

# Factoring

Always look for GCF 1st!

1.  $12x^3 + 16$
2.  $9x^3 - 27x^2 + 36x$
3.  $2x + 5$

$$4(3x^3 + 4)$$
$$9x(x^2 - 3x + 4)$$

not factorable

Factoring quadratic trinomials  $ax^2 + bx + c$

1.  $x^2 + 10x + 24$   
 $(x+6)(x+4)$

2.  $x^2 - 5x - 14$   
 $(x-7)(x+2)$

3.  $5x^2 + 35x + 60 = 5(x^2 + 7x + 12)$   
 $5(x+3)(x+4)$

4.  $2x^2 + 11x + 12$

$$\begin{array}{r|l} 24x^2 & 11x \\ \hline 8x \cdot 3x & 11x \checkmark \end{array}$$

$$2x^2 + 8x + 3x + 12$$
$$(2x^2 + 8x) + (3x + 12)$$
$$2x(x+4) + 3(x+4)$$

identical

$$(2x+3)(x+4)$$

5.  $8x^2 + 2x - 3$   
 $(4x+3)(2x-1)$

6.  $-5x^2 - 7x + 6$   
 $-1(5x^2 + 7x - 6)$   
 $-1(5x-3)(x+2)$

$a \neq 1$  so Guess + Check  
or rewrite middle term  
to use grouping method

- multiply  $a+c$
- determine 2 #s that mult. to " $ac$ " + add to " $b$ "
- rewrite so you have 4 terms + use grouping method
  - group 1st 2 terms + last 2 terms
  - GCF each group
  - Rewrite (GCF)(match)



## 2 Special Binomials : Difference of Squares

$$a^2 - b^2$$

2 terms, subtraction, +  
both terms are perfect squares  
(variables have even exponents)

Take square root of each term + write as  
(a+b)(a-b)

1.  $x^2 - 25$   
 $(x+5)(x-5)$

2.  $4x^2 - 49y^2$   
 $(2x-7y)(2x+7y)$

3.  $9x^2 + 4$   
not factorable

4.  $169v^8 - 1$   
 $(13v^4+1)(13v^4-1)$

## Special Binomials : Sum + Difference of Cubes

$$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$$

Same sign

Opp sign

All plus always

Fill in signs with SOAP

1.  $x^3 + 125$   
 $a = \sqrt[3]{x^3} = x$   $b = \sqrt[3]{125} = 5$   
 $(x+5)(x^2-5x+25)$

2.  $8x^3 - 27$   
 $a = 2x$   $b = 3$   
 $(2x-3)(4x^2+6x+9)$

3.  $2x^3 + 128y^3$   
 $2(x^3 + 64y^3)$   
 $2(x+4y)(x^2-4xy+16y^2)$

4.  $x^6 - y^6$   
 $a = x^2$   $b = y^2$   
 $(x^2-y^2)(x^4+x^2y^2+y^4)$   
 $(x+y)(x-y)(x^4+x^2y^2+y^4)$   
perfect squares + cubes

## 4 Terms : Grouping Method

1.  $2x^3 - 8x^2 - 3x + 12$   
 $(2x^3 - 8x^2) + (-3x + 12)$   
 $2x^2(x-4) - 3(x-4)$   
 $(2x^2-3)(x-4)$

2.  $28mn + 70m + 2n + 5$   
 $(28mn + 70m) + (2n + 5)$   
 $14m(2n + 5) + 1(2n + 5)$   
 $(14m + 1)(2n + 5)$



## 4 Average Rate of Change

The average rate of change between any 2 points on the graph of  $f$  is the slope of the line between those 2 points.

The average rate of change on the interval  $[x_1, x_2]$  is

$$m = \frac{f(x_2) - f(x_1)}{x_2 - x_1} \quad \text{or} \quad \frac{y_2 - y_1}{x_2 - x_1}$$

1. Find the avg. rate of change of  $f(x) = -x^3 + 3x$  on each interval:

a.  $[-2, -1]$   $f(-2) = -(-2)^3 + 3(-2) = 2$   $(-2, 2)$   
 $f(-1) = -(-1)^3 + 3(-1) = -2$   $(-1, -2)$

$$m = \frac{2 - (-2)}{-2 - (-1)} = -4$$

b.  $[0, 1]$   $f(0) = 0$   $f(1) = 2$   $m = \frac{2-0}{1-0} = 2$   
 $(0, 0)$   $(1, 2)$   
 $x_1, y_1$   $x_2, y_2$

2. Find the avg. rate of change of  $f(x) = \frac{x+5}{x-4}$  on the interval  $[-6, 2]$

$$f(-6) = \frac{-6+5}{-6-4} = \frac{1}{10} \quad f(2) = \frac{2+5}{2-4} = \frac{-7}{2} \quad m = \frac{-7/2 - 1/10}{2 - (-6)}$$

