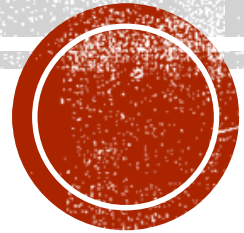
VOLUMES OF REVOLUTION

Honors Calculus

Keeper 38



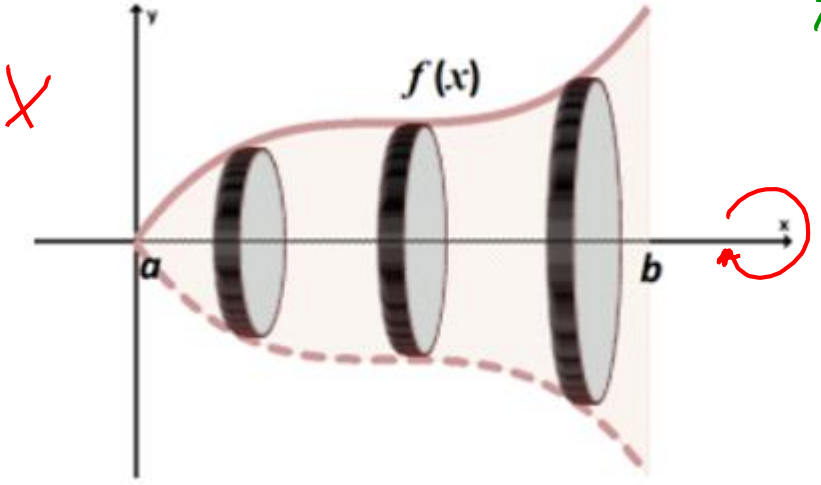
THE DISK METHOD

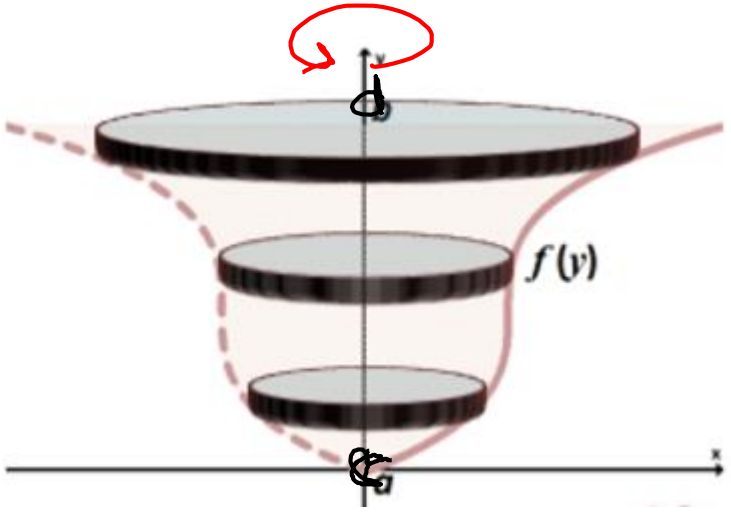
To find the volume of a solid of revolution with the disk method, use one of the following:

Horizontal Axis of Revolution	Vertical Axis of Revolution
$Volume = V = \pi \int_a^b [R(x)]^2 \, dx$	$Volume = V = \pi \int_c^d [R(y)]^2 \, dy$

