

## Keeper 6.3 Virtual Problems – Riemann Sums to Integral

State the integral the sum is approximating.

1. RRS:  $\frac{1}{5} \left[ \ln\left(\frac{11}{5}\right) + \ln\left(\frac{12}{5}\right) + \ln\left(\frac{13}{5}\right) + \ln\left(\frac{14}{5}\right) + \ln(3) \right]$     2. LRS:  $\frac{1}{5} \left[ \ln\left(\frac{11}{5}\right) + \ln\left(\frac{12}{5}\right) + \ln\left(\frac{13}{5}\right) + \ln\left(\frac{14}{5}\right) + \ln(3) \right]$

3. RRS:  $\frac{1}{3} \left[ \sqrt[3]{\frac{4}{3}} + \sqrt[3]{\frac{5}{3}} + \sqrt[3]{2} \right]$

4. LRS:  $\frac{1}{3} \left[ \sqrt[3]{\frac{4}{3}} + \sqrt[3]{\frac{5}{3}} + \sqrt[3]{2} \right]$

Write the limit as a definite integral on the interval  $[a, b]$ , where  $c_i$  is any point in the  $i$ th subinterval.

5.  $\lim_{|\Delta| \rightarrow 0} \sum_{i=1}^n (3c_i + 10) \Delta x_i, \quad [-1, 5]$

6.  $\lim_{|\Delta| \rightarrow 0} \sum_{i=1}^n 6c_i(4 - c_i)^2 \Delta x_i, \quad [0, 4]$

7.  $\lim_{|\Delta| \rightarrow 0} \sum_{i=1}^n \left( \sqrt{c_i^2 + 4} \right) \Delta x_i, \quad [0, 3]$

8.  $\lim_{|\Delta| \rightarrow 0} \sum_{i=1}^n \left( \frac{3}{c_i^2} \right) \Delta x_i, \quad [1, 3]$

Write the summation as a limit.

9.  $\lim_{n \rightarrow \infty} \sum_{k=1}^n \left( \frac{k}{n} \right) \cdot \frac{1}{n}$

10.  $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \left( \frac{1}{1 + \frac{k}{n}} \right)$

$$11. \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(2 + \frac{k}{n}\right)^2 \cdot \frac{1}{n}$$

$$12. \lim_{n \rightarrow \infty} \frac{\pi}{2} \sum_{k=1}^n \left(\frac{k\pi}{2n}\right)$$

$$13. \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(1 + \frac{3k}{n}\right)^3 \cdot \frac{3}{n}$$

$$14. \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\left(\frac{k}{n}\right)^3 + 1\right) \cdot \frac{1}{n}$$

$$15. \lim_{n \rightarrow \infty} \frac{3}{n} \sum_{k=1}^n \left( \left(2 + \frac{3k}{n}\right)^2 - 2 \left(2 + \frac{3k}{n}\right) \right)$$

$$16. \lim_{n \rightarrow \infty} \sum_{k=1}^n \left[ \cos\left(\frac{k\pi}{6n}\right) \right] \cdot \frac{\pi}{6n}$$

$$17. \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{\sqrt{k}}{\sqrt{n^3}}$$

$$18. \lim_{n \rightarrow \infty} \left[ \frac{1}{n^3} + \frac{4}{n^3} + \frac{9}{n^3} + \cdots + \frac{n^2}{n^3} \right]$$