$(-6, 1/10)$   $(2, -7/2)$   
 $x_1, y_1$   $x_2, y_2$

$$m = \frac{-36/10}{8}$$

$$m = \frac{-18 \cdot 1}{8 \cdot 8} = \frac{-9}{20}$$

3.  $f(x) = \sqrt{x+8}$   $[-4, 4]$

$$f(-4) = 2 \quad f(4) = 2\sqrt{3}$$

$(-4, 2)$   $(4, 2\sqrt{3})$   
 $x_1, y_1$   $x_2, y_2$

$$m = \frac{2\sqrt{3} - 2}{8}$$

$$m = \frac{\sqrt{3} - 1}{4}$$



# Writing an Equation of a Line

Slope-Intercept Form: Given the slope ( $m$ ) + the y-intercept ( $b$ )  $y = mx + b$

Point-Slope Form: Given the slope ( $m$ ) + a point ( $x_1, y_1$ )  $y - y_1 = m(x - x_1)$

Standard Form:  $Ax + By = C$   $A, B, +C$  are integers +  $A$  is positive

1. Write the equation of the line that passes through  $(2, 3)$  + has a slope of  $-\frac{1}{2}$ .

$$y - 3 = -\frac{1}{2}(x - 2) \quad \text{Pt. Slope Form}$$

$$y - 3 = -\frac{1}{2}x + 1$$

$$y = -\frac{1}{2}x + 4 \quad \text{Slope-Intercept Form}$$

$$\frac{1}{2}x + y = 4$$

$$x + 2y = 8 \quad \text{Standard Form}$$

2. Passes through  $(3, 2)$  + is perpendicular to  $y = -3x + 2$   
 $\perp m = \frac{1}{3}$   $x_1, y_1$   $m = -3$

$$y - 2 = \frac{1}{3}(x - 3) \quad \text{or} \quad y = \frac{1}{3}x + 1$$

3. Passes through  $(3, 2)$  + parallel to  $y = -3x + 2$

$$y - 2 = -3(x - 3) \quad \text{or} \quad y = -3x + 11$$

4. Passes through  $(-2, -1)$  and  $(3, 4)$   
 $x_1, y_1$   $x_2, y_2$

$$m = \frac{4 - (-1)}{3 - (-2)} = \frac{5}{5} = 1$$

$$y - 4 = 1(x - 3)$$

$$y = x + 1$$

5. Find the standard form of the equation of the line through  $(2, -1)$  + is perpendicular to the line  $2x - 3y = 5$

$$y + 1 = -\frac{3}{2}(x - 2)$$

$$y + 1 = -\frac{3}{2}x + 3$$

$$y = -\frac{3}{2}x + 2$$

$$\frac{3}{2}x + y = 2$$

$$3x + 2y = 4$$

$$-3y = -2x + 5$$

$$y = \frac{2}{3}x - \frac{5}{3}$$



# Domain of Functions

Domain - the set of all possible x-values

- the denominator cannot = 0
- the number under a square (or even) root can't be negative
- you can't take the log (or ln) of a negative # or 0

Find the domain of the following functions:

1.  $f(x) = \sqrt{x-1}$      $x-1 \geq 0$      $[1, \infty)$   
 $x \geq 1$

2.  $f(x) = \sqrt[3]{x+7}$      $(-\infty, \infty)$

3.  $f(x) = \sqrt{x^2+2x-24}$      $x^2+2x-24 \geq 0$      $(-\infty, -6) \cup [4, \infty)$   
 $(x+6)(x-4) \geq 0$      $x = -6 \neq 4$  x-int.  
← above x-axis +    above below above

4.  $f(x) = \frac{x+4}{3x-7}$      $3x-7 \neq 0$      $(-\infty, 7/3) \cup (7/3, \infty)$   
 $x \neq 7/3$

5.  $f(x) = \frac{4x-8}{x^2+3x+10} = \frac{4(x-2)}{(x-2)(x+5)}$      $x \neq 2$  +  $x+5 \neq 0$      $x \neq -5$   
← creates a hole:  
 $(-\infty, -5) \cup (-5, 2) \cup (2, \infty)$

6.  $f(x) = \frac{1}{\sqrt{x-1}}$      $x-1 > 0$      $(1, \infty)$   
 $x > 1$

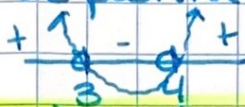
7.  $f(x) = \sqrt{\frac{2x+5}{x+8}}$      $x+8 \neq 0$      $\frac{2x+5}{x+8} \geq 0$      $(-\infty, -8) \cup (-8, -5/2] \cup [-5/2, \infty)$   
 $x \neq -8$      $2x+5 \geq 0$      $x \geq -5/2$



8.  $f(x) = \sqrt{9-x^2}$   $9-x^2 \geq 0$   
 $(3+x)(3-x) \geq 0$   
 $x = -3$   $x = 3$



9.  $f(x) = \ln(x^2 - 4x + 3)$  \* must be positive  
 $(x-3)(x-4) > 0$   
 $x = 3$   $x = 4$



