November Break Extra Credit

1. Find the domain of $f(x) = \sqrt{2x+3}$

$[-3/2, \infty)$

2. Find the domain of the function $f(x) = \ln(5x-2)$ $(2/5, \infty)$

3. Assume $f(x) = \frac{e^x}{e^x - 1}$. Find $f^{-1}(x)$.

$f'(x) = \frac{-e^x}{(e^x-1)^2}$

4. Find the asymptotes of $y = \frac{5x-14}{x^2-4x}$

VA: $x=0$, $x=4$

HA: $y=0$

5. $\lim_{h \to 16} \frac{x-16}{\sqrt{x}-4} =$ 8

6. $\lim_{x \to -3} \frac{1}{(x+3)^2} =$ ∞

7. $\lim_{x \to 0} \frac{\sin 5x}{\sin 8x} =$ $5/8$

8. $\lim_{x \to 7^-} \frac{x+9}{x-7} =$ $-\infty$

9. Which of the following functions are continuous for all real numbers x?

a. $y = \frac{x^2+5}{x^2+2}$

b. $y = \frac{3}{x^2}$

c. $y = |2x-7|$

10. Let f be defined as follows:

$$f(x) = \begin{cases} \dfrac{x^2-9}{x-3}, & x \neq 3 \\ 1, & x = 3 \end{cases}$$

Which of the following are true about f?

a. $\lim_{x \to 3} f(x)$ exists

b. $f(3)$ exists

c. $f(x)$ is continuous at $x = 3$

11. Consider the function

$$f(x) = \begin{cases} x^2+5, & x > 5 \\ 3ax, & x \leq 5 \end{cases}$$

For what value of a is the function continuous?

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = \lim_{x \to 2} f(x) = f(2)$

12. By applying the Intermediate Value Theorem, what interval will $2^x = \sin(2x) + 5$ have a solution?

Somewhere between $\frac{\pi}{2}$ & $3\pi/4$

13. The function f and g are differentiable and have the values shown in the table. If $A = f \cdot g$ the $A'(0) =$

x	f	f'	g	g'
0	6	1	-8	1/3
2	8	3	-5	1
4	14	9	-3	4
6	26	27	-1	16

-6

14. The function f and g are differentiable and have the values shown in the table. If $A = \sqrt{f(x)}$ the $A'(-2) =$

x	f	f'	g	g'
0	6	1	-8	1/3
2	8	3	-5	1
4	14	9	-3	4
6	26	27	-1	16

$\frac{3}{4\sqrt{2}}$

15. A function f is given by the table shown. Estimate $f'(4.3)$.

x	3.7	4.3	4.9	5.5	6.1
$f(x)$	1.8	3.4	4.6	6.4	8.4

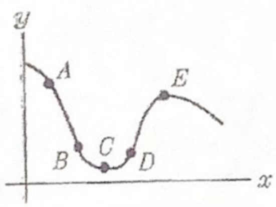
$7/3$

16. A function f is given by the table shown and is differentiable over its domain. What is the best estimate of $f'(0.14)$?

x	0.14	0.34	0.54	0.74	0.94
$f(x)$	9.352	7.044	4.826	2.102	0.288

-11.540

17. At which of the five points shown on the graph are $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ both positive?



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18. Let $f(x) = x^2(x-3)$. Over what interval is the function increasing?

Inc: $(\infty, 0) \cup (2, \infty)$

Dec: $(0, 2)$

19. Let $f(x) = \ln x^x$. Over what interval is the function increasing or decreasing?

Inc: $(\frac{1}{e}, \infty)$

Dec: $(0, \frac{1}{e})$

20. Given the function $f(x) - 1 = x^3$ satisfies the hypothesis of the Mean Value Theorem on the interval $[-2,4]$, find the number C in the interval $(-2,4)$ which satisfies this theorem.

