3x - 10$$
$$y = \tfrac{3}{4}x + \tfrac{5}{2}$$

$$\boxed{y - 7 = -\tfrac{4}{3}(x-5)}$$
or
$$\boxed{y = -\tfrac{4}{3}x + \tfrac{41}{3}}$$

③

Domain

State the domain of the following. Show your work.

1. $f(x) = \frac{3x-5}{x+1}$

$x+1 \neq 0 \qquad (0,\infty-)$

$x \neq -1$

$(-\infty, -1) \cup (-1, \infty)$

2. $f(x) = \sqrt{\frac{3x-5}{x+1}}$

$x \neq -1 \qquad \frac{3x-5}{x+1} \geq 0 \qquad \begin{array}{ccc} + & DNE & + \\ \hline -1 & & 5/3 \end{array}$

$0 \geq 3x-5$

$5/3 \geq x$

$(-\infty, -1) \cup [5/3, \infty)$

3. $f(x) = \ln(x-7)$

$x-7 > 0$

$x > 7$

$(7, \infty)$

4. $f(x) = \ln\left(\frac{x}{x-1}\right)$

$x \neq 1 \qquad \frac{x}{x-1} > 0$

$x = 0 \qquad \begin{array}{ccc} + & - & + \\ \hline 0 & & 1 \end{array}$

$(-\infty, 0) \cup (1, \infty)$

5. $f(x) = \sqrt[3]{3x+2}$

$\mathbb{R}$

$(-\infty, \infty)$

6. $f(x) = \frac{x+1}{x^2+5x+4} = \frac{x+1}{(x+1)(x+4)} = \frac{1}{x+4}$

$x^2+5x+4 \neq 0 \qquad$ hole @ $x=-1$

$(x+4)(x+1)$

$x \neq -4 \quad x \neq -1$

$(-\infty, -4) \cup (-4, -1) \cup (-1, \infty)$

7. $f(x) = \ln\left(\frac{x+1}{x-3}\right)$

$x \neq 3 \qquad \frac{x+1}{x-3} > 0 \qquad \begin{array}{ccc} + & - & + \\ \hline -1 & & 3 \end{array}$

$0 > x+1$

$-1 > x$

$(-\infty, -1) \cup (3, \infty)$

8. $f(x) = \sqrt[4]{x^2-8x-33}$

$x^2-8x-33 \geq 0 \qquad \begin{array}{ccc} + & - & + \\ \hline -3 & & 11 \end{array}$

$(x+3)(x-11) \geq 0$

$(-\infty, -3] \cup [11, \infty)$

9. $f(x) = \frac{x}{x^2-9}$

$x^2-9 \neq 0$

$(x+3)(x-3) \neq 0$

$x \neq \pm 3$

$(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

10. $f(x) = \frac{1}{\sqrt{x-2}}$

$x-2 > 0$

$x > 2$

$(2, \infty)$

(4)

Domain

11. $f(x) = \sqrt{x+1}$

$x+1 \geq 0$

$x \geq -1$

$[-1, \infty)$

12. $f(x) = \frac{1}{x}$

$x \neq 0$

$(-\infty, 0) \cup (0, \infty)$

13. $f(x) = \frac{x+2}{2x-1}$

$2x-1 \neq 0$

$x \neq \frac{1}{2}$

$(-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$

14. $f(x) = |x-2|$

$\mathbb{R}$

$(-\infty, \infty)$

15. $f(x) = \sqrt{x^2 - 16}$

$x^2 - 16 \geq 0$

$(x+4)(x-4) \geq 0$

$(-\infty, -4] \cup [4, \infty)$

16. $f(x) = \log_2(4x - 8) + 3$

$4x - 8 > 0$

$x > 2$

$(2, \infty)$

17. $f(x) = \sqrt{2x-1}$

$2x - 1 \geq 0$

$x \geq \frac{1}{2}$

$[\frac{1}{2}, \infty)$

18. $f(x) = \frac{2}{x^2 - 1}$

$x^2 - 1 \neq 0$

$(x+1)(x-1) = 0$

$x \neq -1 \quad x \neq 1$

$(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

19. $f(x) = \ln(x^2 - 9)$

$x^2 - 9 > 0$

$(x+3)(x-3) > 0$
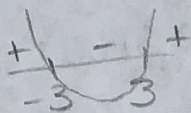
$(-\infty, -3) \cup (3, \infty)$

20. $f(x) = \frac{3x+2}{(3x+2)(x+1)} = \frac{1}{x+1}$

hole at $-\frac{2}{3}$

$x \neq -\frac{2}{3} \quad x \neq -1$

$(-\infty, -1) \cup (-1, -\frac{2}{3}) \cup (-\frac{2}{3}, \infty)$

⑤

Composition of Functions

Given the following functions $f(x)$ and $g(x)$, find the following.

Graph of f
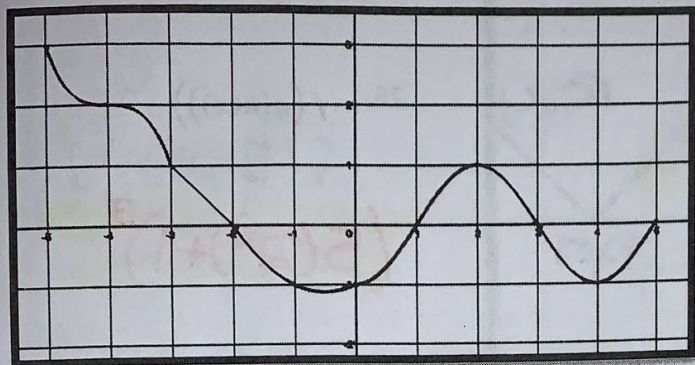
Graph of g
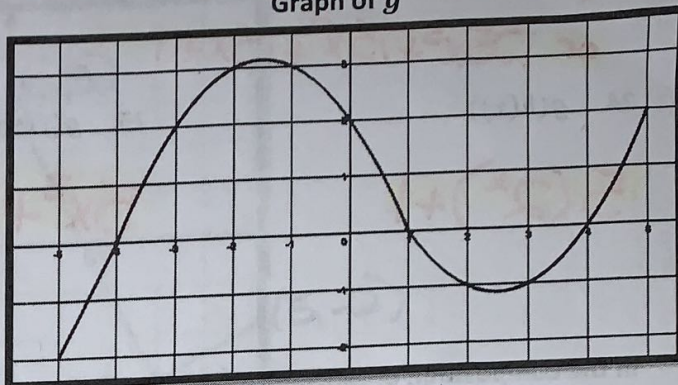
1. $f(g(-2)) = f(3) = 0$

