Characteristics of Polynomial Functions

Domain & Range

Domain and Range are written in interval notation.
Domain describes x-values

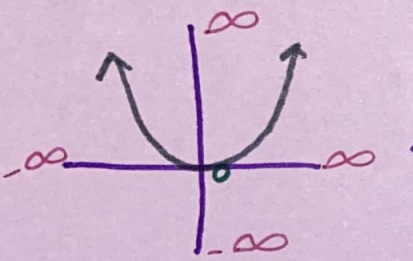
and range describes y-values

Domain: left, right Range: bottom, top

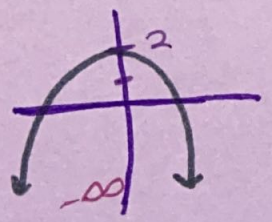
Use (or) when the # value is NOT included in the domain or range

Use [or] when the # value IS included in the domain or range

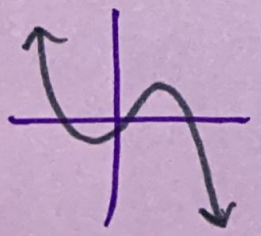
Always use (or) with ∞ or −∞



Domain: $(-\infty, \infty)$
Range: $[0, \infty)$



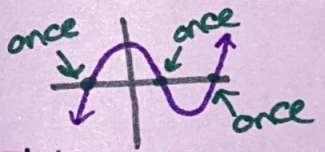
D: $(-\infty, \infty)$
R: $(-\infty, 2]$

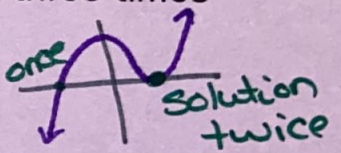


D: $(-\infty, \infty)$
R: $(-\infty, \infty)$

Intercepts

- X-intercepts are where the graph crosses or touches the x-axis
  - Also called roots, zeros or solutions
  - Written as (#, 0)
  - To find algebraically, set function = 0 and solve for x
  - If graph crosses the x-axis, that zero is a zero once
  - If graph "bounces" off the x-axis, that zero is a zero twice
  - If graph "wiggles" through the x-axis, that zero is a zero three times





- Y-intercepts are where the graph crosses or touches the y-axis
  - Written as (0, #)
  - To find algebraically, substitute 0 for x in function and simplify or it is just the constant

$f(x) = x^3 - x^2 - 6x$

x-intercepts $(y=0)$

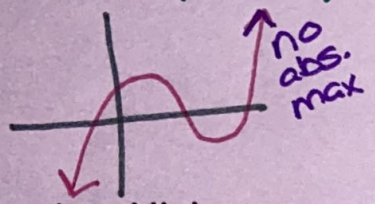
$0 = x^3 - x^2 - 6x$ (0,0)
$0 = x(x^2 - x - 6)$ (3,0)
$0 = x(x-3)(x+2)$ (-2,0)
$x = 0$ $x - 3 = 0$ $x + 2 = 0$
$x = 3$ $x = -2$

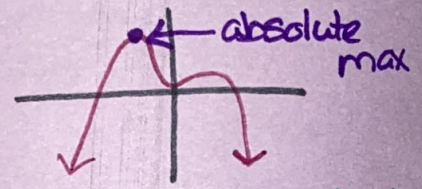
y-int. $(x=0$

$y = (0)^3 - (0)^2 - 6(0)$
$y = 0$ (0,0)

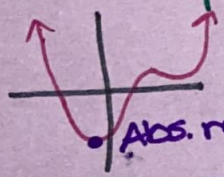
Maximums and Minimums

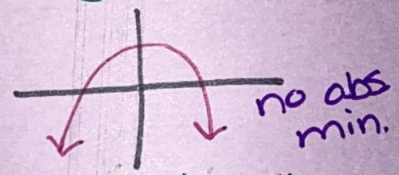
Absolute Maximum: the very highest point (y) that your graph will ever go.



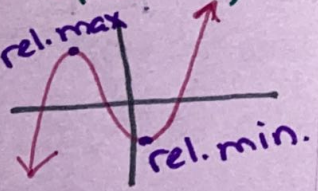


Absolute Minimum: the very lowest point (y) that your graph will go

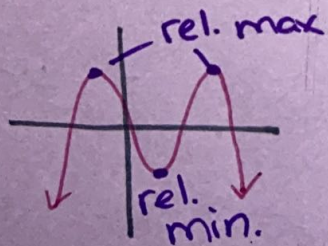




Relative Maximum: highest point in the area (peaks)



Relative Minimum: lowest point in the area (valleys)



