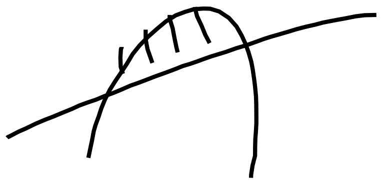
AREA BETWEEN TWO CURVES

Honors Calculus

Keeper 34

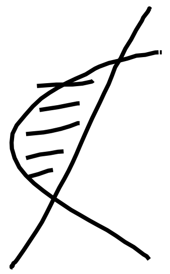
AREA OF A REGION BETWEEN TWO CURVES

$$A = \int_a^b [f(x) - g(x)]dx$$



$$\boldsymbol{TOP - BOTTOM} \;\; \longrightarrow \;\; \mathbf{dx}$$

OR

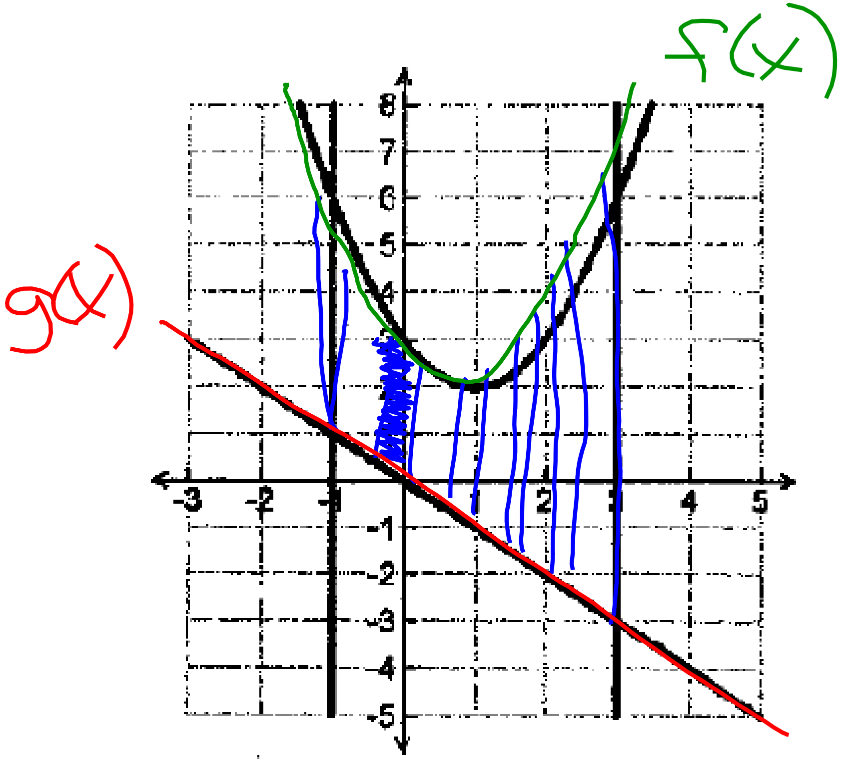


$$\boldsymbol{RIGHT \; - LEFT} \;\; \longrightarrow \;\; \mathbf{dy}$$

FIND THE AREA OF THE REGION BOUNDED BY THE GRAPHS

1. $f(x) = x^2 - 2x + 3$

   $g(x) = -x$



$$A = \int_{-1}^{3} (x^2 - 2x + 3) - (-x)\, dx$$

$$A = \int_{-1}^{3} x^2 - x + 3\, dx$$

$$A = \frac{x^3}{3} - \frac{x^2}{2} + 3x \Big|_{-1}^{3}$$

$$\left(\frac{3^3}{3} - \frac{3^2}{2} + 3 \cdot 3\right) - \left(\frac{-1}{3} - \frac{1}{2} - 3\right)$$