2. $f(g(2)) = f(-1) = -1$

3. $g(f(-1)) = g(-1) = 3$

4. $f(f(5)) = f(0) = -1$

5. $g(g(-2)) = g(3) = -1$

6. All inputs for x for which $g(g(x)) = -1$

$x = -3, -2, -1, 0, 5$

Given the tables for functions f and g, find the following.

x	$f(x)$
-1	2
0	4
1	3
2	0
3	1
4	-1

x	$g(x)$
-1	3
0	4
1	2
2	6
3	2
4	-1

7. $f(g(3))$
$f(2) = 0$

8. $g(f(2))$
$g(0) = 4$

9. $f(f(4))$
$f(-1) = 2$

10. $g(g(4))$
$g(-1) = 3$

11. All inputs x such that $f(g(x)) = 2$
$g(x) = -1$
$x = 4$

Given the tables for functions f and g, find the following.

x	$f(x)$
-2	0
0	1
2	-1
4	2

x	$g(x)$
-1	-2
0	2
1	3
2	1

12. $f(f(-2))$
$f(0) = 1$

13. $f(g(0))$
$f(2) = -1$

14. $g(f(1))$
DNE

15. Domain of $f(g(x))$
$f(-2) = 0$
$f(2) = -1$ $x = -1, 0$
$f(3)$ DNE $\{-1, 0\}$

16. Domain of $g(f(x))$
$g(0) = 2$
$g(1) = 3$
$g(-1) = -2$
$g(2) = 1$
$\{-2, 0, 2, 4\}$

Let $f(x) = 2x - 3$, $g(x) = e^x$, and $h(x) = \ln x$. Find a formula for each function.

17. $f(f(x))$
$2(2x-3)-3$
$4x-6-3$
$4x-9$

18. $f(g(x))$
$2(e^x)-3$
$2e^x - 3$

19. $g(h(x))$
$e^{\ln x}$
x

20. $h(g(x))$
$\ln(e^x)$
x

Let $f(x) = x^3$, $g(x) = 5x + 1$, and $h(x) = 2^x$, find the following.

21. $f(g(x))$

$(5x+1)^3$ or $(5x+1)(25x^2+10x+1)$

or $125x^3+75x^2+15x+1$

22. $h(f(x))$

2^{x^3}

23. $h(g(x))$

2^{5x+1}

24. $g(h(x))$

$5(2^x)+1$

25. $g(f(x))$

$5x^3+1$

26. $f(g(h(x)))$

$f(5(2^x)+1)$

$(5(2^x)+1)^3$

In the composition of functions $f(g(x))$, g is known as the inner function and f is known as the outer function. For each problem below, specify the inner and outer function.

27. $y = (7x - 3)^2$

$f(x) = x^2$
$g(x) = 7x-3$

28. $y = \ln(x^2 + 4)$

$f(x) = \ln x$
$g(x) = x^2+4$

29. $y = 2^{3x-5}$

$f(x) = 2^x$
$g(x) = 3x-5$

30. $y = e^{\ln x}$

$f(x) = e^x$
$g(x) = \ln x$

31. $y = \sqrt{x}$

$f(x) = \sqrt{x}$
$g(x) = x$

32. $y = (\ln x)^2 + 1$

$f(x) = x^2+1$
$g(x) = \ln v$

33. $y = (x^2 - 4)^3$

$f(x) = x^3$
$g(x) = x^2-4$

34. $y = \frac{2}{(x-7)^2}$

$f(x) = \frac{2}{x^2}$
$g(x) = x-7$

35. $y = \sqrt[5]{x + 3}$

$f(x) = \sqrt[5]{x}$
$g(x) = x+3$

Answer each of the following.

36. If $f(g(x)) = \ln(x^2 + 1)$ and $f(x) = \ln x$, what is $g(x)$?

$g(x) = x^2+1$

37. If $f(g(x)) = \sqrt[3]{x - 3}$ and $f(x) = \sqrt[3]{x}$, what is $g(x)$?

$g(x) = x-3$

38. If $f(g(x)) = \sin(3x)$ and $f(x) = \sin x$, what is $g(x)$?

$g(x) = 3x$

Transformations

Draw a graph of each transformation. Let $f(x)$ be the function below. Label the critical points. Finally write a description of the transformation.
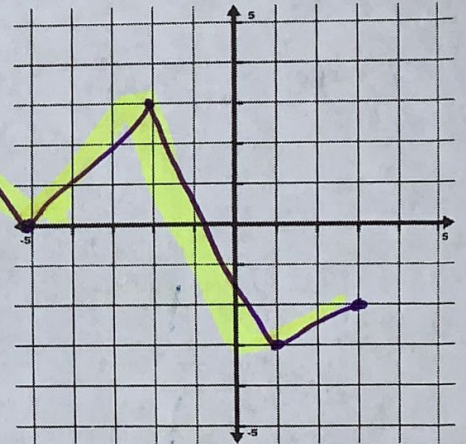