$C = 2$

21. What is the average rate of change over $2 \leq t \leq 4$?

t	2	3	4	5	6
$f(t)$	1.8	3.4	4.6	6.4	8.4

$Aroc_{[2,4]} = 1.4$

22. Given the position function $s = t^2 + 9t - 5$, what is the instantaneous rate of change at $t = 3$?

15

23. Find the point of inflection of $f(x) = x^3 - 3x^2 - x + 7$

$(1,4)$

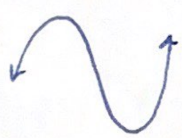
24. Find the interval(s) on which the curve $y = x^3 - 3x^2 - 9x + 6$ is concave upward or concave downward.

CU: $(1, \infty)$

CD: $(-\infty, 1)$

25. Given a continuous function f and the following information, sketch a possible graph of f.

Interval	Sign of f'	Sign of f''
$x < -4$	+	−
$-4 < x < -2$	−	−
$-2 < x < 0$	−	+
$0 < x$	+	+



26. $\lim_{x \to 0} \frac{e^x - e^{-x}}{4x} =$ $\frac{1}{2}$

27. If $y = \ln(x^2 + 3x)$, then $\frac{dy}{dx} =$

$\frac{2x+3}{x^2+3x}$

28. Find $\frac{dy}{dx}$ given $y = \ln(8 - x)^4$

$\frac{4}{x-8}$

29. A curve is defined by $y = e^{\sin 2x}$. Find $\frac{dy}{dx}$.

$2\cos 2x \cdot e^{\sin 2x}$

30. Find the critical numbers, if any, of $f(x) = \frac{x-1}{x+3}$

$x = -3$

31. Given a function defined by $f(x) = 3x^5 - 5x^3 + 12$, for what value(s) of x is there a relative maximum?

$X = 1$

32. Find the relative extrema of the function $f(x) = 3x^5 - 5x^3$?

Min: -2

Max: 2

33. Given $f(x) = x^3 - 3x^2 - 9x$ find the absolute maximum value on the closed interval $[0,6]$.

Abs Min -27

34. If $f(x) = \cos^4 x$, then $f'\left(\frac{\pi}{3}\right) =$

$-\frac{\sqrt{3}}{4}$

35. Given $f(x) = \frac{x}{\tan x}$, find $f'\left(\frac{3\pi}{4}\right)$

$-1 - \frac{3\pi}{2}$

36. Find the derivative, $\frac{dy}{dx}$, of $y = \frac{2x}{1-3x^2}$.

$\frac{6x^2+2}{(1-3x)^2}$

37. If $f(x) = \frac{x^3+3x^2+2}{x}$, then $f'(1) =$

3

38. Find the derivative of $x^2 f(x)$

$x^2 f'(x) + 2x\, f(x)$

39. If $f(x) = \sin x \cos x$, then $f'\left(\frac{\pi}{6}\right) =$

$\frac{1}{2}$

40. Find an equation for the tangent line to the graph of $f(x) = \sqrt{x-7}$ at the point where $x = 16$.

$Y = \frac{1}{6}x + \frac{1}{3}$

41. Find the slope of the tangent line to the graph of $y = \ln(xe^x)$ at the point where $x = 3$.

$4/3$

42. Given $y = \arcsin(5x)$, then $\frac{dy}{dx} =$

$y' = \frac{5}{\sqrt{1-25x^2}}$

43. Find the derivative: $s(t) = \sin\left(\frac{t}{2}\right)$

$s'(t) = \frac{1}{2}\cos\left(\frac{t}{2}\right)$

44. Given $2x = xy + y^2$, then $\frac{dy}{dx} =$

$\frac{dy}{dx} = \frac{2-y}{x+2y}$

45. Given $2x^2 + xy + 3y^2 = 0$, then $\frac{dy}{dx} =$

$\frac{dy}{dx} = \frac{-(4x+y)}{x+6y}$

46. A projectile starts at time $t = 0$ and moves along the x-axis so that its position at any time $t \geq 0$ is $x(t) = t^3 - 6t^2 + 9t + 12$. What is the velocity of the particle at $t = 0$.

9

47. A projectile starts at time $t = 0$ and moves along the x-axis so that its position at any time $t \geq 0$ is $x(t) = t^3 - 6t^2 + 9t + 12$. During what time intervals is the particle moving to the left?