$$9 - \frac{9}{2} + 9 + \frac{1}{3} + \frac{1}{2} + 3$$

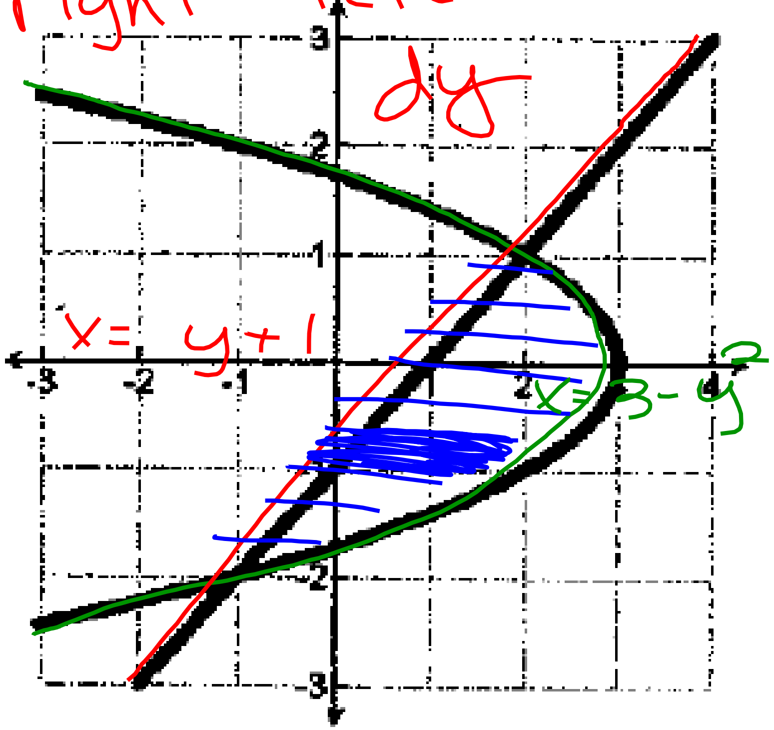
$$A = \frac{52}{3}$$

FIND THE AREA OF THE REGION BOUNDED BY THE GRAPHS

2. $x = 3 - y^2$

$y = x - 1$ $\quad x = y + 1$

right − left



$$A = \int_{-2}^{1} (3 - y^2) - (y+1)$$

$$\int_{-2}^{1} -y^2 - y + 2 \, dy$$

$$-\frac{y^3}{3} - \frac{y^2}{2} + 2y \Big|_{-2}^{1}$$

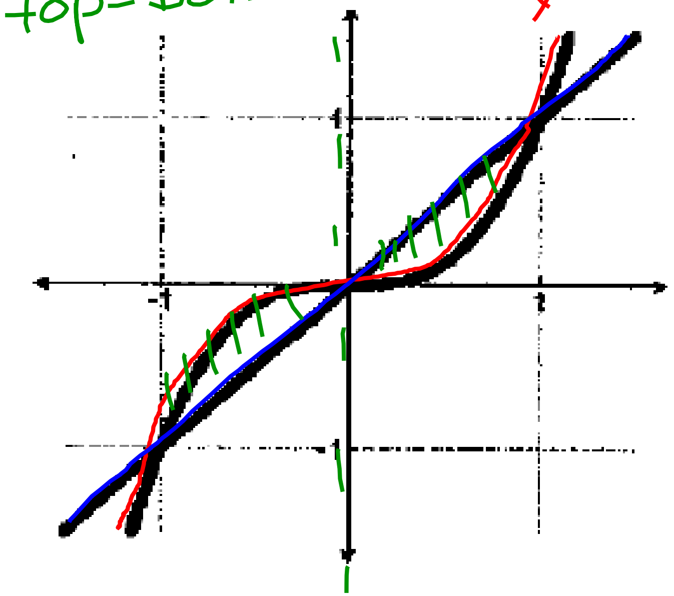
$$\left(-\frac{1}{3} - \frac{1}{2} + 2\right) - \left(\frac{8}{3} - 2 - 4\right)$$

$$A = \frac{9}{2} \text{ or } 4.5$$

FIND THE AREA OF THE REGION BOUNDED BY THE GRAPHS

3. $f(x) = x^3$

$g(x) = x$



$$A = \int_{-1}^{0} x^3 - x \;+\; \int_{0}^{1} x - x^3 \; dx$$

$$\left.\frac{x^4}{4} - \frac{x^2}{2}\right|_{-1}^{0} + \left.\frac{x^2}{2} - \frac{x^4}{4}\right|_{0}^{1}$$

