Solving Logarithmic Equations

Logs on One Side

And

Logs on Both Sides

1 log = 1 log

Property of Equality: If $\log_x a = \log_x b$, then $a = b$.

1. Condense each side, if necessary.
2. Use the Property of Equality to solve the equation.
3. If it is a quadratic equation, you will need to factor to solve.
4. Be sure to go back and check your answer(s) in the expressions.
5. The expressions cannot be a zero or a negative.

* Check answers in orig. problem. you can't take log of 0 or −#

1. $\log_3(x^2 + 3x) = \log_3(x + 15)$

$x^2 + 3x = x + 15$

$x^2 + 2x - 15 = 0$

$(x+5)(x-3) = 0$

$x+5=0 \quad x-3=0$

$x=-5 \quad x=3$

$\log_3[(-5)^2+3(-5)] = \log_3(-5+15)$

$\log_3 10 = \log_3 10$ ✓

$\log_3[3^2+3(3)] = \log_3(3+15)$

$\log_3 18 = \log_3 18$ ✓

Condense 1st so you have 1 log on ea. side

2. $\log_3(x+5) \oplus \log_3(x-2) = \log_3 18$

$\log_3(x+5)(x-2) = \log_3 18$

$(x+5)(x-2) = 18$

$x^2 + 3x - 10 = 18$

$x^2 + 3x - 28 = 0$

$(x+7)(x-4) = 0$

$x+7=0 \quad x-4=0$

$\cancel{x=-7} \quad x=4$

extraneous bc $\log_3(-7+5)$ $\log_3(-2)$

3. $\log_6 x = \frac{1}{2}\log_6 9 + \frac{1}{3}\log_6 27$

Condense right side to a single log

$\log_6 x = \log_6 9^{1/2} \cdot 27^{1/3}$

$\cancel{\log_6} x = \cancel{\log_6} \sqrt{9} \cdot \sqrt[3]{27}$

$x = \sqrt{9} \cdot \sqrt[3]{27}$

$x = 3 \cdot 3$

$x = 9$

logs on 1 side

If $\log_{base}$ # = exponent, then $base^{exponent}$ = number.

1. Use the properties to condense the equation, if necessary.
2. If the equation only has logs on one side of the equal sign, you will have to rewrite it as an exponential equation.
3. If it is a quadratic equation, you will need to factor to solve.
4. Be sure to go back and check your answer(s) in the expressions.
5. The expressions cannot be a zero or a negative.

1. $\log_4(2x) \oplus \log_4(x-2) = 2$ condense

$\log_4(2x)(x-2) = 2$

$\log_4(2x^2-4x) = 2^{exp}$

$4^2 = 2x^2 - 4x$

Take out GCF $\rightarrow 0 = 2x^2 - 4x - 16$

$0 = x^2 - 2x - 8$

$0 = (x-4)(x+2)$

① Isolate $\log_b \# = exp$

② Rewrite as $b^{exp} = \#$

③ Solve + check

$x=4 \quad x=\cancel{-2}$ extraneous

2. $\log_2(4x) \ominus \log_2(x-2) = 3$

condense 1st

$\log_2\left(\frac{4x}{x-2}\right) = 3^{exp}$

$2^3 = \frac{4x}{x-2}$

$\frac{8}{1} = \frac{4x}{(x-2)}$

cross multiply

$4x = 8(x-2)$

$4x = 8x - 16$

$-4x = -16$

$x = 4$

① Isolate $\log_b \# = exp$

② Rewrite $b^{exp} = \#$

③ Solve

3. $\log(x^2 + 21x) = 2$

understood 10

$10^2 = x^2 + 21x$

$100 = x^2 + 21x$

$0 = x^2 + 21x - 100$

$0 = (x+25)(x-4)$

$x+25=0 \quad x-4=0$

$x=-25 \quad x=4$

check:

$(-25)^2 + 21(-25)$

$625 - 525$

100