$(1, 3)$

48. A point moves along the curve $y = \sqrt{x}$ in such a way that the y-value is increasing at the rate of 2 units per second. At what rate is x changing when $x = \frac{1}{2}$?

$\frac{dx}{dt} = 2\sqrt{2}$ u/sec

49. A man $2m$ tall walks away from a lamppost whose light is $5m$ above the ground. If he walks at a speed of $1.4\,m/s$, at what rate is his shadow growing when he is $10m$ from the lamppost?

$\frac{ds}{dt} = \frac{14}{15}$ m/s

50. A balloon rises vertically at the rate of $10\,ft/sec$. A person watches the balloon ascend from a point on the ground $100ft$ away from the spot below the rising balloon.

Skip

51. Evaluate $\int \frac{2x^2+3x^{\frac{1}{2}}+4}{x^{\frac{1}{2}}}dx$

$\frac{4}{5}x^{5/2} + 3x + 8x^{1/2} + C$

52. Evaluate $\int_0^{16} 2\sqrt[4]{x^3}\,dx$

$\frac{1024}{7}$

53. Evaluate $\int_2^7 |x-4|dx$

$\frac{13}{2}$

54. Suppose $\int_0^5 f(x)dx = 7$ and $\int_2^5 f(x)dx = -1$ find $\int_0^2 f(x)dx$

8

55. Evaluate $\int x(x^2-1)^4 dx$

$\frac{(x^2-1)^5}{10} + c$

56. If $\frac{dy}{dx} = \cos^7 x \sin x$, then $y =$

$\frac{-\cos^8 x}{8} + c$

57. Evaluate $\int x\sqrt{x+1}\,dx$

$\frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} + C$

58. Evaluate $\int \frac{7e^x}{e^x+5} dx$

$7\ln(e^x+5) + C$

59. Find the indefinite integral: $\int \frac{x}{16+x^4} dx$

$\frac{1}{8}\tan^{-1}\left(\frac{x^2}{4}\right) + c$

60. On the planet Mathematica the population in the year 2000 was about 8 billion. If the population is growing according to $P(t) = 8e^{0.021t}$ then write a definite integral that gives the population for the 10-year period starting from the year 2000. Assume $t = 0$ at the beginning of the year 2000.

$\int_0^{10} 8e^{.021t}\,dt$

61. Evaluate $\frac{d}{dx}\int_1^x \sqrt{3\cos^2 t + 4}\,dt$

$\sqrt{3\cos^2 x + 4}$

61. Evaluate $\frac{d}{dx}\int_{x^6}^1 \frac{dt}{2t-5}$

$\frac{-6x^5}{2x^6-5}$

63. Find the equation of the family of curves $\frac{dy}{dx} = 12x^2 + 4x$ that passes through the point $(-1,-5)$

$y = 4x^3 + 2x^2 - 3$

64. Evaluate $\int x^2 \ln x\, dx$

$\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + c$

Calculator Section

65. Which of the following is an approximate root of $y = \sin(3x) + 1$

$x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

66. If $f(7) = 2$ and $f'(7) = 9$, then the tangent line approximation at $x = 7$ is what?

$\mathcal{L}(x) = 9x - 61$

67. If $f(5) = 9$ and $f'(5) = -2$, then $f(5.011) =$

8.978

68. Consider the integral $\int \frac{1}{x} dx$ from $x = 1$ to $x = 4$. Using a Riemann sum with 6 sub-intervals calculate the area under the curve, and above the x-axis, using right endpoints. Answer to 3 decimal places.

≈ 1.218

69. Use a Trapezoidal approximation for $\int_1^3 x^3\, dx$ with $n = 4$.

$41/2$

70. The following table shows selected coordinates for $y = f(x)$. Given that f is continuous on $[1,4]$, find a trapezoidal approximation with $n = 3$, for the area under the curve from $x = 1$ to $x = 4$.

x	1	2	3	4
y	1.2	2.3	2.5	4.9

7.85

71. Approximate $\int_{0.3}^{7.6} \frac{1}{\sqrt[4]{x}} dx$

5.5626