$(-\infty, 3) \cup (4, \infty)$

10.  $f(x) = \log_5(2x+3) - 4$   
 $2x+3 > 0$   
 $x > -3/2$

$(-3/2, \infty)$

11.  $f(x) = \log_2\left(\frac{2x-1}{x-6}\right)$   
 $x \neq 6$

$\frac{2x-1}{x-6} > 0$   
 $2x-1 > 0$   
 $x > 1/2$



$(-\infty, 1/2) \cup (1/2, 6) \cup (6, \infty)$



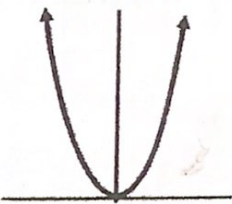
# Transformations of Functions

$f(x) = a \text{ number}$



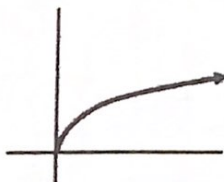
Domain:  $(-\infty, \infty)$   
Range: the number

Parabola:  $f(x) = x^2$



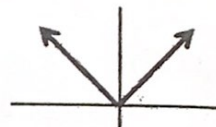
Domain:  $(-\infty, \infty)$   
Range:  $[0, \infty)$

$f(x) = \sqrt{x}$



Domain:  $[0, \infty)$   
Range:  $[0, \infty)$

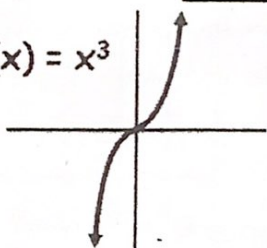
$f(x) = |x|$



Domain:  $(-\infty, \infty)$   
Range:  $[0, \infty)$

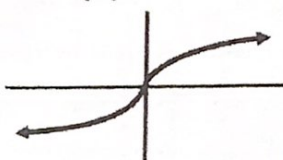
## Library Function Graphs

$f(x) = x^3$



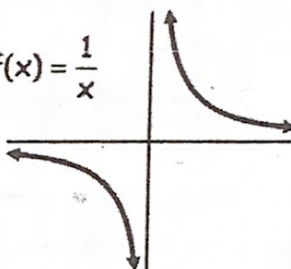
Domain:  $(-\infty, \infty)$   
Range:  $(-\infty, \infty)$

$f(x) = \sqrt[3]{x}$



Domain:  $(-\infty, \infty)$   
Range:  $(-\infty, \infty)$

$f(x) = \frac{1}{x}$



Domain:  $(-\infty, 0) \cup (0, \infty)$   
Range:  $(-\infty, 0) \cup (0, \infty)$

Transformation	Function	Description
Horizontal Shift <i>*opposite sign of h</i>	$f(x + h)$	Shift left $h$ units
	$f(x - h)$	Shift right $h$ units
Vertical Shift	$f(x) + k$	Shift up $k$ units
	$f(x) - k$	Shift down $k$ units
Reflection	$-f(x)$	Reflect across x-axis
	$f(-x)$	Reflect across y-axis
Vertical Stretch/Compress	$a f(x), a > 1$	Stretch vertically by a factor of $a$
	$a f(x), 0 < a < 1$	Compress vertically by a factor of $a$
Horizontal Stretch/Compress <i>multiply by reciprocal</i>	$f(ax), a > 1$	Compress horizontally by a factor of $\frac{1}{a}$
	$f(ax), 0 < a < 1$	Stretch horizontally by a factor of $\frac{1}{a}$



cont...

Describe the transformation + sketch the graph.

1.  $g(x) = \frac{2}{3}x^2 - 1$     $a = \frac{2}{3}$     $h = 0$     $k = -1$

vertical shrink by  $\frac{2}{3}$  + shift down 1

2.  $g(x) = 2|x-1|$     $a = 2$     $h = 1$     $k = 0$

vertical stretch by 2 + right 1

3.  $g(x) = -2(x+1)^2 + 3$     $a = -2$     $h = -1$     $k = 3$

reflect over x-axis, stretch vert. by 2, left 1, + up 3

4.  $g(x) = -3x - 2$     $a = -3$     $h = 0$     $k = -2$

reflect over x-axis, stretch vert. by 3, + down 2

Write the equations described...

1. Absolute value - vertical shift up 5, horizontal shift right 3.

$$g(x) = |x-3| + 5$$

2. Linear - vertical compression by  $\frac{2}{5}$  + shift up 2.

$$g(x) = \frac{2}{5}x + 2$$

3. Quadratic - vertical stretch by 5, horizontal shift left 8, + reflected over the x-axis.

$$g(x) = -5(x+8)^2$$



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# Compositions of Functions

Substituting a function or its value into another function.

