Introduction to Logarithms

Can you solve for x in this equation? $3^x = 27$

Exponential equation has a variable for exp.

yes $x = 3$ bc $3^3 = 27$

Can you solve for x in this equation? $3^x = 35$

no so you need to use logarithms

Some equations require logarithms to solve for the variable.

A logarithm is the inverse of an exponential equation.

Exponential Form	Rewrite	Logarithmic Form
base ^{exp} **= #**	⟷	$\log_{\text{base}}$ **# = exp**

$3^4 = 81$ (base, exponent, #)	$\log_3 81 = 4$ (base, #, exp)
$5^{-2} = \frac{1}{25}$ (b, exp, #)	$\log_5 \frac{1}{25} = -2$
$16^{\frac{3}{2}} = 64$ (b, exp, #)	$\log_{16} 64 = \frac{3}{2}$
$a^b = c$ (base, exp, #)	$\log_a c = b$
$e^3 = 20.09$ (e is a #)	$\log_e 20.09 = 3$
$5^2 = 25$	$\log_5 25 = 2$ (b, #, exp)
$11^0 = 1$	$\log_{11} 1 = 0$ (b, #, exp)
$81^{3/4} = 27$	$\log_{81} 27 = \frac{3}{4}$ (b, #, exp)

Exponents can be....

- Whole numbers: $3^0 = 1$ or $4^3 = 64$
- Fractions: $4^{1/2} = 2$ or $125^{1/3} = 5$
- Zero: $7^0 = 1$
- Negative: $4^{-1} = \frac{1}{4}$ or $2^{-3} = \frac{1}{8}$

  $\left(\frac{1}{2}\right)^{-1} = 2$

Reminders:

When going from a big base to a small number, your exponent is a fraction.
*Think of roots

When going from a whole number to a fraction or a fraction to a whole number, your answer will be negative.

Special Logs:

- Common Log – Log with base 10.
  Often written as log. *the base 10 is understood
- Natural Log – Log with base e. $\log_e$
  Often written as ln.
  *base e is understood

To evaluate log equations:

1) Set expression equal to x.
2) Rewrite it into exponential form. (base exp = #)
3) Solve without a calculator.

Evaluate without a calculator: "2 to what exponent equals 32?"

1) $\log_{2} 32 = x$ (b, #, exp) $2^x = 32$, $x = 5$
2) $\log_{6} 36 = x$ (b, #, exp) $6^x = 36 \rightarrow x = 2$
3) $\log_{5} 125 = x$ (b, #, exp) $5^x = 125$, $x = 3$
4) $\log_{25} 5 = x$ (b, #, exp) $25^x = 5$, $x = 1/2$ — $\sqrt{25} = 5$, rewrite $\sqrt{25}$ as $25^{1/2}$
5) $\log_{16} 2 = x$ $16^x = 2$, $x = 1/4$ — $\sqrt{16} = 4$, $\sqrt[3]{16} = ?$, $\sqrt[4]{16} = 2$ ✓ $16^{1/4}$
6) $\log_{3} \frac{1}{9} = x$ (b, #, exp) $3^x = \frac{1}{9}$, $x = -2$ — $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$
7) $\log_{\frac{1}{2}} 8 = x$ (b, #, exp) $\frac{1}{2}^x = 8$, $x = -3$ — $\left(\frac{1}{2}\right)^{-1} = 2^1 = 2$, $\left(\frac{1}{2}\right)^{-2} = 2^2 = 4$, $\left(\frac{1}{2}\right)^{-3} = 2^3 = 8$ ✓
8) $\log_{65} 1 = x$ $65^x = 1$, $x = 0$
9) $\log 100$ — 10 understood $10^x = 100$, $x = 2$