Area of disk
πr^2

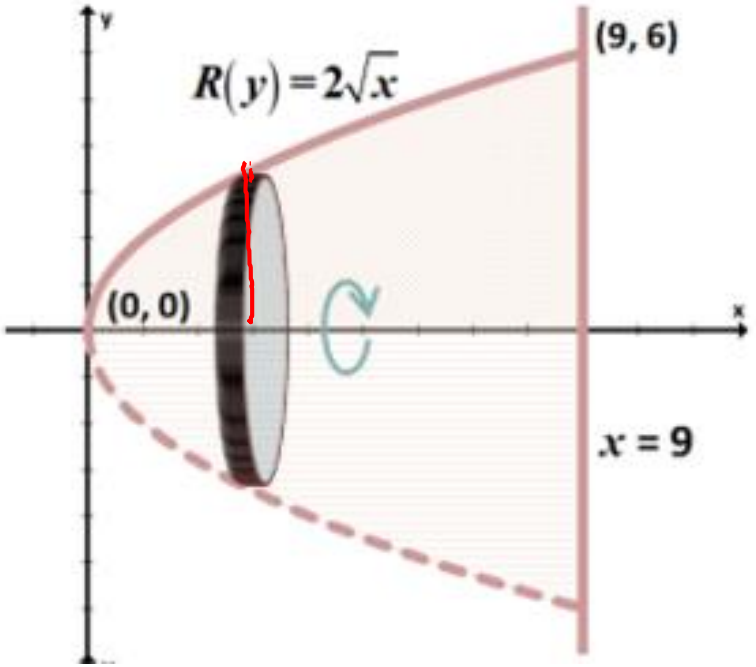
over x
dx





over y
dy

1. Find the volume of the solid formed by revolving the region bounded by the graph of $f(x) = 2\sqrt{x}, y = 0$ & $x = 9$ about the $x - axis$.



top − bottom = $2\sqrt{x} - 0 = 2\sqrt{x}$

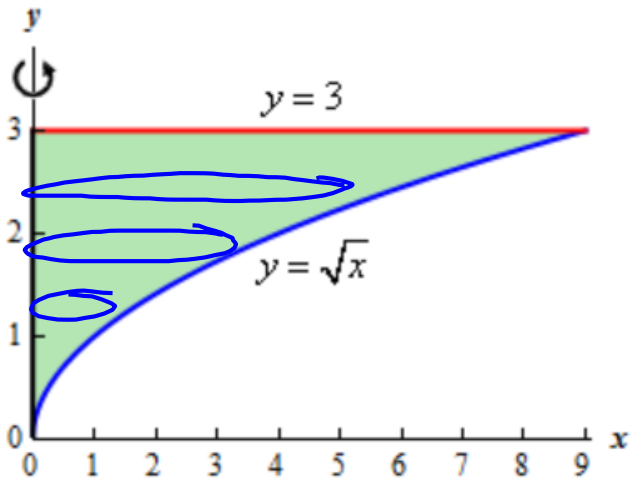
$$V = \pi \int_0^9 (2\sqrt{x})^2 \, dx$$