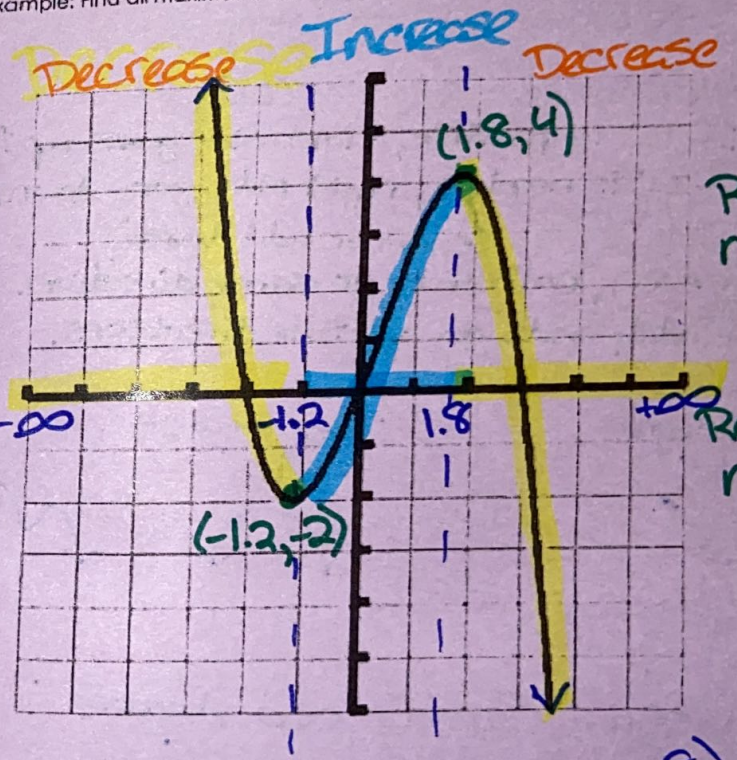
Intervals of Inc/Dec

Increasing/Decreasing/Constant Intervals

****Always write intervals in terms of X coordinate. Use interval notation with parentheses, not brackets.***

All minimum and maximum points are written as ordered pairs.

Example: Find all maximums and minimums. Write the intervals of increase and decrease.



Relative max = 4

Relative min = -2

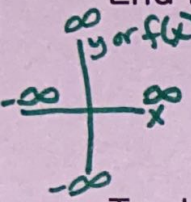
Int. of Increase: (-1.2, 1.8)

Int. of Decrease: (-∞, -1.2) U (1.8, ∞)

End Behavior

The End Behavior describes the direction that the "tails" of the graph are going

End behavior is written as:



As $x \to \infty$, $f(x) \to \infty$ or $-\infty$ (right end; up or down)

As $x \to -\infty$, $f(x) \to \infty$ or $-\infty$ (left end; up or down)

To determine end behavior by looking at function:

- Look at leading coefficient (LC)
  - If LC is positive, right side goes up ↑
  - If LC is negative, right side goes down ↓
- Look at degree (even or odd degree)
  - If even, both ends go same direction.
  - If odd, ends go opposite directions.

Examples:

Even degree, + LC, $3x^2$

$x \to +\infty$, $f(x) \to \infty$

$x \to -\infty$, $f(x) \to \infty$

Even degree, − LC, $-3x^4$

$x \to +\infty$, $f(x) \to -\infty$

$x \to -\infty$, $f(x) \to -\infty$

Odd degree, + LC, $2x$ or $2x^3$

$x \to \infty$ (right), $f(x) \to \infty$ (up)

$x \to -\infty$ (left), $f(x) \to -\infty$ (down)

Odd degree, − LC, $-2x$ or $-2x^3$

$x \to \infty$ (right), $f(x) \to -\infty$ (down)

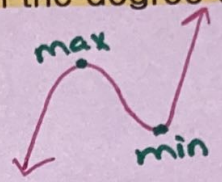
$x \to -\infty$ (left), $f(x) \to \infty$ (up)

Extrema

The graph of a polynomial function can have "turns" or extrema. Maximums are like mountains, minimums are like valleys.

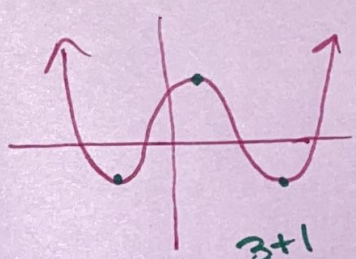
The highest number of extrema (turns) in the graph is one less than the degree of the function.

Examples:



2 extrema (or turns)

What is the least possible degree? turns + 1



3 turns, so the least possible degree is 4

How many "turns" are possible? degree − 1

$f(x) = -75x^5 + 2x^3 - 3$

$5 - 1 = 4$ turns or extrema

↑ degree