Lesson 1.4 and 1.5 Notes Complex Numbers

Today's Question: How do we take the square root of negative numbers?

$$\boxed{\sqrt{-1} = i}$$

Examples:

1. $\sqrt{-16}$ $\sqrt{16}\cdot\sqrt{-1}$ $4i$

2. $\sqrt{-81}$ $\sqrt{81}\cdot\sqrt{-1}$ $9i$

3. $\sqrt{-45}$ $\sqrt{45}\ \sqrt{-1}$ $3\sqrt{5}\cdot i$ $3i\sqrt{5}$

4. $\sqrt{-200}$ $\sqrt{200}$ $10\sqrt{2}$ $10i\sqrt{2}$

Powers of *i*

$i = i$

$i^2 = -1$

$i^3 = -i$

remainder is 0 → $i^4 = 1$

memorize

$(i)^2 = (\sqrt{-1})^2$

$i^2 = -1$

$i^2 \cdot i = i^3$

$-1 \cdot i = -i$

$(i^2)^2 = (-1)^2$

Always divide the exponent by 4.

- If it divides evenly, then the answer is 1.

★If it doesn't ÷ evenly, the remainder is your exp. for i

Examples:

5. i^{13} $4\overline{)13}$ 3 R1 so i^1 $= i$

6. i^{27} R3 → i^3 $-i$

7. i^{54} $4\overline{)54}$ 13 R2 so i^2 -1

8. i^{72} $4\overline{)72}$ 18 R0 so i^4 1

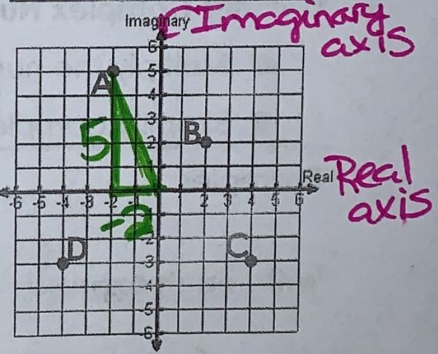
Complex Numbers & The Complex Plane

- ◎ A complex number has a real part & an imaginary part.
- ◎ Standard form is $a + bi$ real imaginary

A $-2+5i$ B $2+2i$

C $4-3i$ D $-4-3i$

Identify the points



Absolute Value of Complex Numbers – distance from origin

$$|a + bi| = \sqrt{a^2 + b^2}$$ pythagorean theorem

Find the absolute value of the points A, B, C and D labeled on the graph above.

$|A| = \sqrt{(-2)^2 + (5)^2} = \sqrt{29}$

$|B| = \sqrt{(2)^2 + (2)^2} = \sqrt{8}$ $2\sqrt{2}$

$|C| = \sqrt{(4)^2 + (-3)^2} = \sqrt{25} = 5$

$|D| = \sqrt{(-4)^2 + (-3)^2} = \sqrt{25} = 5$

Add and Subtract Complex Numbers

- ◎ Add or subtract the real parts, and then, add or subtract the imaginary parts.

9. $(3+2i)+(7+6i)$ $= 10+8i$

10. $(6-5i)-(1+2i)$
$6-5i-1-2i = 5-7i$

11. $(9-4i)-(-2+3i)$
$9-4i+2-3i$
$11-7i$

12. $9-(10+2i)-5i$ $\quad 9+0i \quad 0-5i$
$9-10-2i-5i$
$-1-7i$

13. $(11i^4+4i^3)-(2i^4-6i^3)$
$9i^4+10i^3$
$9(1)+10(-i)=9-10i$

Multiplying Complex Numbers

Write answers → $a+bi$

- ◎ Treat the *i*'s like variables, then change any that are not to the first power.

Examples: $x \cdot x = x^2$ so $i \cdot i = i^2 = -1$

14. $-i(3+i)$
$-3i-i^2 \leftarrow -1$
$-3i-(-1)$
$1-3i$

15. $(2+3i)(-6-2i)$
$-12-4i-18i-6\overset{(-1)}{i^2}$
$-12-22i+6$
$-6-22i$

16. $(-3+i)(8+5i)$
$-24-15i+8i+5i^2$
$-24-7i-5$
$-29-7i$

17. $(4+3i)(4-3i)$
$16-\cancel{12i+12i}-9i^2$
$16+9=25$

18. $-2i(1+4i)$
$-2i-8i^2$
$-2i+8$
$8-2i$

19. $(3-2i)(-5-9i)$
$-15-27i+10i+18i^2$
$-15-17i-18$
$-33-17i$

Conjugates

- ◎ Two complex numbers of the form *a* + *bi* and *a* – *bi* are complex *conjugates*.
- ◎ The product is always a real number.

Example $(2+4i)(2-4i)$

Dividing Complex Numbers

- ◎ Multiply the numerator and denominator by the conjugate of the denominator.
- ◎ Simplify completely.

Examples:

Write each expression as a complex number in standard form.

20. $\dfrac{5-2i}{3+8i}$

21. $\dfrac{3+11i}{-1-2i}$

22. $\dfrac{5}{1+i}$

23. $\dfrac{8+3i}{1-2i}$

24. $\dfrac{6-3i}{2i}$