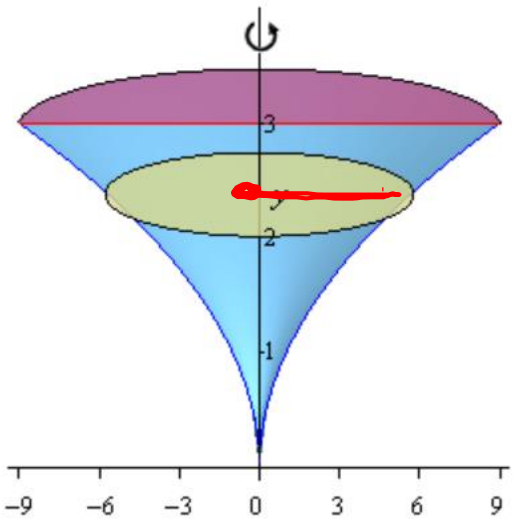
$$V = \pi \int_0^9 4x \, dx$$

$$V = \pi \left(2x^2 \Big|_0^9\right)$$

$$V = 162\pi$$

2. Determine the volume of the solid by rotating the region bounded by $y = \sqrt{x}\ and\ y = 3$, about the $y - axis$.





over $y \rightarrow dy$

$y = (\sqrt{x})^2$

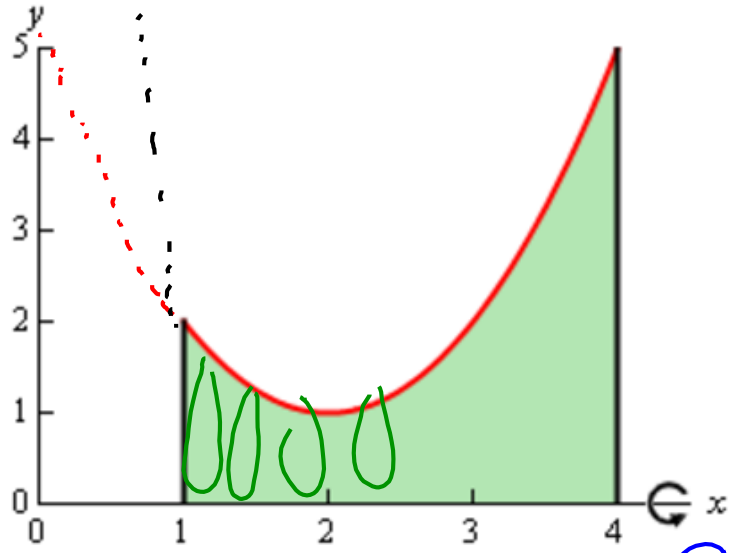
$x = y^2$

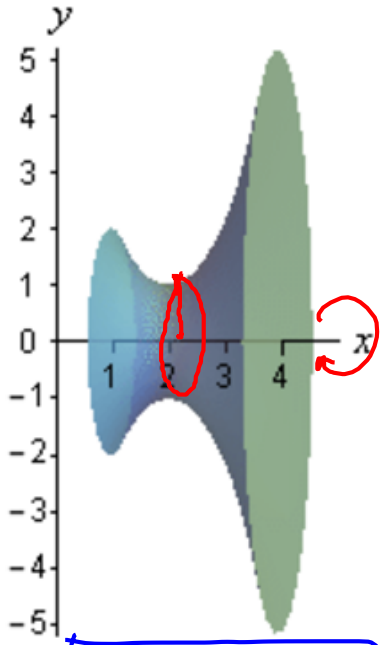
$$V = \pi \int_0^3 (y^2)^2\, dy$$

$$V = \pi \left(\frac{y^5}{5} \Big|_0^3 \right)$$

$$V = \frac{243\pi}{5}$$

3. Determine the volume of the solid by rotating the region bounded by $y = x^2 - 4x + 5, x = 1, x = 4$, and the $x - axis$ about the $x - axis$.