(-5,2) (0,3) (-3,0) (5,-2) (3,-3)
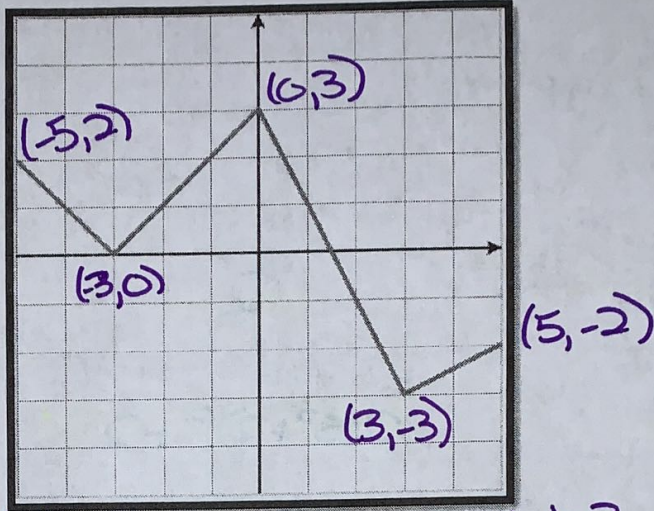
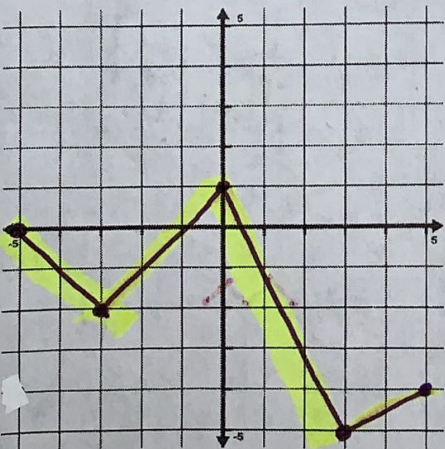1. $f(2x)$ — **shrink horizontally by 2 or multiply x by ½**
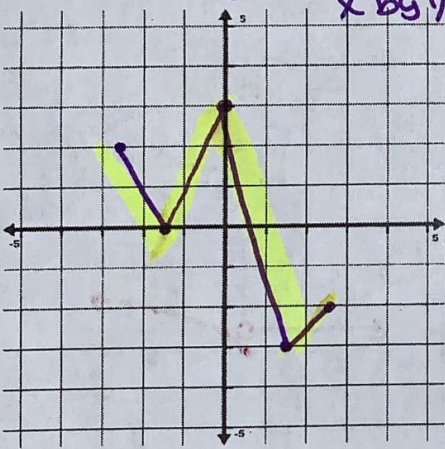
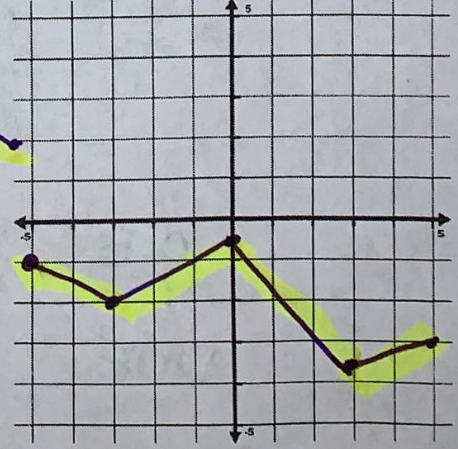2. $2f(x)$ — **stretch vertically by 2 or mult. y by 2**
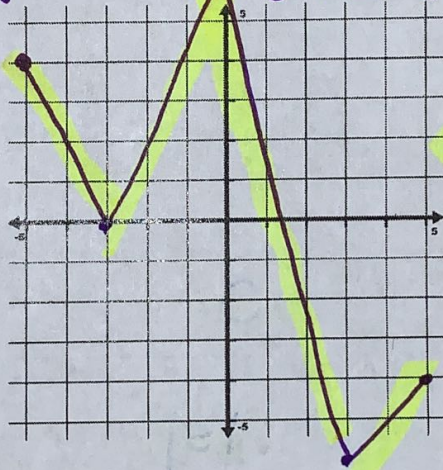
3. $f(x+2)$ — **left 2**

4. $f(x) - 2$ — **shift down 2**

5. $-(f(x-2))$ — **right 2 + reflect over x-axis (mult. y by -1)**
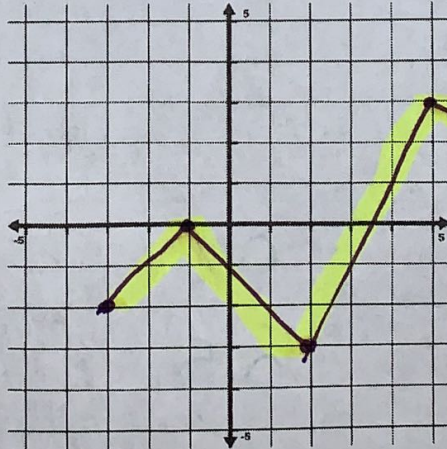
6. $\frac{1}{2}f(x) - 2$ — **shrink vertically by ½ + down 2**

8

Piecewise Functions

Piecewise functions

1. $f(x) = \begin{cases} x^2 + 2, & x \leq 1 \\ 2x^2 + 2, & x > 1 \end{cases}$

Domain: $\mathbb{R}$

Range: $[2, \infty)$

Find:

$f(-2)$	6	$f(0)$	2
$f(1)$	3	$f(s^2 + 2)$ will be ≥ 2 $2(s^2+2)^2 + 2$	
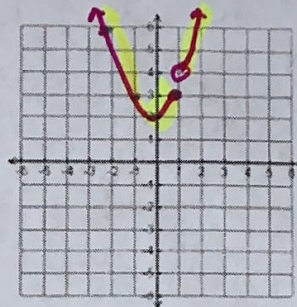
2. $f(x) = \begin{cases} 1, & x < 0 \\ \sqrt{x}, & x \geq 0 \end{cases}$

Domain: $\mathbb{R}$

Range: $[0, \infty)$

Find:

$f(-2)$	1	$f(0)$	0
$f(1)$	1	$f(s^2)$ will be ≥ 0 $\sqrt{s^2}$ $|s|$	
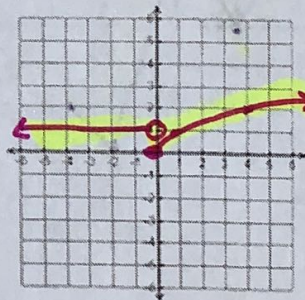
3. $f(x) = \begin{cases} x + 2, & -2 \leq x \leq -1 \\ 1 - \sqrt{1 - x^2}, & -1 < x < 1 \\ -x + 2, & 1 \leq x \leq 2 \end{cases}$

Domain: $[-2, 2]$

Range: $[0, 1]$

Find:

$f(-1.5)$	0.5	$f(0)$	0
$f(3)$	DnE	$f(1.5)$	0.5
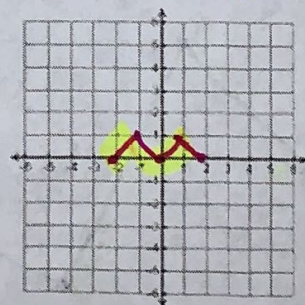
Piecewise Functions

4.

$$f(x) = \begin{cases} 4 - x^2, & x < 1 \\ \frac{3}{2}x + \frac{3}{2}, & 1 \le x \le 3 \\ -x + 2, & x > 3 \end{cases}$$

Domain: **ℝ**

Range: **$(-\infty, 6]$**

Find:

$f(5)$	$f(0)$
-3	**4**
$f(3)$	$f(2)$
6	**9/2**
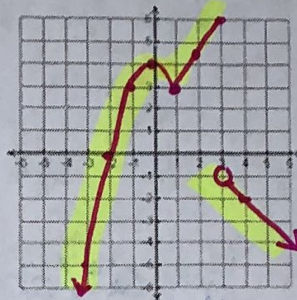
6.

$$f(x) = \begin{cases} x^2, & x < 0 \\ x^3, & 0 \le x \le 1 \\ 2x - 1, & x > 1 \end{cases}$$

Domain: **ℝ**

Range: **$[0, \infty)$**

$f(1)$	$f(0)$
1	**0**
$f(3)$	$f(-2)$
5	**4**
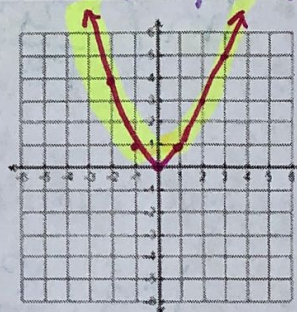
Find the formula for the following:

11.
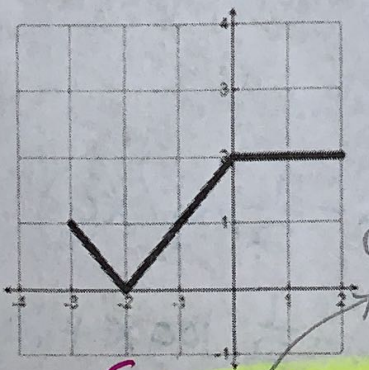
could also be
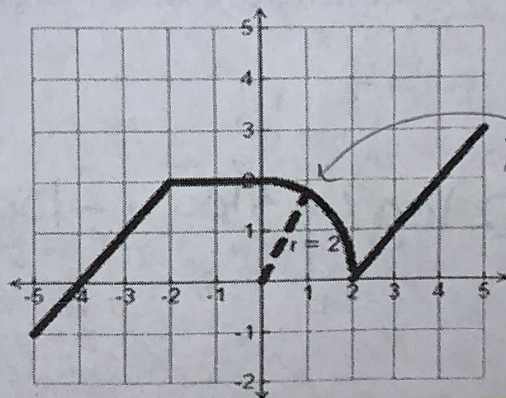$-x-2, \ -3 \le x \le -2$
$x+2, \ -2 < x \le 0$

$$f(x) = \begin{cases} |x+2|, & -3 \le x \le 0 \\ 2, & x > 0 \end{cases}$$

12.

$x^2 + y^2 = 4$
$y = \sqrt{4 - x^2}$

$$f(x) = \begin{cases} x+4, & -5 \le x \le -2 \\ 2, & -2 < x < 0 \\ \sqrt{4-x^2}, & 0 \le x \le 2 \\ x-2, & 2 < x \le 5 \end{cases}$$

(10)

Exponentials and Logarithms

Find the value of the following without using your calculator.

1. $3 \ln e + \ln \left(\frac{1}{e}\right)$

$$3(1) + \ln(e^{-1})$$
$$3 - 1 \cdot \ln e$$
$$3 - 1 = 2$$

2. $\ln e^2 + e^{-\ln e}$

$$2 \cdot \ln e + \frac{1}{e^{\ln e}}$$
$$2(1) + \frac{1}{e^1} = 2 + \frac{1}{e}$$

Simplify to a single ln or e expression, or to a single number.

3. $2 \ln a - 3 \ln b + \ln(ab)$

$$\ln \frac{a^2(ab)}{b^3} = \ln\left(\frac{a^3}{b^2}\right)$$

4. $\ln(e^2 \ln(e \ln e))$

$$\ln(e^2 \ln(e))$$
$$\ln e^2$$
$$2 \cdot \ln e = 2$$

Express each of the following as the logarithm of a single expression.

5. $2 \ln x + 4 \ln y - \ln 13$

$$\ln\left(\frac{x^2 y^4}{13}\right)$$

6. $\ln 7 + 5 \ln y - \frac{1}{2} \ln x$

$$\ln\left(\frac{7y^5}{x^{1/2}}\right) \text{ or } \ln\left(\frac{7y^5}{\sqrt{x}}\right)$$

7. $\log(x^2 - 16) - 3\log(x + 4) + 2 \log x$

$$\log\left(\frac{x^2(x^2 - 16)}{(x+4)^3}\right) = \log \frac{x^2(x+4)(x-4)}{(x+4)(x+4)^2} = \log\left(\frac{x^2(x-4)}{(x+4)^2}\right)$$

Use the properties of logarithms to express the following as an algebraic expression involving $\log x$, $\log y$, **and/or** $\log z$

8. $\log \frac{x^2 y^3}{z}$

$$2 \log x + 3 \log y - \log z$$

9. $\log \frac{z}{\sqrt{xy}} = \log \frac{z}{(xy)^{1/2}}$

$$\log z - \frac{1}{2} \log(xy)$$
$$\log z - \frac{1}{2}(\log x + \log y)$$

Exponentials and Logarithms

Solve for x.

