# AVERAGE FUNCTION VALUE AND MEAN VALUE THEOREM FOR INTEGRATION

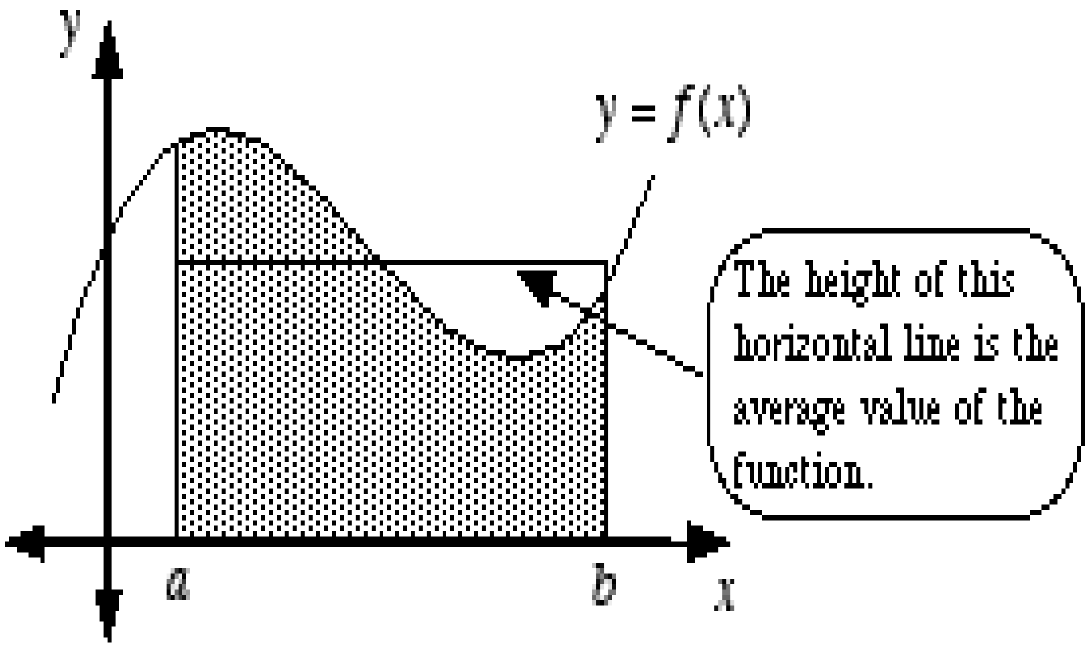
Honors Calculus

Keeper 42

# AVERAGE FUNCTION VALUE FOR INTEGRALS

Average value = $\frac{1}{b-a}\int_a^b f(x)\,dx$

The rectangle has the same area as the shaded region under the curve.

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# AVERAGE FUNCTION VALUE FOR INTEGRALS

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)dx$$

The area under the curve f(x) for the interval of [a, b] is equal to the area of the rectangle whose width is b-a and the height is equal to the average value.

# FIND THE AVERAGE VALUE OF THE FUNCTION OVER THE GIVEN INTERVAL

1. $f(x) = -x^2 + 2x + 1 \qquad [1,4]$

a b

$$f_{ave} = \frac{1}{4-1}\int_1^4 -x^2+2x+1\,dx$$

$$= \frac{1}{3}\left(-\frac{x^3}{3}+x^2+x \Big|_1^4\right)$$

$$= \frac{1}{3}\left(\left(-\frac{64}{3}+16+4\right)-\left(-\frac{1}{3}+1+1\right)\right)$$