Second  
 $f(g(x))$   
 First  
 (inside parentheses always first)

OR

$f \circ g(x)$

$$f(g(x)) = f \circ g(x)$$

$$g(f(x)) = g \circ f(x)$$

Given the table, evaluate the following:

x	-2	-1	0	1	2	3
f(x)	-3	-2	1	4	-1	0
g(x)	-2	0	1	3	-1	2

$$1. f(-1) = -2$$

$$2. g(2) = -1$$

$$3. f(-3) = \text{DNE}$$

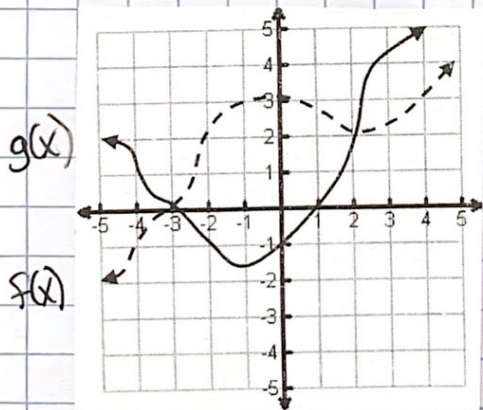
$$4. g^{-1}(0) \text{ means inverse, so what } g(x) \text{ value makes } g(x) = 0 \text{ so } x = -1$$

$$5. f^{-1}(-2) \text{ } f(x) = -2 \text{ so } x = -1$$

$$6. f(g(-1)) = f(0) = 1$$

$$7. (f \circ g)(3) = f(g(3)) = f(2) = -1$$

$$8. (g \circ g)(1) = g(g(1)) = g(3) = 2$$



Given the graph of  $f(x)$  +  $g(x)$  evaluate the following:

$$1. (f \circ g)(1) = f(g(1)) = f(0) = 3$$

$$2. (g \circ f)(-2) = g(f(-2)) = g(-2) = 2$$

$$3. (f \circ f)(-3) = f(f(-3)) = f(0) = 3$$

$$4. (g \circ g)(-4) = g(g(-4)) = g(0) = 3$$

Given  $f(x) = 3x^2 - 2x$   $g(x) = \ln x$   $h(x) = e^x$   $j(x) = x + 4$

$$1. f(j(x)) = 3(x+4)^2 - 2(x+4) = 3(x^2 + 8x + 16) - 2x - 8 = 3x^2 + 22x + 40$$

$$2. (g \circ h)(x) = \ln(e^x) = x$$

$$3. h(j(g(x))) = e^{\ln(x+4)} = x+4$$



# Piecewise Functions

A function that is defined using 2 or more equations for different intervals of the domain

Evaluate given  $f(x) = \begin{cases} \sqrt{20+x}, & \text{if } -8 < x < -1 \\ 3x^2 - 2x, & \text{if } -1 \leq x \leq 16 \\ 15 - 2x, & \text{if } x > 16 \end{cases}$

a)  $f(16)$

$3(16)^2 - 2(16)$   
 $736$

b)  $f(-2)$

$\sqrt{20-2} = \sqrt{18}$   
 $3\sqrt{2}$

c)  $f(28)$

$15 - 2(28)$   
 $51$

Graph the following:

2.  $f(x) = \begin{cases} -3x+2, & x \leq 2 \\ \frac{1}{2}x^2 - 4, & x > 2 \end{cases}$

$y = -3x+2$

x	y
2	-4
1	-1
0	2

$y = \frac{1}{2}x^2 - 4$

x	y
2	-2
3	1/2
4	4



3.  $f(x) = \begin{cases} |x+1|, & \text{if } x < 1 \\ \sqrt{x+3}, & \text{if } 1 \leq x \leq 6 \\ \frac{2}{3}x - 5, & \text{if } x > 6 \end{cases}$

$y = |x+1|$

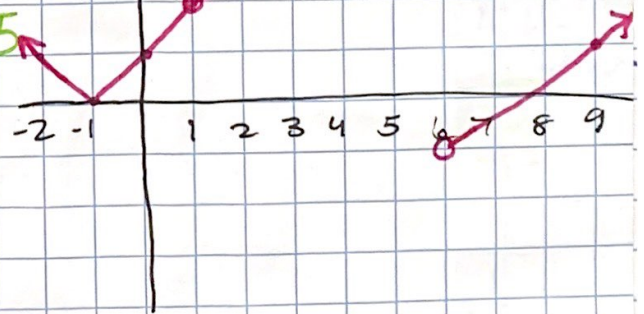
x	y
0	1
0	1
-1	0
-2	1

$y = \sqrt{x+3}$

x	y
1	2
2	$\sqrt{5}$
6	$\sqrt{9}$
6	3

$y = \frac{2}{3}x - 5$

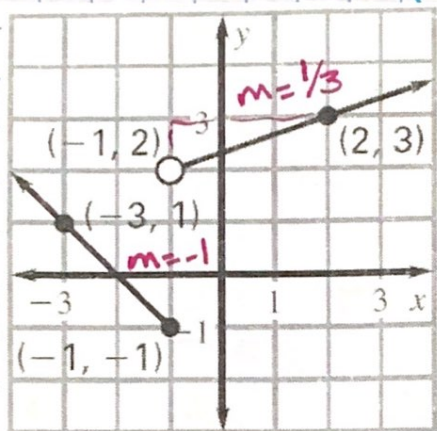
x	y
6	-1
7	-1/3
9	1





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4. Write the equation of the piecewise function