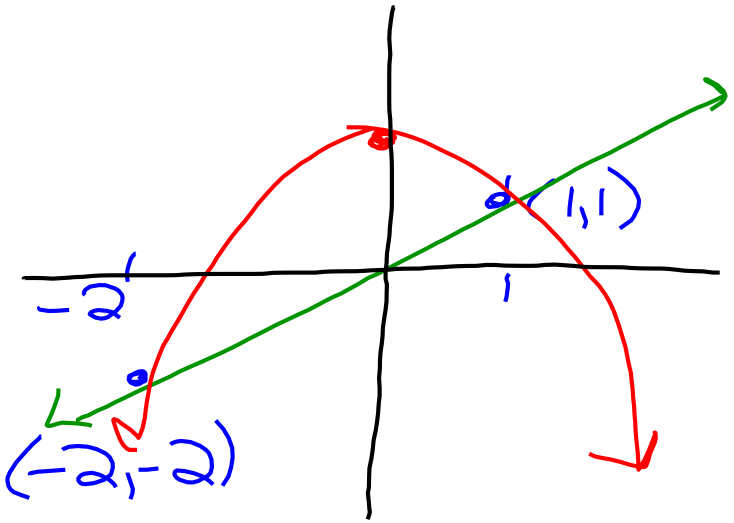
$$(0-0) - \left(\frac{1}{4} - \frac{1}{2}\right) \qquad \left(\frac{1}{2} - \frac{1}{4}\right) - (0)$$

$$-\frac{1}{4} + \frac{1}{2}$$

$$\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

FIND THE AREA OF THE REGION BOUNDED BY THE GRAPHS

4. $f(x) = 2 - x^2, \qquad g(x) = x$



$2 - x^2 = x$

$0 = x^2 + x - 2$

$0 = (x+2)(x-1)$

$x = -2 \quad x = 1$

$$\left. -\frac{x^3}{3} - \frac{x^2}{2} + 2x \right|_{-2}^{1}$$

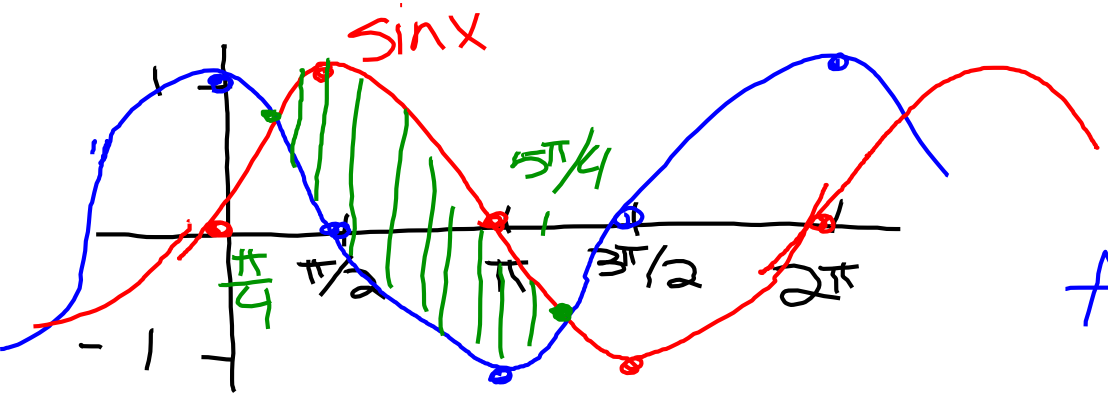
$$\left(-\frac{1}{3} - \frac{1}{2} + 2\right) - \left(\frac{8}{3} - 2 - 4\right)$$