10. $2^x + 5 = 17$

$$2^x = 12$$
$$\log_2 12 = x$$
$$x \approx 3.585$$

11. $5e^{x+1} = 27$

$$e^{x+1} = 27/5$$
$$\ln 27/5 = x+1$$
$$\ln 27/5 - 1 = x$$
$$x \approx .686$$

12. $3 \cdot 2^{2x+1} = 24$

$$2^{2x+1} = 8$$
$$\log_2 8 = 2x+1$$
$$3 = 2x+1$$
$$x = 1$$

13. $4e^{2x-3} - 5 = e$

$$e^{2x-3} = \frac{e+5}{4}$$
$$\ln\left(\frac{e+5}{4}\right) = 2x-3$$
$$x = \frac{\ln\left(\frac{e+5}{4}\right)+3}{2} \approx 1.829$$

14. $2P = Pe^{kx}$

$$\frac{2P}{P} = \frac{Pe^{kx}}{P}$$
$$2 = e^{kx}$$
$$\ln 2 = kx$$
$$x = \frac{\ln 2}{k}$$

15. $3(3^x) - 5x(3^x) + 2x^2(3^x) = 0$
hint: factor out GCF

$$(3^x)(3-5x+2x^2) = 0$$
$$(3^x)(2x^2-5x+3) = 0$$
$$(3^x)(2x-3)(x-1) = 0$$
$$3^x = 0 \quad 2x-3=0 \quad x-1=0$$
$$\text{no Sol.} \quad x = 3/2 \quad x = 1$$

16. $\log_3(x-6) + 4 = 6$

$$\log_3(x-6) = 2$$
$$3^2 = x-6$$
$$9 = x-6$$
$$x = 15$$

17. $\ln(x^2 + 2x) = \ln(6 + x)$

$$x^2 + 2x = 6+x$$
$$x^2 + x - 6 = 0$$
$$(x+3)(x-2) = 0$$
$$x = -3 \quad x = 2$$

18. $\log_3(\log_3(2x)) = 1$

$$3^1 = \log_3(2x)$$
$$3^3 = 2x$$
$$27 = 2x$$
$$x = 27/2$$

19. $\ln x + \ln(x + 1) = \ln 12$

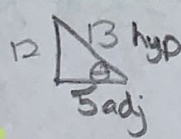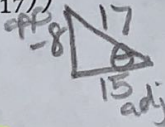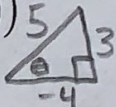$$\ln x(x+1) = \ln 12$$
$$x(x+1) = 12$$
$$x^2 + x - 12 = 0$$
$$(x+4)(x-3) = 0$$
$$x \neq -4 \quad x = 3$$

⑫

Inverse Trig and Solving Trig Equations

Evaluate each expression.

1. $\arcsin\left(\frac{\sqrt{3}}{2}\right)$ $\pi/3$	2. $\arccos(-1)$ π	3. $\tan^{-1}(-1)$ $-\frac{\pi}{4}$
4. $\sin^{-1}(-1)$ $-\pi/2$	5. $\arcsin\left(-\frac{1}{2}\right)$ $-\pi/6$	6. $\arctan(-\sqrt{3})$ $-\pi/3$
7. $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ $3\pi/4$	8. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ $-\pi/4$	9. $\tan^{-1}(0)$ 0
10. $\arcsin\left(-\frac{\sqrt{3}}{2}\right)$ $-\pi/3$	11. $\cos^{-1}\left(-\frac{1}{2}\right)$ $2\pi/3$	12. $\arccos\left(-\frac{\sqrt{3}}{2}\right)$ $5\pi/6$
13. $\sin^{-1}\left(\sin\frac{3\pi}{2}\right)$ $\sin^{-1}(-1)$ $-\pi/2$	14. $\cos^{-1}\left(\sin\left(-\frac{\pi}{6}\right)\right)$ $\cos^{-1}(-1/2)$ $2\pi/3$	15. $\sin^{-1}\left(\cos\left(-\frac{\pi}{6}\right)\right)$ $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ $2\pi/3$
16. $\cos\left(\sin^{-1}\frac{12}{13}\right)$ 12 $\boxed{}$ 13 hyp 5 adj $5/13$	17. $\tan\left(\sin^{-1}\left(-\frac{8}{17}\right)\right)$ 17 -8 15 adj $-8/15$	18. $\sin\left(\cos^{-1}\left(-\frac{4}{5}\right)\right)$ 5 $\boxed{}$ 3 -4 $3/5$

(13)

Solve for x, where $0 \le x < 2\pi$

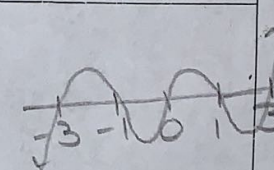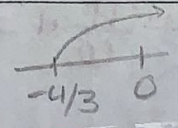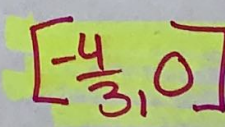| 22. $2\sin x - 1 = 0$	23. $2\cos x + 1 = 0$
$2\sin x = 1$ $\sin x = 1/2$ $x = \sin^{-1}\left(\frac{1}{2}\right)$ $x = \frac{\pi}{6} + \frac{5\pi}{6}$	$\cos x = -1/2$ $x = \cos^{-1}\left(-\frac{1}{2}\right)$ $x = \frac{2\pi}{3} + \frac{4\pi}{3}$
24. $4\sin x + 2\sqrt{3} = 0$	25. $2\tan x + \sqrt{3} = -\tan x$
$\sin x = -\dfrac{2\sqrt{3}}{4}$ $x = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ $x = \frac{4\pi}{3} + \frac{5\pi}{3}$	$3\tan x = -\sqrt{3}$ $x = \tan^{-1}\left(\frac{-\sqrt{3}}{3}\right)$ $x = \frac{5\pi}{6} + \frac{11\pi}{6}$
26. $2\cos^2 x = 3\cos x + 2$	27. $2\cos^2 x = \sin x + 1$
$2\cos^2 x - 3\cos x - 2 = 0$ $(2\cos x + 1)(\cos x - 2) = 0$ $\cos x = -\frac{1}{2} \qquad \cos x = 2$ $\qquad\qquad\qquad DnE$ $x = \cos^{-1}\left(-\frac{1}{2}\right)$ $x = 2\pi/3 + 4\pi/3$	$\checkmark$ pythagorean id. $\quad \sin^2 x + \cos^2 x = 1$ $\qquad\qquad\qquad\qquad \cos^2 x = 1 - \sin^2 x$ $2(1 - \sin^2 x) = \sin x + 1$ $2 - 2\sin^2 x = \sin x + 1$ $0 = 2\sin^2 x + \sin x - 1$ $0 = (2\sin x - 1)(\sin x + 1)$ $2\sin x - 1 = 0 \qquad \sin x + 1 = 0$ $\sin x = 1/2 \qquad \sin x = -1$ $x = \sin^{-1}\left(\frac{1}{2}\right) \qquad x = \sin^{-1}(-1)$ $x = \frac{\pi}{6} + \frac{5\pi}{6} \qquad x = \frac{3\pi}{2}$
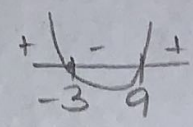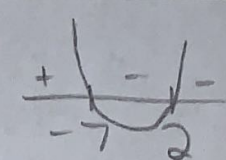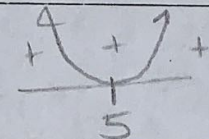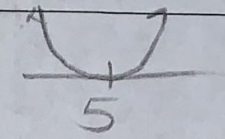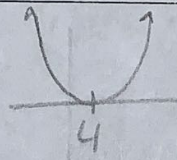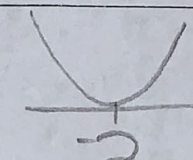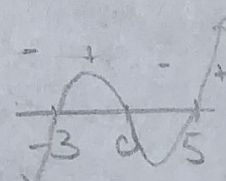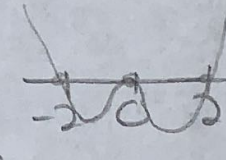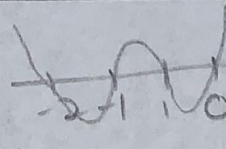
Inequalities