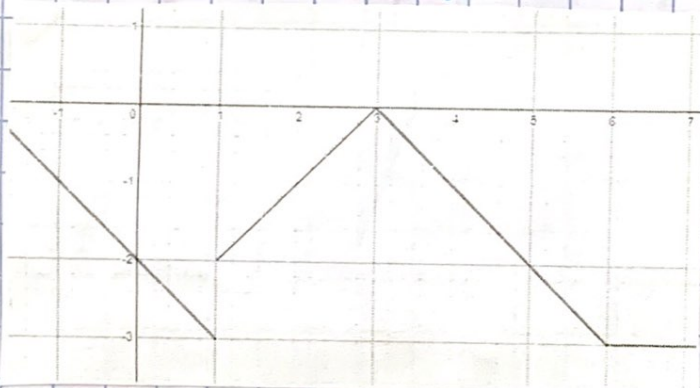
$$f(x) = \begin{cases} -x - 2; & x \leq -1 \\ \frac{1}{3}x + \frac{7}{3}; & x > -1 \end{cases}$$

$$y - 3 = \frac{1}{3}(x - 2)$$

$$y - 3 = \frac{1}{3}x - \frac{2}{3}$$

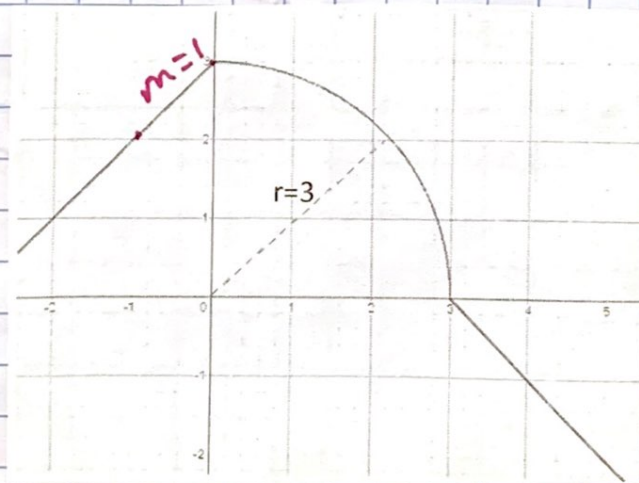
$$y = \frac{1}{3}x + \frac{7}{3}$$

5. Write the equation



$$f(x) = \begin{cases} -x - 2; & x < 1 \\ -|x - 3|; & 1 \leq x \leq 6 \\ -3; & x > 6 \end{cases}$$

6. Write the equation of the following:



$$f(x) = \begin{cases} x + 3; & x \leq 0 \\ \sqrt{9 - x^2}; & 0 < x < 3 \\ -x + 3; & x \geq 3 \end{cases}$$

Circle:

$$x^2 + y^2 = 3^2$$

$$y = \sqrt{9 - x^2}$$



# Exponentials & Logarithms

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Properties of Exponents

Let  $a$  and  $b$  be real #'s and let  $m$  and  $n$  be integers.

Product of Powers

$$a^m \cdot a^n = a^{m+n}$$

$$\log_e x = \ln x$$

$y = e^x$  and  $y = \ln x$  are inverses

$$e^{-x} = \frac{1}{e^x}$$

Power of a Power

$$(a^m)^n = a^{mn}$$

$$\ln e = 1$$

$$\ln a^c = c \ln a$$

$$e^{\ln x} = x$$

Power of a Product

$$(ab)^m = a^m b^m$$

$$\ln 1 = 0$$

$$\ln(ab) = \ln a + \ln b$$

$$a^{x+y} = a^x \cdot a^y$$

Negative Exponent

$$a^{-m} = \frac{1}{a^m}, a \neq 0$$

$$\ln e^b = b$$

$$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$$

$$a^{x-y} = \frac{a^x}{a^y}$$

Zero Exponent

$$a^0 = 1, a \neq 0$$

Quotient of Powers

$$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$$

Find the value without a calculator.

$$1. \quad 5 \ln e + 2 \ln\left(\frac{1}{e}\right)$$

$$5(1) + 2 \ln e^{-1}$$

$$5 - 2(1) = 3$$

$$2. \quad e^{-\ln e} - e^{\ln \sqrt{e}}$$

$$\frac{1}{e^{\ln e}} - e^{\ln e^{1/2}} = \frac{1}{e} - e^{1/2}$$

$$\frac{1}{e} - \sqrt{e}$$

Simplify.

$$3. \quad 2 \ln a - 3 \ln b + \ln(ab)$$

$$\ln\left(\frac{a^2(ab)}{b^3}\right) = \ln \frac{a^3 b}{b^3} = \ln \frac{a^3}{b^2}$$

$$4. \quad 3(\ln e^2 \ln(e \ln e))$$

$$3(\ln e^2) = 3 \cdot 2 \ln e = 6$$

Express as a single logarithm.

