

Composition of Functions

Ways to show functions are being composed:

- * open circle : $f \circ g$ (means g is inside of f)
- * one function written inside of the other $f(g(x))$

outside
↓
inside
↓

What does it mean to compose two functions?

It means that you put one function inside of the other function where it has a variable

↑
outside ↑ inside

Always start with the _____

DANGER:

$f \circ g$ does not always equal $g \circ f$
 $f(g(x))$ $g(f(x))$

Example: $f(x) = 2x+3$ and $g(x) = x^2$

"x" is just a placeholder, and to avoid confusion let's just call it "input":

$$f(\text{input}) = 2(\text{input})+3.$$

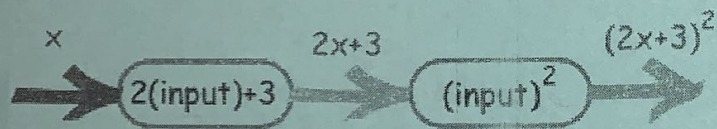
$$g(\text{input}) = (\text{input})^2$$

So, let's start:

outside → inside

$$(g \circ f)(x) = g(f(x))$$

First we apply f, then apply g to that result:



$$\begin{aligned} (g \circ f)(x) &= (2x+3)^2 \\ &= (2x+3)(2x+3) \quad \text{FOIL} \\ &= 4x^2 + 6x + 6x + 9 \\ &= 4x^2 + 12x + 9 \end{aligned}$$

Given: **Examples**

$$f(x) = 5x+1 \quad g(x) = x^2+2x \quad h(x) = x-1$$

1. $f(g(x)) = 5(x^2+2x) + 1$ Then simplify

$$\uparrow \text{out in}$$
$$5x^2 + 10x + 1$$

2. $g(f(x)) = (5x+1)^2 + 2(5x+1)$

$$\begin{aligned} &= (5x+1)(5x+1) + 2(5x+1) \\ &= 25x^2 + 5x + 5x + 1 + 10x + 2 \\ &= 25x^2 + 20x + 3 \end{aligned}$$

3. $g \circ h(x)$ means $g(h(x))$

$$\begin{aligned} &(x-1)^2 + 2(x-1) \\ &(x-1)(x-1) + 2(x-1) \\ &x^2 - 1x - 1x + 1 + 2x - 2 = x^2 - 1 \end{aligned}$$

4. $f(g(7))$ you can do $f(g(x))$ like #1 & then plug in 7 to the answer or ...

start inside

$$g(7) = (7)^2 + 2(7) = 49 + 14$$

$$g(7) = 63 \quad \text{now plug in 63 to } f$$

$$f(63) = 5(63) + 1 = 316$$