DERIVATIVES OF LOGARITHMIC AND EXPONENTIAL FUNCTIONS

Keeper 23

Honors Calculus

PROPERTIES OF LOGS REVIEW

Also on p.13

The Product Rule: $log_a MN = log_a M + log_a N$

The Quotient Rule: $\log_a \frac{M}{N} = \log_a M - \log_a N$

The Power Rule: $\log_a M^p = p \cdot \log_a M$

DERIVATIVE OF NATURAL LOGS

Basic: $\frac{d}{dx}\ln x = \frac{1}{x}$

Other natural logs:

$$\frac{d}{dx}\ln(f(x)) = \frac{\mathbf{1}}{\boldsymbol{f}(\boldsymbol{x})} \cdot \boldsymbol{f}'(\boldsymbol{x})$$

***In other words, "1 over the function times the derivative of the function"

FIND THE DERIVATIVE

function

1. $y = \ln(2x^2 + 1)$

$y' = \frac{1}{fxn} \cdot \frac{d}{dx} fxn$

$y' = \frac{1}{2x^2+1} \cdot \frac{4x}{1}$

$y' = \frac{4x}{2x^2+1}$

2. $y = \ln(\tan x)$

$y' = \frac{1}{\tan x} \cdot \sec^2 x$

$y' = \frac{\sec^2 x}{\tan x}$

3. $y = \overset{\text{1st}}{(\cos x)}\overset{\text{2nd}}{(2\ln x)}$ Product

1st $\cdot \frac{d}{dx}$ 2nd + 2nd $\cdot \frac{d}{dx}$ 1st

$y' = (\cos x)\left(2\cdot\frac{1}{x}\right) + (2\ln x)(-\sin x)$

$y' = \frac{2}{x}\cos x - 2\ln x \sin x$

4. $y = \ln\left(\overset{fxn}{\frac{3x}{3-x}}\right)$

$$\frac{1}{\frac{3x}{3-x}} = 1 \div \frac{3x}{3-x}$$

$$1 \cdot \frac{3-x}{3x} = \frac{3-x}{3x}$$

$$y' = \frac{1}{fxn} \cdot \frac{d}{dx}\, fxn$$

$$\frac{d}{dx} = \frac{3x}{3-x} \quad \text{Quotient Rule}$$

$$y' = \left(\frac{3-x}{3x}\right)\left(\frac{\overset{9-3x}{(3-x)(3)} - \overset{+3x}{(3x)(-1)}}{(3-x)^2}\right)$$

$$\frac{\cancel{(3-x)}}{\cancel{3}x} \cdot \frac{\cancel{9}\,3}{(3-x)^{\cancel{2}}}$$

$$y' = \frac{3}{x(3-x)}$$

5. $y = \ln(7-x)^4$

use prop. of logs to Rewrite 1st!

$y = 4\ln(7-x)$

$y' = 4 \cdot \frac{1}{7-x} \cdot -1$

$y' = \frac{-4}{7-x}$

6. $y = \ln e^{x^7}$

Prop. of Logs

$y = x^7 \cdot \ln e$

$\ln e = 1$

$y = x^7$

x^7

$\ln e^{\sin x}$

$y' = 7x^6$

$y = \sin x$

$e^{\ln x^7} = x^7$

$\ln e^{\sqrt{x+2}} = \sqrt{x+2}$

IF YOU HAVE A LOG OF ANY OTHER BASE…USE THE CHANGE OF BASE FORMULA

Change of Base formula: $log_b a = \dfrac{\ln a}{\ln b}$

$$log_7 x = \frac{\ln x}{\ln 7}$$

FIND THE DERIVATIVE

1. $log_8 x$

Rewrite: $y = \frac{\ln x}{\ln 8}$

$y = \frac{1}{\ln 8} \ln x$ constant

$y' = \frac{1}{(\ln 8)} \cdot \frac{1}{x}$

$y' = \frac{1}{x \ln 8}$

Log of any other base...

rewrite w/ change of base formula $\ln_b a = \frac{\ln a}{\ln b}$

$\frac{\ln \#}{\ln \text{base}}$

like $2 \ln x$

$2 \cdot \frac{1}{x} = \frac{2}{x}$

2. $log_5(cosx)$

DERIVATIVE OF EXPONENTIALS RULES

$$\frac{d}{dx}a^u = a^u \cdot \ln a \cdot \frac{d}{dx}(u)$$

***In other words…copy the problem, ln of the base, times the derivative of the exponent

Easiest one… $\frac{d}{dx}e^x = e^x \cdot 1 = e^x$

FIND THE DERIVATIVE

1. $y = 3^x$

copy $\cdot$ ln base $\cdot$ $\frac{d}{dx}$ exp.

$y' = 3^x \cdot \ln 3 \cdot 1$

$y' = 3^x \ln 3$

1st 2nd

$2.\ y = 2^x \cdot (x^2 + 1)$ Product

1st $\cdot \frac{d}{dx}$ 2nd + 2nd $\cdot \frac{d}{dx}$ 1st

copy ln base $\frac{d}{dx}$ exp

$y = 2^x \cdot 2x + (x^2+1)\, 2^x \ln 2 \cdot 1$

3. $y = 3^{\ln x}$

ln base $\qquad \frac{d}{dx} \ln x$

$$y' = 3^{\ln x} \cdot \ln 3 \cdot \frac{1}{x}$$

$$y' = \frac{3^{\ln x} \ln 3}{x}$$

4. $y = e^{2x}$

copy · ln base · $\frac{d}{dx}$ exp

$y' = e^{2x} \cdot \cancel{\ln e}^{\,1} \cdot 2$

$y' = 2 \cdot e^{2x}$

5. $y = e^{\cos x}$

$y' = e^{\cos x} \cdot \cancel{\ln e'} \cdot -\sin x$

$y' = -\sin x \cdot e^{\cos x}$

or

$-e^{\cos x} \sin x$

6. $y = e^{x^3+5x}$

$y' = e^{x^3+5x} \cdot \cancel{\ln e} \cdot (3x^2+5)$

$y' = e^{x^3+5x}(3x^2+5)$

7. $y = e^{\ln x^5}$

Same as $y = x^5$ → bc

$y' = 5x^4$

$e^{\ln \ldots}$

or

$\ln e^{\ldots}$

$y = e^{\ln x^5}$

$\ln y = \ln x^5 \ln e$

$\cancel{\ln} y = \cancel{\ln} x^5 \cancel{\ln e}$

$y = x^5$

$y = e^{\ln \cos x} \rightarrow y = \cos x$

$y = e^{\ln(x^2+1)} \rightarrow y = x^2+1$