$R = x^2 - 4x + 5$

$V = \pi \int_1^4 (x^2 - 4x + 5)^2 \, dx$

$V = \frac{78\pi}{5}$

Calc: MATH 9

THE WASHER METHOD

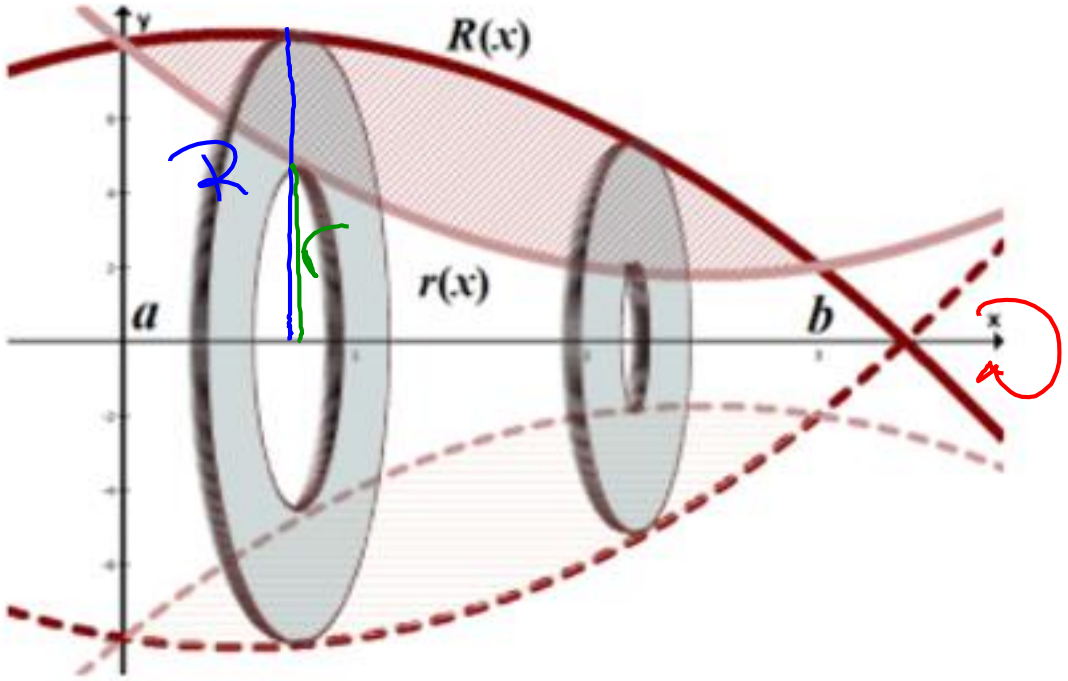
Use the washer method for solids of revolution with holes.

R = outside radius

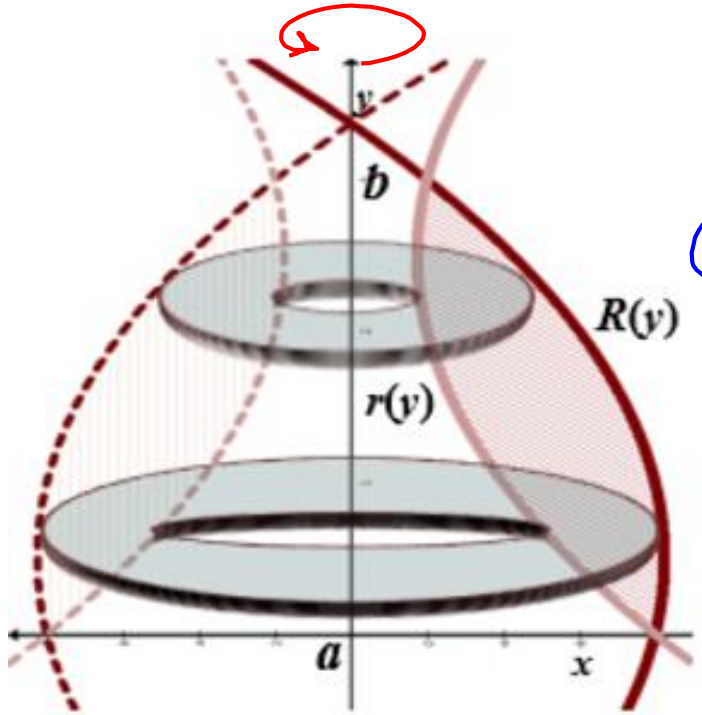
r = inside radius

$$V = \pi \int_a^b ([R(x)]^2 - [r(x)]^2)\, dx$$

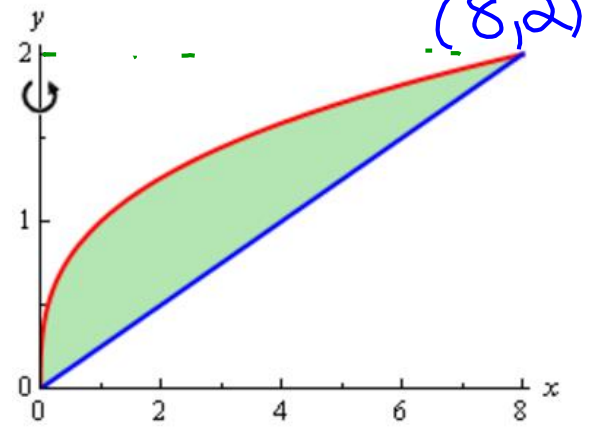
dx



dy



1. Determine the volume of the solid by rotating the portion of the region bounded by $y = \sqrt[3]{x}$ and $y = \frac{x}{4}$ that lies in the first quadrant about the $y-axis$.