1. $x^2 - 6x - 30 > -3$

$x^2 - 6x - 27 > 0$

$(x-9)(x+3) > 0$

$(-\infty, -3] \cup [9, \infty)$

2. $x^2 + 5x + 6 < 20$

$x^2 + 5x - 14 < 0$

$(x+7)(x-2) < 0$

$(-7, 2)$

3. $x^2 - 10x + 25 > 0$

$(x-5)^2 > 0$

$(-\infty, 5) \cup (5, \infty)$

4. $x^2 - 10x + 25 \leq 0$

$(x-5)^2 \leq 0$

$\{5\}$

5. $2x^2 + 16 \leq x^2 + 8x$

$x^2 - 8x + 16 \leq 0$

$(x-4)^2 \leq 0$

$\{4\}$

6. $-x^2 \geq 4x + 4$

$0 \geq x^2 + 4x + 4$

$0 \geq (x+2)^2$

$(x+2)^2 \leq 0$

$\{-2\}$

7. $x^2(5-x)(x+3) < 0$

$x=0 \quad x=5 \quad x=-3$

$(-\infty, -3) \cup (0, 5)$

8. $x^4 < 4x^2$

$x^4 - 4x^2 < 0$

$x^2(x^2-4) \leq 0$

$x^2(x+2)(x-2) < 0$

$(-2, 0) \cup (0, 2)$

9. $x^4 - 5x^2 \geq -4$

$x^4 - 5x^2 + 4 \geq 0$

$(x^2-4)(x^2-1)$

$(x+2)(x-2)(x+1)(x-1) \geq 0$

$(-\infty, -2] \cup [-1, 1] \cup [2, \infty)$

10. $x^5 + 9x \leq 10x^3$

$x^5 - 10x^3 + 9x \leq 0$

$x(x^4 - 10x^2 + 9) \leq 0$

$x(x^2-9)(x^2-1) \leq 0$

$x(x+3)(x-3)(x+1)(x-1) \leq 0$

$(-\infty, -3] \cup [-1, 0] \cup [1, 3]$

11. $\sqrt{3x+4} \geq 2$

$3x + 4 \geq 4$

$x \geq 0$

$3x + 4 \geq 0$

$x \geq -\frac{4}{3}$

$[0, \infty)$

12. $\sqrt{3x+4} \leq 2$

$3x + 4 \leq 4$

$x \leq 0$

$3x + 4 \geq 0$

$x \geq -4/3$

$\left[-\frac{4}{3}, 0\right]$

13. $\sqrt{2x+1} + 3 < 0$
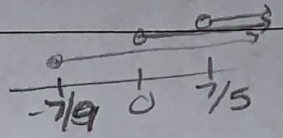
$\sqrt{2x+1} < -3$

* Always positive

~~No Solution~~

$2x+1 \geq 0$

$x \geq -1/2$

14. $\sqrt{9x+7} < \sqrt{14x}$

$9x+7 < 14x$

$-5x < -7$

$x > 7/5$

$9x+7 \geq 0 \qquad 14x > 0$

$x \geq -\frac{7}{9} \qquad x > 0$

$(7/5, \infty)$

15. $\frac{x^2 - 11x + 28}{x+3} \geq 0 \qquad \frac{(x-7)(x-4)}{x+3} \geq 0$
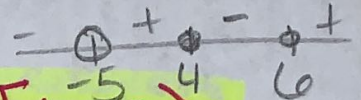
$x \neq -3$
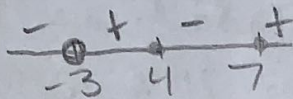
$(-3, 4] \cup [7, \infty)$

