

Compound Interest

Compounded Interest

Regular Compounding

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

$$A = P e^{rt}$$

(Compounded Continuously)

A = accumulated or end amount

P = principal (initial amount, investment)

r = interest rate or growth/decay rate
must be in decimal form (not %)

n = # of times interest compounded
in a year

t = time in years

| n = | Compounding Periods |
|----------------------|---------------------|
| 1 | annually (yearly) |
| 2 | semi-annually |
| 4 | quarterly |
| 12 | monthly |
| 52 | weekly |
| 365 | daily |
| use $A = Pe^{rt}$ | Continuous |

Reg. Compounding

Bob is depositing \$400 in a savings account with 7% interest, compounded monthly. How much will be in the savings account in 10 years?

$$A = ?$$

$$P = 400$$

$$r = 7\% = .07$$

$$n = 12$$

$$t = 10$$

$$A = P(1 + \frac{r}{n})^{nt}$$

$$A = 400(1 + \frac{.07}{12})^{12 \cdot (10)}$$

$$A = \$803.86$$

Janet started a bank account 12 years ago which paid 6% interest, compounded quarterly. and then forgot about it. She just remembered she had the account and checked the balance. She now has \$10,217.39. How much was her initial deposit?

$$A = 10,217.39$$

$$P = ?$$

$$r = 6\% = .06$$

$$n = 4$$

$$t = 12$$

$$A = P(1 + \frac{r}{n})^{n \cdot t}$$

$$10,217.39 = P(1 + \frac{.06}{4})^{48}$$

$$\frac{10,217.39}{(1 + \frac{.06}{4})^{48}} = \frac{P(1 + \frac{.06}{4})^{48}}{(1 + \frac{.06}{4})^{48}}$$

$$P = \frac{10,217.39}{(1 + \frac{.06}{4})^{48}}$$

$$P = \$5000$$

How long will it take for Robert to earn \$1200 on \$500 that earns 9% interest, compounded daily?

Reg. compounding

$$A = 1200$$

$$P = 500$$

$$r = 9\% = .09$$

$$n = 365$$

$$t = ?$$

$$A = P(1 + \frac{r}{n})^{nt}$$

$$\frac{1200}{500} = \frac{500}{500}(1 + \frac{.09}{365})^{365t}$$

$$2.4 = (1 + \frac{.09}{365})^{365t}$$

$$\log(1 + \frac{.09}{365})^{365t} = \log 2.4$$

$$365t \log(1 + \frac{.09}{365}) = \log 2.4$$

$$t = 9.7 \text{ years}$$

Exp. Eq. so rewrite as Log

Michael decides to invest his graduation money (\$2000) in a money market account which pays 7.2%, compounded continuous.

Continuous compounding formula

How much will he have in 5 years?

$$A = ?$$

$$P = 2000$$

$$r = 7.2\% = .072$$

$$t = 5$$

$$A = Pe^{rt}$$

$$A = 2000e^{.072(5)}$$

$$A = \$2866.16$$

When will the money double in value?

$$A = 4000$$

$$P = 2000$$

$$r = .072$$

$$t = ?$$

$$A = Pe^{rt}$$

$$\frac{4000}{2000} = \frac{2000e^{.072t}}{2000}$$

$$2 = e^{.072t}$$

$$\ln 2 = .072t$$

$$t = 9.6 \text{ years}$$

Exp. Eq. b exp = # Rewrite as log # = exp

Exponential Growth + Decay

Exponential Growth : $y = a(1+r)^t$
end amt. \uparrow beginning amt.
use when "increased", "appreciates",
"spreading"

Exponential Decay : $y = a(1-r)^t$

use when "decreased", "depreciate",
"decline", "lose value"