$$5. \quad 2 \ln x - 4 \ln y - \ln 13$$

$$\ln\left(\frac{x^2}{13y^4}\right)$$

$$6. \quad \log_3 7 + \frac{1}{2} \log_3 x - 5 \log_3 y$$

$$\log_3 \frac{7x^{1/2}}{y^5} = \log_3 \frac{7\sqrt{x}}{y^5}$$

Expand the following logarithms.

$$7. \quad \log \frac{x^2 y^3}{w z^3}$$

$$\log x^2 + \log y^3 - \log w - \log z^3$$

$$8. \quad \log \frac{b^3}{\sqrt{ac}} = \log \frac{b^3}{(ac)^{1/2}}$$

$$\log b^3 - \log (ac)^{1/2}$$

$$2 \log x + 3 \log y - \log w - 3 \log z$$

$$\text{or } 3 \log b - \frac{1}{2}(\log a + \log c)$$

$$3 \log b - \frac{1}{2} \log a - \frac{1}{2} \log c$$



# 14 Solving Equations with Common Bases

\* If  $b^x = b^y$ , then  $x = y$

$$1. \left(\frac{1}{3}\right)^{-x+7} = \left(\frac{1}{3}\right)^{3x-1}$$

$$-x+7 = 3x-1$$

$$-4x = -8$$

$$x = -2$$

$$2. 8^{x+2} = 16^{2x+7}$$

$$((2^3)^{x+2}) = (2^4)^{(2x+7)}$$

$$3(x+2) = 4(2x+7)$$

$$3x+6 = 8x+28$$

$$x = -\frac{22}{5}$$

Solving Equations when you can't get a common base  $\rightarrow$  rewrite as a log equation.

$$1. 3e^{4x} = 45$$

$$e^{4x} = 15$$

$$\frac{\ln 15}{4} = \frac{4x}{4}$$

$$x \approx .67 \text{ calc}$$

$$2. 2(5^{2x}) - 1 = 47$$

$$5^{2x} = 24$$

$$\log_5 24 = 2x$$

$$\frac{\log 24}{2} \div \frac{\log 5}{2} = \frac{2x}{2}$$

$$x \approx .987$$

$$3. 4 - 2e^x = -23$$

$$-2e^x = -27$$

$$e^x = 13.5$$

$$\ln 13.5 = x$$

$$x \approx 2.603$$

## Solving Log Equations

$$1. \log_3(5x-1) = \log_3(x+7)$$

$$5x-1 = x+7$$

$$4x = 8$$

$$x = 2$$

$$2. \log(x+6) = \log(8x) - \log(3x+2)$$

condense!

$$\log(x+6) = \log \frac{8x}{3x+2}$$

$$x+6 = \frac{8x}{3x+2}$$

$$3. \log_5(3x+1) = 2$$

$$5^2 = 3x+1$$

$$25 = 3x+1$$

$$x = 8$$

$$(3x+2)(x+6) = 8x$$

$$3x^2 + 20x + 12 = 8x$$

$$3x^2 + 12x + 12 = 0$$

$$3(x^2 + 4x + 4) = 0$$

$$3(x+2)(x+2) = 0$$

$$x = -2$$

$$4. 4 \ln(x+2) = 6$$

$$\ln(x+2) = \frac{3}{2}$$

$$e^{\frac{3}{2}} = x+2$$

$$x \approx 2.482$$

No Solution

$$5. \log_2 4x + \log_2(x+3) = 4$$

condense  
1#!