16. $\frac{x^2 - 10x + 24}{x+5} \geq 0 \qquad \frac{(x-4)(x-6)}{x+5} \geq 0$

$x \neq -5$

$(-5, 4] \cup [6, \infty)$

17. $32^{5x+2} \geq 16^{5x}$

$2^{5(5x+2)} \geq 2^{4(5x)}$

$25x + 10 \geq 20x$

$5x \geq -10$

$x \geq -2$

$[-2, \infty)$

18. $2^{4x-5} > \left(\frac{1}{2}\right)^{x-5}$

$2^{4x-5} > 2^{-1(x-5)}$

$4x - 5 > -x + 5$

$5x > 10$

$x > 2$

$(2, \infty)$

19. $\log_3 x < 2$

$x < 3^2$

$x < 9$

$x > 0$

$(0, 9)$

20. $\log_5 x \leq 1 \qquad x > 0$

$x \leq 5^1$

$x \leq 5$

$(0, 5]$

21. $\log_4 x \leq \frac{3}{2}$

$x > 0 \qquad x \leq 4^{3/2}$

$x \leq \sqrt[3]{4^3}$

$x \leq 8$

$(0, 8]$

22. $\log_5 x - 7 \geq -6$

$\log_5 x \geq 1 \qquad x > 0$

$x \geq 5$

$[5, \infty)$

23. $2\log_4(x+3) - 1 > 5$

$\log_4(x+3) > 3$

$x + 3 > 4^3$

$x + 3 > 0 \qquad x > 61$

$x > -3$

$(61, \infty)$

24. $\log_3 4x + 7 \leq 12 \qquad x > 0$

$\log_3 4x \leq 5$

$4x \leq 3^5$

$4x \leq 243$

$x \leq \frac{243}{4}$

$\left[0, \frac{243}{4}\right]$

Unit 1: Pre-Calculus Practice Test

Answer the questions below. Please be sure to highlight your final answer. To earn full credit you must show all of your work.

1. Factor Completely: $x^2 - x - 6$

$$(x-3)(x+2)$$

2. Factor Completely: $x^4 - 17x^2 + 16$

$$(x^2-16)(x^2-1)$$
$$(x+4)(x-4)(x+1)(x-1)$$

3. Factor Completely: $6x^3 + 6$

$6(x^3+1)$ $a=x$
$b=1$

$$6(x+1)(x^2-x+1)$$

4. Factor Completely: $6x^2 - 7x - 3$

$$(3x+1)(2x-3)$$

5. Find the equation of the line that passes through $(2, -3)$ and is perpendicular to $4x - 5y = 18$

$4x-5y=18$
$y=\frac{4}{5}x-\frac{18}{5}$
$m=\frac{4}{5}$

$Lm=-5/4$
$(2,-3)$
$x_1 \ y_1$

$$y+3=-\frac{5}{4}(x-2)$$

6. Find the average rate of change of $\frac{2x+3}{x^2+2}$ on the interval $[-1,1]$

$x_1 \ y_1$
$f(-1)=\frac{2(-1)+3}{(-1)^2+2}=\frac{1}{3}$ $f(1)=\frac{2(1)+3}{(1)^2+2}=\frac{5}{3}$

$(-1,\frac{1}{3})$ $(1,\frac{5}{3})$
$x_1 \ y_1$ $x_2 \ y_2$

$2/3$

$m=\frac{5/3-1/3}{1+1}=\frac{4/3}{2}=\frac{4}{3}\div\frac{2}{1}=\frac{4}{3}\cdot\frac{1}{2}=\frac{4}{6}$

7. State the domain: $f(x) = \ln(x^2 - 8x + 12)$

$x^2-8x+12=0$
$(x-6)(x-2)=0$ can't be neg. or 0
$x=6 \ x=2$

$$(-\infty,2) \cup (6,\infty)$$

8. State the domain: $f(x) = \frac{x+5}{x^2-6x+8}$

$x^2-6x+8 \neq 0$ can't $=0$
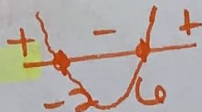$(x-2)(x-4)\neq 0$ in denom.
$x\neq 2 \ x\neq 4$

$$(-\infty,2)\cup(2,4)\cup(4,\infty)$$

9. State the domain: $f(x) = \sqrt[6]{x^2 - 4x - 12}$

$x^2-4x-12>0$
$(x-6)(x+2)=0$ Can't be negative. It can $=0$
$x=6 \ x=-2$

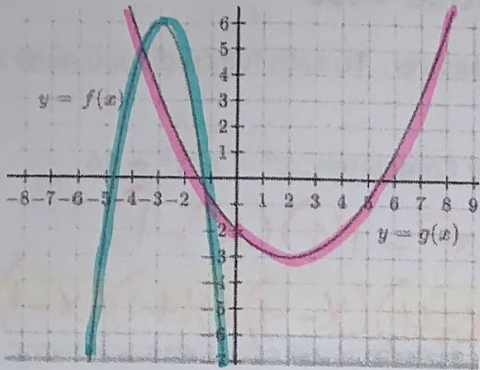$$(-\infty,-2] \cup [6,\infty)$$

10. Describe the transformations from the parent function:

a. $f(x) = -3(x-5)^3$ $-1.3 \ h=5$ Reflect over x-axis Vertical stretch by 3 Right 5

b. $f(x) = -\frac{1}{2}x^2 + 3$ Reflect over x-axis Vertical shrink by ½ Up 3

c. $f(x) = \sqrt{x+4} - 9$ $h=-4 \ K=-9$ Left 4 Down 9

F19

11.

$y = f(x)$

$y = g(x)$

Evaluate:

a. $f(g(-3))$

$f(2)$

DNE

b. $g(f(-4))$

$g(3)$

about -2.5

12. Perform the indicated operation:

$f(x) = 3(2x - 1)^2 + 1$ and $g(x) = x + 2$

Find $(g \circ f)(x)$

$3(2x-1)^2 + 1 + 2$
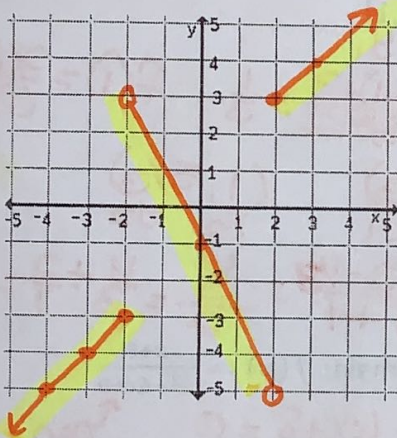
$3(2x-1)^2 + 3$

$3(2x-1)(2x-1) + 3$

$3(4x^2 - 4x + 1) + 3$

$12x^2 - 12x + 3 + 3$

$12x^2 - 12x + 6$

13. Graph the function:

$$f(x) = \begin{cases} -1 + x, & x \le -2 \\ -2x - 1, & -2 < x < 2 \\ |x| + 1, & x \ge 2 \end{cases}$$

