Exponentials and Logarithms

Honors Calculus

Keeper 7

Properties of Exponents	Let *a* and *b* be real #'s and let *m* and *n* be integers.	
Product of Powers	$a^m \cdot a^n = a^{m+n}$	$x^2 x^3 = x^5$
Power of a Power	$(a^m)^n = a^{mn}$	$(x^2)^3 = x^6$
Power of a Product	$(ab)^m = a^m b^m$	$(xy)^3 = x^3 y^3$
Negative Exponent	$a^{-m} = \frac{1}{a^m}, a \neq 0$	$x^{-3} = \frac{1}{x^3}$
Zero Exponent	$a^0 = 1, a \neq 0$	$x^0 = 1$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\frac{x^3}{x^2} = x$

Properties of Log & Exponentials

- $\log_e x = \ln x$
- $\ln e = 1$
- $\ln 1 = 0$
- $\ln e^b = b$

- $y = e^x$ & $y = \ln x$ are inverses

prop. of logs for condensing + expanding

- $\ln a^c = c \ln a$
- $\ln(ab) = \ln a + \ln b$
- $\ln \frac{a}{b} = \ln a - \ln b$

- $e^{-x} = \frac{1}{x}$
- $e^{\ln x} = x$
- $a^{x+y} = a^x \cdot a^y$
- $a^{x-y} = \frac{a^x}{a^y}$

Find the value without a calculator

1. $5\ln e + 2\ln\left(\frac{1}{e}\right)$

$5(1) + 2\cancel{\ln e}^{-1}$

$5 - 2 = \boxed{3}$

$\sqrt[2]{x^{1}}$

$x^{1/2}$

2. $e^{-\ln e} - e^{\ln\sqrt{e}}$

$\frac{1}{\cancel{e^{\ln}}e} - e^{\frac{1}{2}\cancel{\ln e}^{1/2}}$

$\boxed{\frac{1}{e} - e^{1/2} \text{ or } \frac{1}{e} - \sqrt{e}}$

Simplify

$2\ln a - 3\ln b + \ln a + \ln b$

3. $2\ln a - 3\ln b + \ln(ab)$

$\ln\left(\frac{a^2(ab)}{b^{\cancel{3}2}}\right)$ $\boxed{\ln\left(\frac{a^3}{b^2}\right)}$

$3\ln a - 2\ln b$

$\ln\frac{a^3}{b^2}$

4. $3\ln(e^2\ln(e\,\cancel{\ln e}))$

$3\ln(e^2\,\cancel{\ln e}) = 3\ln e^2$

$2\cdot 3\,\cancel{\ln e}$ $\boxed{6}$

Express as a single logarithm

(condense)

5. $2\ln x - 4\ln y - \ln 13$

$\ln\left(\frac{x^2}{13y^4}\right)$

6. $\log_3 7 + \frac{1}{2}\log_3 x - 5\log_3 y$

$\log_3\left(\frac{7\sqrt{x}}{y^5}\right)$

$x^{1/2} = \sqrt{x}$

$y^{-5} = \frac{1}{y^5}$

Expand the following Logarithms

7. $\log \dfrac{x^2 y^3}{w z^3}$

$\log x^2 + \log y^3 - \log w - \log z^3$

$\boxed{2\log x + 3\log y - \log w - 3\log z}$

8. $\log \dfrac{b^3}{\sqrt{ac}} = \log \dfrac{b^3}{(ac)^{1/2}}$

$\boxed{3\log b - \frac{1}{2}(\log a + \log c) \quad \text{or} \quad -\frac{1}{2}\log a - \frac{1}{2}\log c}$

Solve Exponential & Logarithmic Equations

Solving Equations with Common Bases:

If $b^x = b^y$

Then $x = y$

$\log_b b^x = \log_b b^y$

$x \log_b \cancel{b} = y \log_b \cancel{b}$

$x = y$

Example 1: Solve the Equation

$$\left(\frac{1}{3}\right)^{-x+7} = \left(\frac{1}{3}\right)^{3x-1}$$

Same base, set exponents equal & solve

$$-x+7=3x-1$$

$$-4x=-8$$

$$x=2$$

Example 2: Solve the Equation

$$8^{x+2} = 16^{2x+7}$$

Rewrite with smaller base

$$\left(2^3\right)^{(x+2)} = \left(2^4\right)^{(2x+7)}$$

$$3(x+2) = 4(2x+7)$$

$$3x + 6 = 8x + 28$$

$$-5x = 22$$

$$x = -22/5$$

SOLVING EXPONENTIAL EQUATIONS WHEN YOU CAN'T GET A COMMON BASE

$$1.\ 3e_b^{4x^{\text{exp}}} = 45$$

$$\frac{3e^{4x}}{3} = \frac{45}{3}$$

$$e_b^{4x^{\text{exp}}} = \overset{\#}{15}$$

$$\log_e 15 = 4x$$

$$\ln(15) = 4x$$

$$x \approx .677$$

Isolate $\quad base^{exp} = \#$

Rewrite as log $\quad \log_{base}\# = exp$

2. $2(5^{2x}) - 1 = 47$

$b^{exp} = \#$

$\log_b \# = exp$

$\overset{exp}{5^{2x}} = \overset{\#}{24}$

$\log_5 24 = 2x$

$\cancel{\log_5} 5^{2x} = \log_5 24$

$2x = \log_5 24$

calc:

Math: A

or Alpha: Window 5

or $\log 24 \div \log 5 = 2x$

Ans $\div 2$

$x \approx .987$

3. $4 - 2e^x = -23$

$-4 \qquad -4$

$$\frac{-2e^x}{-2} = \frac{-27}{-2}$$

$e^x = 13.5$

$\ln 13.5 = x$

$x \approx 2.603$

Remember your Logarithm Properties!!!!

The Product Rule: $log_a MN = log_a M + log_a N$

The Power Rule: $\log_a M^p = p \cdot \log_a M$

The Quotient Rule: $\log_a \frac{M}{N} = \log_a M - \log_a N$

Solving Logarithmic Equations

1. $\log_3(5x - 1) = \log_3(x + 7)$

$\log_b x = \log_b y$

$x = x$

$5x - 1 = x + 7$

$4x = 8$

$x = 2$

$\log_3(5 \cdot 2 - 1) = \log_3(2+7)$

$\log_3 9 = \log_3 9$ ✓

condense to a single log 1st

2. $\log(x+6) = \log(8x) - \log(3x+2)$

$\cancel{\log}(x+6) = \cancel{\log}\dfrac{8x}{3x+2}$

$\dfrac{(x+6)}{1} = \dfrac{8x}{(3x+2)}$

$8x = (x+6)(3x+2)$

$8x = 3x^2 + 20x + 12$

$0 = 3x^2 + 12x + 12$

$0 = 3(x^2 + 4x + 4)$

$0 = 3(x+2)(x+2)$

$x + 2 = 0$

$x \neq -2$

No Solution

you can't take a log of 0 or − #

3. $\log_{5}(\overset{\#}{3x+1}) = \overset{exp}{2}$

b

$5^2 = 3x+1$

$25 = 3x+1$

$x = 8$

$\log_b \# = exp$

$\downarrow$

$b^{exp} = \#$

4. $\frac{4\ln(x+2)}{4} = \frac{6}{4}$

Isolate

$\log_b \# = exp$

Rewrite

$b^{exp} = \#$

$\ln(x+2) = 1.5$

$\log_e (x+2) = 1.5$

$e^{1.5} = x+2$

calc

$x \approx 2.482$

$$5.\ \log_2 4x + \log_2(x+3) = 4$$

$\log_2 4x(x+3) = 4$

$\log_2 (4x^2+12x) = 4$

b # exp

$2^4 = 4x^2+12x$

$16 = 4x^2+12x$

$0 = 4x^2+12x-16$

$0 = 4(x^2+3x-4)$

$\frac{0}{4} = \frac{4}{4}(x+4)(x-1)$

$x+4=0$

$x \neq -4$

$x-1=0$

$x=1$