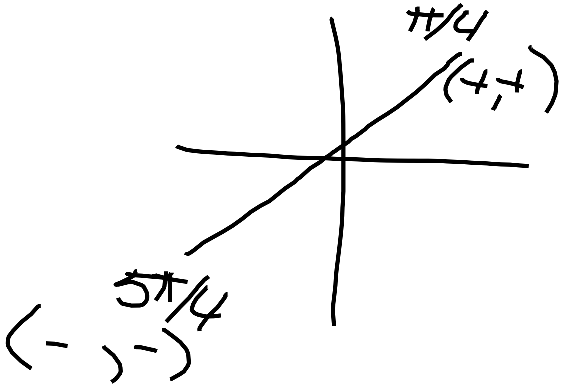
$$A = \frac{9}{2} \text{ or } 4.5$$

FIND THE AREA OF THE REGION BOUNDED BY THE GRAPHS

5. $f(x) = \sin x\,,\quad g(x) = \cos x$

Find the area for one of the repeated regions!!!





$$A = \int_{\pi/4}^{5\pi/4} \sin x - \cos x \; dx$$

$$A = -\cos x - \sin x \Big|_{\pi/4}^{5\pi/4}$$

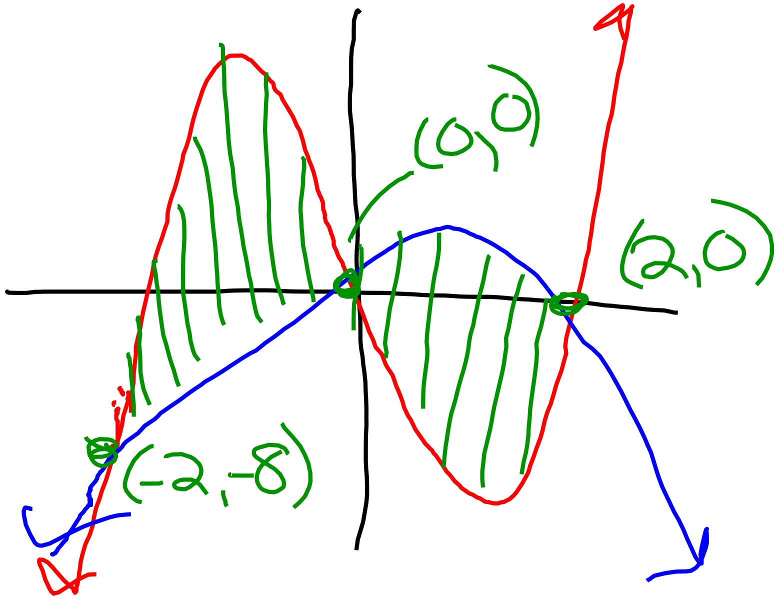
$$\left(-\cos\tfrac{5\pi}{4} - \sin\tfrac{5\pi}{4}\right) - \left(-\cos\tfrac{\pi}{4} - \sin\tfrac{\pi}{4}\right)$$

$$\left(\tfrac{\sqrt{2}}{2} + \tfrac{\sqrt{2}}{2}\right) - \left(-\tfrac{\sqrt{2}}{2} - \tfrac{\sqrt{2}}{2}\right)$$

$$\frac{\cancel{2}\sqrt{2}}{\cancel{2}} - -\frac{\cancel{2}\sqrt{2}}{\cancel{2}} = \sqrt{2} + \sqrt{2} = 2\sqrt{2}$$

FIND THE AREA OF THE REGION BOUNDED BY THE GRAPHS

6. $f(x) = 3x^3 - x^2 - 10x, \qquad g(x) = -x^2 + 2x$



$$\int_{-2}^{0} (3x^3-x^2-10x)-(-x^2+2x)\,dx$$

$$\int_{-2}^{0} 3x^3-12x\,dx = \frac{3x^4}{4} - \frac{\overset{6}{\cancel{12}}x^2}{\cancel{2}}\Bigg|_{-2}^{0}$$

$$\cancel{(0-0)}-(12-24) = 12$$ ← 1st section

$$\int_{0}^{2} (-x^2+2x)-(3x^3-x^2-10x)\,dx$$

$$\int_{0}^{2} -3x^3+12x\,dx = \frac{-3x^4}{4}+6x^2\Bigg|_{0}^{2}$$

$$(-12+24)-(0-0)= 12$$ ← 2nd section

$A = 12 + 12$

$A = 24$