$y = -1 + x$

x	y
-2	-3
-3	-4
-4	-5

$y = -2x - 1$

x	y
0	-2
0	-1
0	-5

$y = |x| + 1$

x	y
2	3
3	4
4	5

14. Evaluate the following:

$$f(x) = \begin{cases} 2 \cdot 3^x + 1, & x \le -2 \\ \sqrt{8x} + 1, & -2 < x \le 2 \\ -|x + 1|, & x > 4 \end{cases}$$
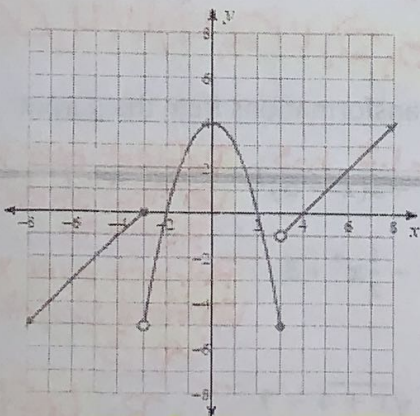
a. $f(-2) = 2 \cdot 3^{-2} + 1$

$2(\frac{1}{9}) + 1 = \frac{11}{9}$

b. $f(2)$ $\sqrt{8 \cdot 2} + 1 = \sqrt{16} + 1 = 5$

c. $f(6)$ $-|6 + 1| = -7$

d. $f(0)$ $\sqrt{8 \cdot 0} + 1 = 1$

$\sqrt{0} + 1$

15. Write the equation for the graph:

$f(x) = \begin{cases} x + 3, & x \le -3 \\ -x^2 + 4, & -3 < x \le 3 \\ x - 4, & x > 3 \end{cases}$

16. Simplify to a single expression:

a. $\ln(\ln e(e^7(5 \ln e^{-x})$

$\ln e^7 \cdot -5x \ln e$

$7 \cdot -5x = -35x$

b. $\log (5x) - 2 \log 3 + \frac{1}{2}\log (x-3)$

$\log \dfrac{5x\sqrt{x-3}}{3^2} = \log \dfrac{5x\sqrt{x-3}}{9}$

17. Solve: $16^{2x-1} = 8^{x-5}$

$2^{4(2x-1)} = 2^{3(x-5)}$

$4(2x-1) = 3(x-5)$

$8x-4 = 3x-15$

$5x = -11$

$\boxed{x = -11/5}$

18. Solve: $\ln(4x^2 - 15) \oplus \ln(3) = \ln(6)$

$\ln(4x^2-15)(3) = \ln 6$

$12x^2 - 45 = 6$

$\dfrac{12x^2}{12} = \dfrac{51}{12}$ reduce fraction

$\sqrt{x^2} = \sqrt{\dfrac{17}{4}}$ $\boxed{x = \pm\dfrac{\sqrt{17}}{2}}$

19. Solve: $3 \cdot 6^{x-1} - 8 = 22$

$3 \cdot 6^{x-1} = 30$

$6^{x-1} = 10$

$\text{calc} \to \dfrac{\log_6 10 = x - 1}{+1 \qquad +1}$

$\boxed{x \approx 2.285}$

20. Solve: $\log_2(7x - 5) + 2 = 6$

$\log_2(7x-5) = 4$

$2^4 = 7x - 5$

$16 = 7x - 5$

$21 = 7x$

$\boxed{x = 3}$

21. Find the exact value of $\sec\left(\dfrac{41\pi}{6}\right)$

$\dfrac{41\pi}{6} - 2\pi = \dfrac{29\pi}{6} - 2\pi = \dfrac{17\pi}{6} - 2\pi = \dfrac{5\pi}{6}$

$\sec\left(\dfrac{5\pi}{6}\right) \to \cos\dfrac{5\pi}{6} = -\dfrac{\sqrt{3}}{2}$

$\dfrac{-2}{\sqrt{3}}\dfrac{\sqrt{3}}{\sqrt{3}} = \boxed{\dfrac{-2\sqrt{3}}{3}}$

22. Find the exact value of $\sin\left(-\dfrac{14\pi}{3}\right)$

$-\dfrac{14\pi}{3} + 2\pi = -\dfrac{8\pi}{3} + 2\pi = -\dfrac{2\pi}{3} + 2\pi = \dfrac{4\pi}{3}$

$\sin\left(\dfrac{4\pi}{3}\right) = \boxed{-\dfrac{\sqrt{3}}{2}}$

23 Find the exact value of $\cot\left(\dfrac{11\pi}{6}\right)$

$\dfrac{11\pi}{6} \to \left(\dfrac{\sqrt{3}}{2}, -\dfrac{1}{2}\right)$ $-\sqrt{3}$
$\qquad\quad \cos \quad \sin$

$\dfrac{\sqrt{3}}{2} \div -\dfrac{1}{2} = \dfrac{\sqrt{3}}{2} \cdot -2 = -\sqrt{3}$

24. Find the exact value of $\csc(-3\pi)$

$\dfrac{+2\pi}{-\pi} + 2\pi = \pi$

$\csc(\pi)$

$\pi \to (-1, 0)$ $\sin\pi = \dfrac{0}{1}$
$\quad\;\; \cos \;\, \sin$ $\csc\pi = \dfrac{1}{0}$

$\boxed{\text{undefined}}$

25. Solve: $6\cos\theta - 3 = 0$

$6\cos\theta = 3$

$\cos\theta = \dfrac{1}{2}$

$\theta = \cos^{-1}\left(\dfrac{1}{2}\right)$

$\boxed{\theta = \dfrac{\pi}{3} + \dfrac{5\pi}{3}}$

26. Solve: $2\sin^2\theta - \sin\theta = 1$

$2\sin^2\theta - \sin\theta - 1 = 0$

$(2\sin\theta + 1)(\sin\theta - 1) = 0$

$2\sin\theta + 1 = 0 \qquad \sin\theta - 1 = 0$

$\sin\theta = -\dfrac{1}{2} \qquad\quad \sin\theta = 1$

$\theta = \sin^{-1}\left(-\dfrac{1}{2}\right) \qquad \theta = \sin^{-1}(1)$

$\boxed{\theta = \dfrac{7\pi}{6}, \dfrac{11\pi}{6}, \dfrac{\pi}{2}}$