

12.5-1.

Exponential

equations  
to solve  
with  
logarithms

12.5-2.

**SOLVE**

# **EXPONENTIAL EQUATIONS**

## **Exponential Equations**

Many exponential equations can be solved by taking logarithms.

## **Equations Requiring log**

## **Equations Requiring ln**

# If you can get the Same base on both sides

To solve exponential equations:

1. Rewrite the bases as the same number
2. Use the power to the power property to simplify the exponents on each side
3. Set the exponents equal to each other and solve

$$\boxed{\text{base}^{\text{exp}} = \text{base}^{\text{exp}}}$$

EXAMPLE 1:

$$6^{2b} = 6^{b-1}$$
$$2b = b - 1$$
$$b = -1$$

Same base on each side  
so set exponents equal  
to each other

EXAMPLE 2:

$$\left(\frac{1}{25}\right)^{3n} = 625$$
$$25^{-1(3n)} = 25^2$$

$$-1(3n) = 2$$
$$-3n = 2$$
$$n = -\frac{2}{3}$$

Example 3:

$$5^{3x} = \frac{1}{125}$$
$$5^{3x} = 5^{-3}$$
$$3x = -3$$
$$x = -1$$

Example 4:

$$\left(\frac{1}{9}\right)^{-n-1} = 81^{-n}$$
$$9^{-1(-n-1)} = 9^{2(-n)}$$
$$-1(-n-1) = 2(-n)$$
$$n+1 = -2n$$
$$\frac{1}{3} = -\frac{3n}{3}$$
$$n = -\frac{1}{3}$$

## Exponential Equations

## Equations Requiring log

## Equations Requiring ln

# When you can't get the same base on each side

Equations Requiring Logs:

4. Isolate the base with its exponent  $\text{base}^{\text{exp}} = \#$

3. Rewrite into log form  $\log_{\text{base}} \# = \text{exp}$

3. Solve by using change of base

\*to enter in calculator  $\log(\#) \div \log(\text{base})$  or Math A: LOGBASE

Example 5:

$$13^m = 2^{\#}$$

$$\log_{13} 2^{\#} = m \quad \text{Solve in calculator}$$

$$\log \# \div \log \text{base}$$

$$\text{calc: } \log 2 \div \log 13 = m$$

$$m \approx .270$$

Example 6:

$$2^{-5.6b} - 5 = 28$$

$$+5 +5$$

$$2^{-5.6b} = 33$$

$$\log_2 33 = -5.6b$$

$$\text{calc } \log 33 \div \log 2 = -5.6b$$

$$\text{calc Ans} \div -5.6b = b$$

$$b \approx -.901$$

Example 7:

$$20^{5n+5} - 3 = 93$$

$$+3 +3$$

$$20^{5n+5} = 96^{\#}$$

$$\log_{20} 96 = 5n + 5$$

$$\log 96 \div \log 20 = 5n + 5$$

$$Ans - 5 = 5n$$

$$Ans \div 5 = n$$

$$n = -.695$$

Keep  
decimals  
in calc

Example 8:

$$-3 \cdot 8^{k+0.1} = -81$$

$$-3 \quad b \quad -3$$

$$8^{k+0.1} = 27$$

$$\log_8 27 = k + 0.1$$

calc

$$Ans - 0.1 = k$$

$$K = 1.485$$

## Equations Requiring log

## Equations Requiring ln

### Equations Requiring LN:

1. Isolate the base with its exponent
2. Rewrite into log form
3. Solve by using LN

\* Remember  
 $\log_e \rightarrow \ln$

### EXAMPLE 7:

$$\begin{aligned} e^{6m} &= 48 \\ \text{base} & \quad \text{exp} \\ \log_e 48 &= 6m \\ \downarrow & \\ \ln \frac{48}{6} &= \frac{6m}{6} \\ m &\approx .645 \end{aligned}$$

### Example 8:

$$\begin{aligned} 6e^{a+2.7} &= 47 \\ \frac{6e^{a+2.7}}{6} &= \frac{47}{6} \\ e^{a+2.7} &= \frac{47}{6} \\ \ln \frac{47}{6} &= a+2.7 \\ \text{Ans} - 2.7 &= -2.7 \\ a &\approx -0.642 \end{aligned}$$

### EXAMPLE 9:

$$\begin{aligned} e^{-7x-2} - 7 &= 62 \\ +7 & \quad +7 \\ \hline e^{-7x-2} &= 69 \\ \log_e 69 &= -7x-2 \\ \downarrow & \\ \ln 69 &= -7x-2 \\ \text{calc.} & \\ \text{Ans} + 2 &= -7x \\ \text{Ans} \div -7 &= x \\ x &= -0.891 \end{aligned}$$

### Example 10:

$$\begin{aligned} 4e^{0.8n+5} + 6 &= 104 \\ -6 & \quad -6 \\ \hline 4e^{0.8n+5} &= 98 \\ \frac{4e^{0.8n+5}}{4} &= \frac{98}{4} \\ e^{0.8n+5} &= 24.5 \\ \ln 24.5 &= 0.8n+5 \\ \text{Ans} - 5 & \\ \text{Ans} \div 0.8 & \\ n &= -2.252 \end{aligned}$$

# Equations Requiring Ln