$$\log_2 4x^2 + 12x = 4$$

$$4x^2 + 12x = 2^4$$

$$4x^2 + 12x - 16 = 0$$

$$4(x^2 + 3x - 4) = 0$$

$$4(x+4)(x-1) = 0$$

$$x \neq -4$$

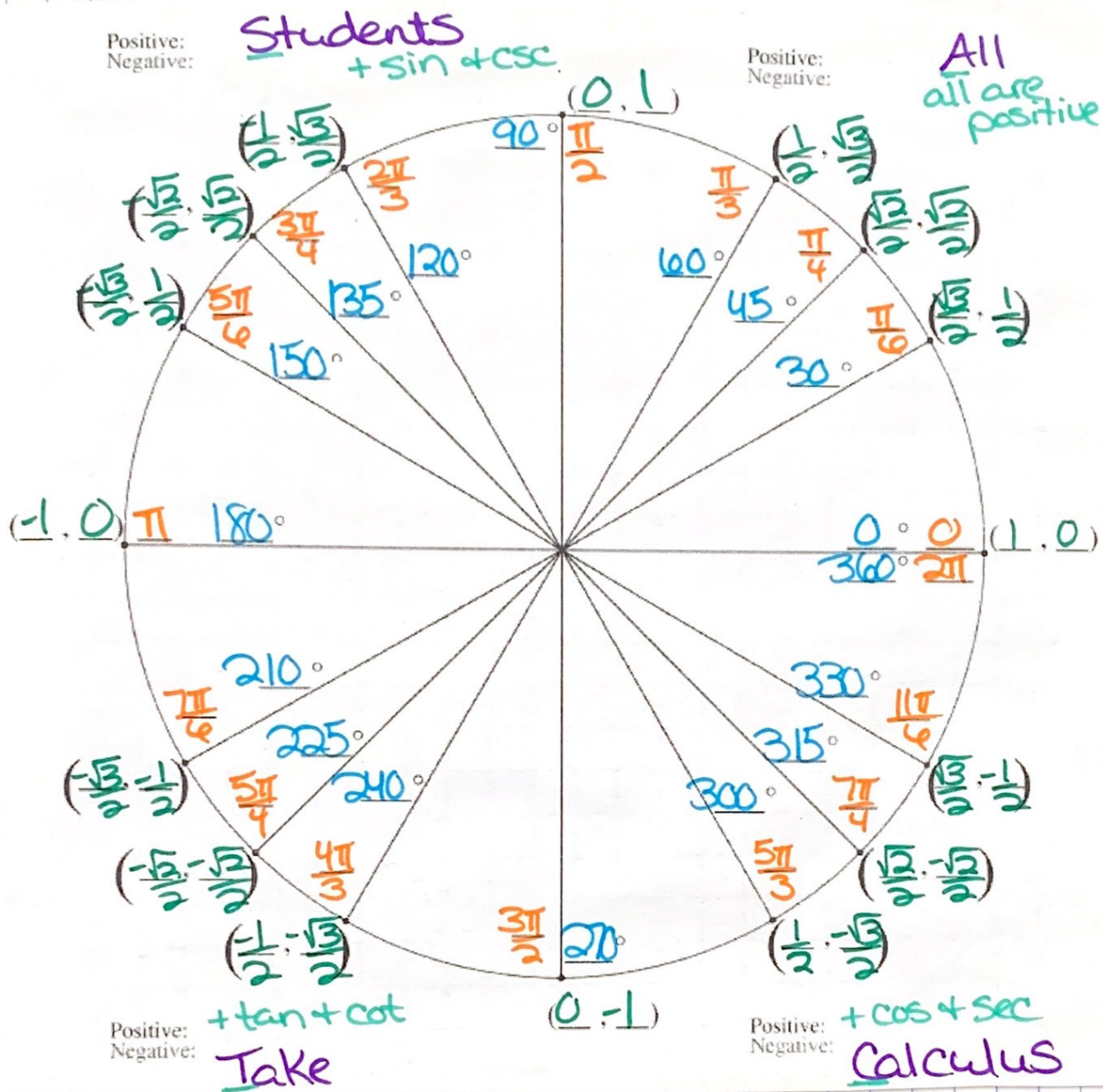
$$x = 1$$

Can't  
take  
log  
of 0



# The Unit Circle

15



$(\cos, \sin)$

$\cos = x$ -coordinate

$\sin = y$ -coordinate

$$\tan = \frac{\sin}{\cos} \text{ or } \frac{y}{x}$$

$$\cot = \frac{\cos}{\sin} \text{ or } \frac{x}{y}$$

$$\sec = \frac{1}{\cos} \text{ or } \frac{1}{x}$$

$$\csc = \frac{1}{\sin} \text{ or } \frac{1}{y}$$



16 Find the exact values of the trig function:

1.  $\cos\left(\frac{4\pi}{3}\right)$   $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right) = -\frac{1}{2}$     2.  $\tan(210^\circ)$   $\frac{-\sqrt{3}}{-1} = \sqrt{3}$

3.  $\csc\left(\frac{11\pi}{6}\right)$   $\sin\frac{11\pi}{6} = -\frac{1}{2} \Rightarrow -2$     4.  $\sec(270^\circ)$   $\frac{1}{0}$  DNE

5.  $\sin(-120^\circ)$   $= -\frac{\sqrt{3}}{2}$    
  $\frac{+360^\circ}{\sin 240^\circ}$     6.  $\cot\left(\frac{17\pi}{6}\right)$   $-\frac{\sqrt{3}}{1} = -\sqrt{3}$    
  $-\frac{\sqrt{3}}{2} \div \frac{-1}{2} = \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \sqrt{3}$

Trigonometric Identities

Reciprocal Identities

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

Quotient Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Solve the trig equations. Give answers in radians ( $0 \leq \theta < 2\pi$ )

1.  $2 \cos \theta - 1 = 0$

$2 \cos \theta = 1$  Inverse trig  
 $\cos \theta = 1/2$  Find an angle with  $\cos = 1/2$   
 $\theta = \cos^{-1}(1/2)$   
 $\theta = \pi/3 + 5\pi/3$

2.  $4 \sin^2 \theta = 3$

$\sqrt{\sin^2 \theta} = \sqrt{3/4}$   
 $\sin \theta = \pm \frac{\sqrt{3}}{2}$      $\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{4\pi}{3}, \frac{7\pi}{6}$