$$\frac{1}{3}(-3) = -1$$

2. $f(x) = -2e^{2x+4}$  $[-3, -2]$

$\frac{1}{-2+3}\int_{-3}^{-2} -2e^{2x+4}\,dx$

$u = 2x+4$

$du = 2\,dx$

$dx = \frac{du}{2}$

$u_{-3} = -2$

$u_{-2} = 0$

$\int_{-3}^{-2} -\cancel{2}e^{u}\,\frac{du}{\cancel{2}}$

Quick

$\frac{-\cancel{2}e^{2x+4}}{\cancel{lne}\ \cancel{2}}$

$-e^{2x+4}\Big|_{-3}^{-2}$

$-e^{0} - -e^{-2}$

$-e^{u}\Big|_{-2}^{0} = -e^{0} - -e^{-2}$

$-1 + \frac{1}{e^{2}}$

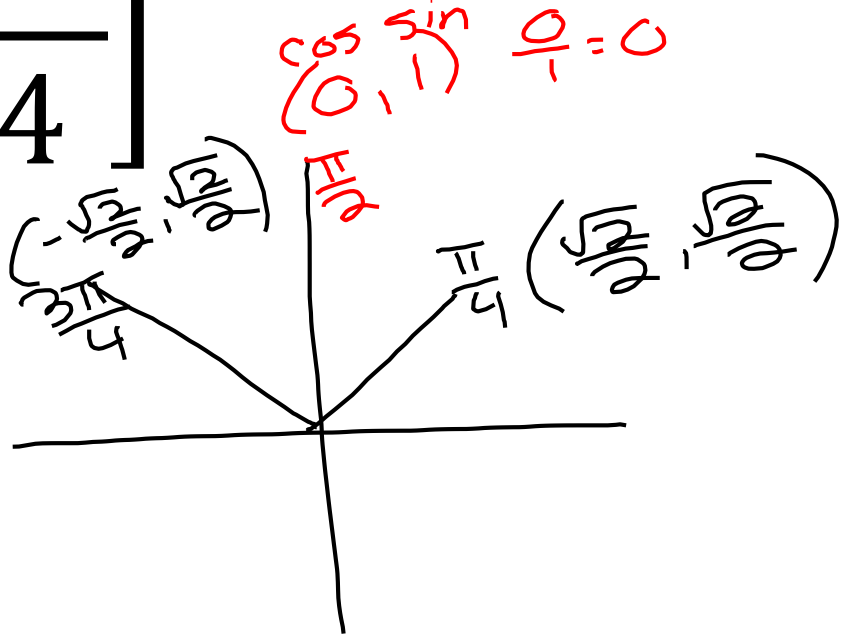
3. $f(x) = \csc^2 x$ $\quad \left[\frac{\pi}{2}, \frac{3\pi}{4}\right]$

$$\frac{1}{\frac{3\pi}{4}-\frac{\pi}{2}}\int_{\pi/2}^{3\pi/4} \csc^2 x \, dx$$

$$\frac{1}{\frac{\pi}{4}}\left(-\cot x \Big|_{\pi/2}^{3\pi/4}\right)$$

$$\frac{4}{\pi}\left(-\cot\frac{3\pi}{4} + \cot\frac{\pi}{2}\right)$$

$$\frac{4}{\pi}\left(-(-1)+0\right) = \frac{4}{\pi}$$



# THE MEAN VALUE THEOREM FOR INTEGRALS

$f(c) = \frac{1}{b-a} \cdot \int_a^b f(x)dx$

$(b-a)f(c) = \int_a^b f(x)dx$

$$\int_a^b f(x)dx = f(c)(b-a)$$

There is a rectangle whose area is precisely equal to the area of the region under the curve of the entire interval.

# FIND THE VALUE OF C GUARANTEED BY THE MVT

$$1.\ f(x) = \frac{5}{x^2} \quad [1,4]$$

$$(b-a)\,f(c) = \int_a^b f(x)\,dx$$

$$(4-1)\,\frac{5}{c^2} = \int_1^4 \frac{5}{x^2}\,dx \qquad \frac{5x^{-2+1}}{-1}$$

$$\frac{15}{c^2} = \left.\frac{-5}{x}\right|_1^4$$

$$\frac{15}{c^2} = \frac{-5}{4} + \frac{+5}{1}$$

$$\frac{15}{c^2} = \frac{15}{4}$$

$$15c^2 = 60$$

$$c^2 = 4$$

$$c = \pm 2$$

$c = 2$ only bc $-2$ isn't in the int. $[1,4]$

# 2. $f(x) = -x + 1 \; [-6, -5]$

$(-5 - -6)(-c+1) = \int_{-6}^{-5} -x+1 \, dx$

$-c+1 = \frac{-x^2}{2} + x \Big|_{-6}^{-5}$

$-c+1 = \left(-\frac{25}{2} - 5\right) - \left(-18 - 6\right)$

$-c+1 = 6.5 \quad -c = 5.5$

$+c = -5.5$

$$3.\ f(x) = 3x^{\frac{1}{2}} \quad [0, 9]$$

$$(9-0)\,3c^{1/2} = \int_0^9 3x^{1/2}\,dx$$

$$27\sqrt{c} = \tfrac{2}{3} 3x^{3/2} \rightarrow 2x^{3/2}\Big|_0^9$$

$$27\sqrt{c} = 2\sqrt{9}^3 - 2\sqrt{0}^3$$

$$27\sqrt{c} = 54$$

$$\sqrt{c} = 2$$

$$c = 4$$