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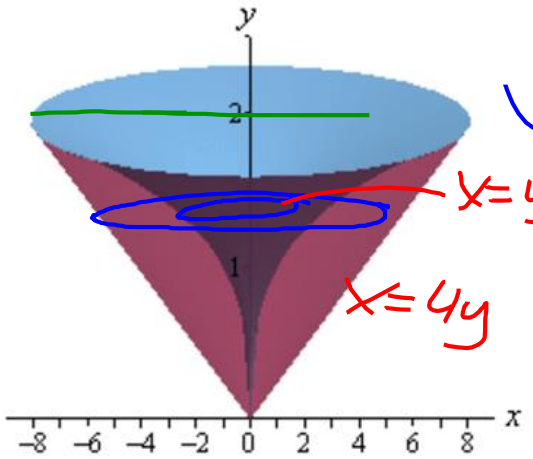


$x = y^3$

$x = 4y$

washer

$y^3 = \sqrt[3]{x}^3$

$y^3 = x$

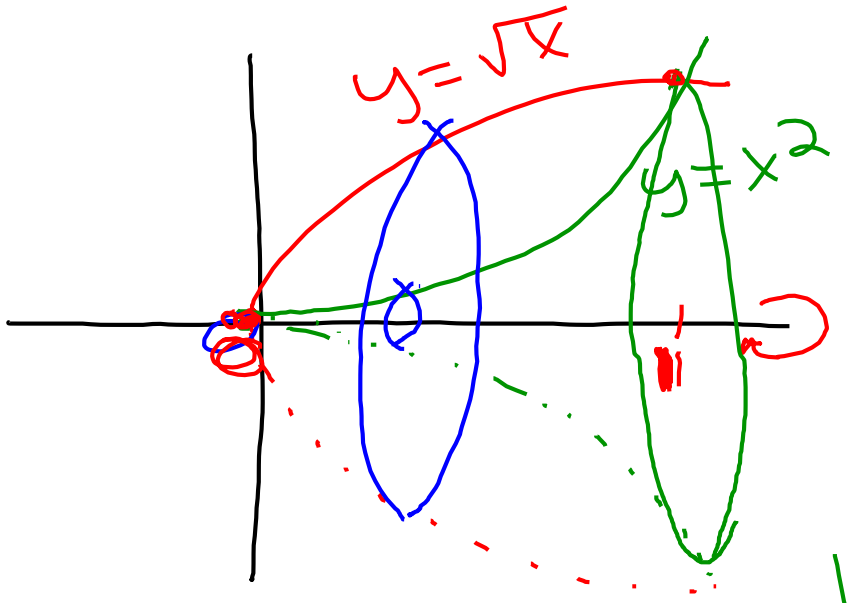
$4 \cdot y = \frac{x}{4} \cdot 4$

$x = 4y$

$$V = \pi \int_0^2 (4y)^2 - (y^3)^2 \, dy$$

$$V = \frac{512\pi}{21}$$

2. Find the volume of the solid formed by revolving the region bounded by the graphs of $y = \sqrt{x}$ and $y = x^2$ about the **x-axis**.



$R = \sqrt{x}$ $\quad r = x^2$

Intersection: $(x^2)^2 = (\sqrt{x})^2$

$x^4 = x$

$x^4 - x = 0$

$x(x^3 - 1) = 0$

$x = 0 \quad x = 1$

$$V = \pi \int_0^1 (\sqrt{x})^2 - (x^2)^2 \, dx$$

$$V = \frac{3\pi}{10}$$