3.  $2 \sin^2 \theta - \sin \theta = 3$

$2 \sin^2 \theta - \sin \theta - 3 = 0$   
 $(2 \sin \theta - 3)(\sin \theta + 1) = 0$   
 $\sin \theta = 3/2$      $\sin \theta = -1$   
 $\theta = 3\pi/2$

4.  $2 \sin^2 \theta = \cos \theta - 1$

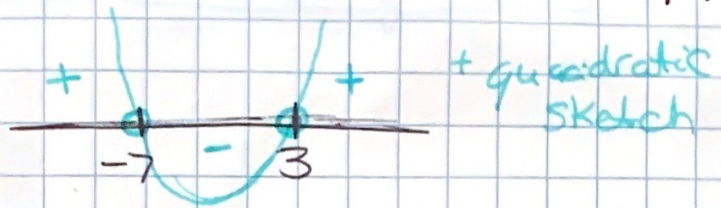
$2(1 - \cos^2 \theta) = \cos \theta - 1$   
 $2 - 2 \cos^2 \theta = \cos \theta - 1$   
 $0 = 2 \cos^2 \theta + \cos \theta - 3$   
 $0 = (2 \cos \theta + 3)(\cos \theta - 1)$   
 $\cos \theta = -3/2$      $\cos \theta = 1$   
 $\theta = 0$

Use trig identities to have only 1 trig function  
 $\sin^2 \theta + \cos^2 \theta = 1$   
 $\sin^2 \theta = 1 - \cos^2 \theta$



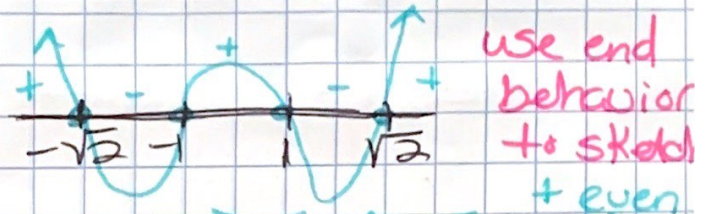
# Inequalities

1.  $x^2 + 4x + 7 < 28$   
 $x^2 + 4x - 21 < 0$   
 $(x+7)(x-3) < 0$   
 $x+7=0 \quad x-3=0$   
 $x=-7 \quad x=3$   
 $(-7, 3)$



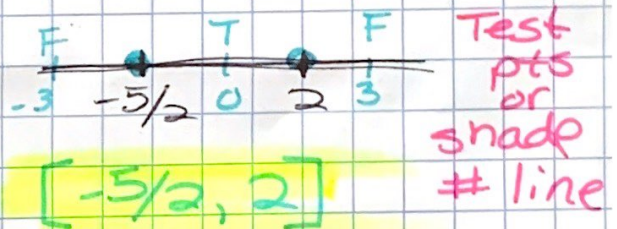
These are the zeros (x-int) of graph  
 so plot on # line + test pts of sketch the graph

2.  $x^4 - 3x^2 + 2 \geq 0$   
 $(x^2 - 2)(x^2 - 1) \geq 0$   
 $(x^2 - 2)(x+1)(x-1) \geq 0$   
 $x = \pm\sqrt{2}, -1, 1$  x-int.

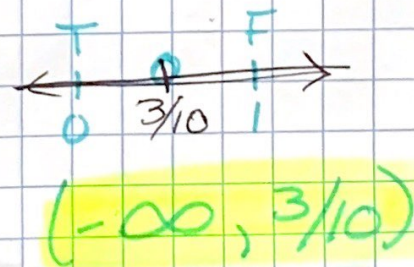


$(-\infty, -\sqrt{2}] \cup [-1, 1] \cup [\sqrt{2}, \infty)$

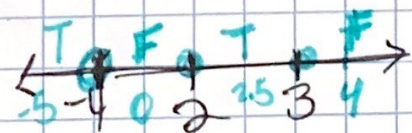
3.  $\sqrt{2x+5} \leq 3$   
 $2x+5 \geq 0$  consider domain restrictions  
 $x \geq -5/2$   
 $\sqrt{2x+5} \leq 3$  solve for x-int.  
 $2x+5 \leq 9$   
 $x \leq 2$



4.  $8^{2x+1} > 4^{8x}$   
 $(2^3)^{2x+1} > (2^2)^{8x}$  solve for x.  
 $3(2x+1) > 2(8x)$  No domain restrictions  
 $6x+3 > 16x$   
 $3 > 10x$   
 $3/10 > x$  or  $x < 3/10$



5.  $\frac{x^2 - 5x + 6}{x+4} \leq 0$   
 $x+4 \neq 0$  Find domain restriction  
 $x \neq -4$   
 $\frac{x^2 - 5x + 6}{x+4} \leq 0$  Find x-intercepts  
 $0 = x^2 - 5x + 6 \quad x=3$   
 $0 = (x-3)(x-2) \quad x=2$



$(-\infty, -4) \cup [2, 3]$