

Composition of Functions

Ways to show functions are being composed:

* $(f \circ g)(x)$ OR $(g \overset{\text{open}}{\circ} f)(x)$
* $f(g(x))$ OR $g(f(x))$
↑ out ↑ inside

What does it mean to compose two functions?

to plug one function inside of another function to create the composition

Always start with the innermost parenthesis to plug into outside

DANGER:

$$(f \circ g)(x) \neq (g \circ f)(x)$$

Example: $f(x) = 2x+3$ and $g(x) = x^2$

"x" is just a placeholder, and to avoid confusion let's just call it "input":

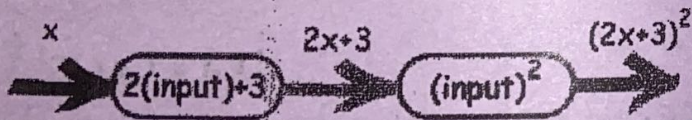
$$f(\text{input}) = 2(\text{input})+3$$

$$g(\text{input}) = (\text{input})^2$$

So, let's start:

$$(g \circ f)(x) = g(f(x))$$

First we apply f, then apply g to that result:



$$(g \circ f)(x) = (2x+3)^2$$

$$(2x+3)^2$$

$$(2x+3)(2x+3)$$

$$4x^2 + 6x + 6x + 9$$

$$4x^2 + 12x + 9$$

Examples

Given:

$$f(x) = x-3 \quad g(x) = \sqrt{x} + 1 \quad h(x) = 2x+3$$

1. $(g \circ h)(x)$ means plug in h to g
 $g(h(x))$

$$\sqrt{2x+3} + 1$$

2. $f(g(x))$

$$(\sqrt{x} + 1) - 3$$

$$\sqrt{x} + 1 - 3 = \sqrt{x} - 2$$

$$= \sqrt{x} - 2$$

3. $(h \circ f)(x)$ or $h(f(x))$

$$2(x-3) + 3$$

$$2x - 6 + 3 = 2x - 3$$

$$= 2x - 3$$

4. $f(g(4))$

plug 4 into g + then plug in the result to f

$$g(4) = \sqrt{4} + 1$$

$$g(4) = 3$$

$$f(3) = 3 - 3$$

$$f(g(4)) = 0$$

5. $(h \circ f)(-1)$

$$f(-1) = -1 - 3 = -4$$

$$h(-4) = 2(-4) + 3 = -5$$