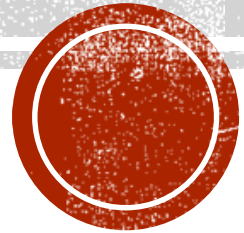
THE POWER AND SUM-DIFFERENCE RULES

Honors Calculus

Keeper 22



NOTATIONS

When y is a function of x, we will denote the derivative using

$$y' \text{ or } f'(x) \text{ or } \frac{dy}{dx}$$

The formula $\frac{dy}{dx}$ is read as the derivative of y with respect to x.

THE POWER RULE

For any real number k,

$$\frac{dy}{dx} x^k = k \cdot x^{k-1}$$

EXAMPLES – FIND THE DERIVATIVE

1. $y = x^5$ $= 5x^{5-1}$ $y' = 5x^4$
2. $y = x^1$ $1x^{1-1} = 1x^0$ $y' = 1$
3. $y = x^{-4}$ $-4x^{-4-1} = -4x^{-5}$ $y' = \frac{-4}{x^5}$
4. $y = \sqrt[2]{x}^1$ $y = x^{1/2}$ $y' = \frac{1}{2}x^{1/2-1}$ $y' = \frac{1}{2}x^{-1/2}$ $y' = \frac{1}{2x^{1/2}}$ or $y' = \frac{1}{2\sqrt{x}}$

THE DERIVATIVE OF A CONSTANT

The derivative of a constant function is 0.

$$\frac{d}{dx}c = 0$$

EXAMPLE – FIND THE DERIVATIVE

1. $y = 94$ same as $y = 94x^0$ $y' = 0$

$0 \cdot 94$
0

2. $y = \pi^2$ $y' = 0$

This is a constant

THE DERIVATIVE OF A CONSTANT AND A FUNCTION

The derivative of a constant times a function is the constant times the derivative of the function.

$$\frac{d}{dx}[c \cdot f(x)] = c \cdot \frac{d}{dx} f(x)$$

EXAMPLE – FIND THE DERIVATIVE

1. $f(x) = 7x^4$ $7 \cdot \frac{dy}{dx} x^4$ $7 \cdot 4x^3$ $f'(x) = 28x^3$
2. $f(x) = -9x^1$ $-9 \cdot 1x^{1-1}$ $-9x^0$ $f'(x) = -9$
3. $f(x) = \frac{1}{5x^2}$ $\frac{1}{5} \cdot \frac{dy}{dx} x^{-2}$ $f(x) = \frac{1}{5} x^{-2}$ $y' = -\frac{2}{5} x^{-3}$ $f'(x) = \frac{-2}{5x^3}$

DERIVATIVE - SUM-DIFFERENT RULE

Sum: The derivative of a sum is the sum of the derivatives.

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

Different: The derivative of a difference is the difference of the derivative.

$$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

EXAMPLES – FIND THE DERIVATIVE

1. $y = 5x^3 - 12$

$y' = 3 \cdot 5x^{3-1} - 0$ ← deriv. of constant

$y' = 15x^2$

EXAMPLE – FIND THE DERIVATIVE

$$2.\ y = 24x - \sqrt{x} + \frac{5}{x^1}$$

$y = 24x^1 - x^{1/2} + 5x^{-1}$

$y' = 24x^0 - \frac{1}{2}x^{-1/2} - 5x^{-2}$

$y' = 24 - \frac{1}{2\sqrt{x}} - \frac{5}{x^2}$

also $\frac{1}{2x^{1/2}}$

EXAMPLE – FIND THE DERIVATIVE

3. $y = 3x^5 + 2\sqrt[3]{x}' + \frac{1}{3x^2} + \sqrt{5}$

$y = 3x^5 + 2x^{1/3} + \frac{1}{3}x^{-2} + \sqrt{5}$

$y' = 15x^4 + \frac{2}{3}x^{-2/3} - \frac{2}{3}x^{-3} + 0$

or $y' = 15x^4 + \frac{2}{3x^{2/3}} - \frac{2}{3x^3}$

$y' = 15x^4 + \frac{2}{3\sqrt[3]{x^2}} - \frac{2}{3x^3}$

FIND THE DERIVATIVE

4. $y = 3x\left(x^2 - \frac{2}{x}\right)$ Multiply

$y = 3x^3 - \frac{6x}{x}$

$y = 3x^3 - 6$

$y' = 9x^2$

FIND THE DERIVATIVE

$$5.\ y = \frac{x^3 - 3x^2 + 4}{x^2}$$

Simplify 1st when dividing by a monomial

$$y = \frac{x^3}{x^2} - \frac{3x^2}{x^2} + \frac{4}{x^2}$$

$$y = 1x^1 - 3 + 4x^{-2}$$

$$y' = 1x^0 - 0 - 8x^{-3}$$

$$y' = 1 - \frac{8}{x^3}$$

FIND THE DERIVATIVES

6. $y = \dfrac{(x+1)^2}{4x}$ $(x+1)(x+1)$ Find $y', y'', and\ y'''$

$y = \dfrac{x^2+2x+1}{4x}$

$y = \dfrac{1}{4}x^2 + \dfrac{2x}{4x} + \dfrac{1}{4x}$

$y = \dfrac{1}{4}x + \dfrac{1}{2} + \dfrac{1}{4}x^{-1}$ $-\frac{1}{4}x^{-2}$

$y' = \dfrac{1}{4} - \dfrac{1}{4x^2}$

$y' = \frac{1}{4} - \frac{1}{4}x^{-2}$

$y'' = 0 + \frac{1}{2}x^{-3}$

$y'' = \dfrac{1}{2x^3}$

$y''' = \frac{1}{2}x^{-3}$

$y''' = \frac{-3}{2}x^{-4}$

$y''' = \dfrac{-3}{2x^4}$

7. $g(t) = \frac{t^3 - 4t^2 + 6}{\sqrt{t}}$ Find $\frac{dg}{dt}$ at (4, 3)

$g(t) = \frac{t^3}{t^{1/2}} - \frac{4t^2}{t^{1/2}} + \frac{6}{t^{1/2}}$

$g(t) = t^{5/2} - 4t^{3/2} + 6t^{-1/2}$

$g'(t) = \frac{5}{2}t^{3/2} - 6t^{1/2} - 3t^{-3/2}$

$g'(t) = \frac{5\sqrt{t^3}}{2} - 6\sqrt{t} - \frac{3}{\sqrt{t^3}}$

$g'(4) = \frac{5\sqrt{4^3}}{2} - 6\sqrt{4} - \frac{3}{\sqrt{4^3}}$

$20 - 12 - \frac{3}{8}$

$\frac{61}{8}$