DEFINITION OF A DERIVATIVE

Keeper 20

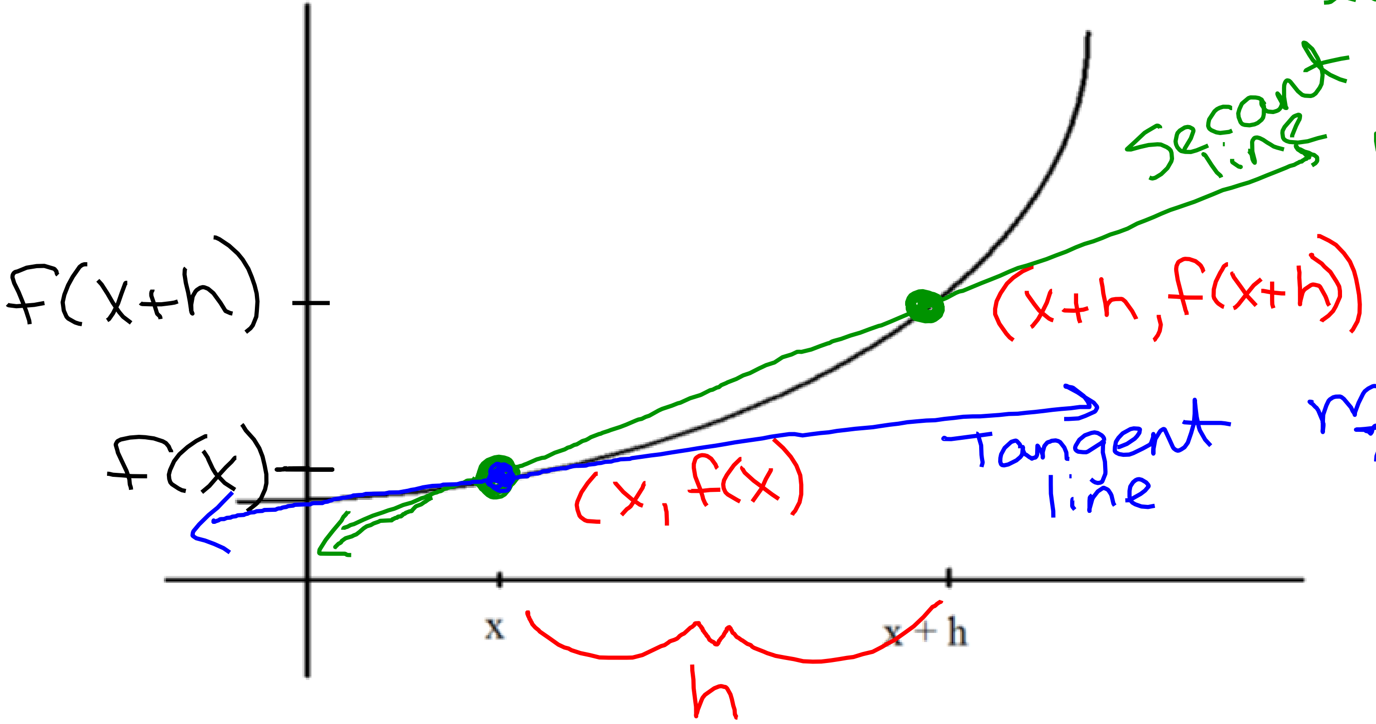
Honors Calculus

Label the following on the picture below:

$f(x)$, $f(x+h)$, h, $f(x+h) - f(x)$ and a segment whose slope represents $\frac{f(x+h)-f(x)}{h}$ and $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$$m_{sec} = \frac{f(x+h) - f(x)}{\cancel{x}+h - \cancel{x}}$$

$$m_{sec} = \frac{f(x+h) - f(x)}{h}$$



$$m_{Tan} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

SLOPE OF THE SECANT LINE

$$\xcancel{m_{sec} = \frac{f(x+h) - f(x)}{h}}$$

This formula represents the **AVERAGE** rate of change

SLOPE OF THE TANGENT LINE

$$m_{tan} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

This formula represents the **INSTANTANEOUS** rate of change

~~Compare and contrast the~~ average rate of change with the instantaneous rate of change (derivative).

Similarities

Both are rates of change

Both are slopes

$$\frac{f(x+h) - f(x)}{h}$$

Differences

IROC is a limit; AROC isn't

One is an avg. rate + the other is instantaneous

AROC uses 2 pts on graph + IROC uses only 1 pt

DERIVATIVE

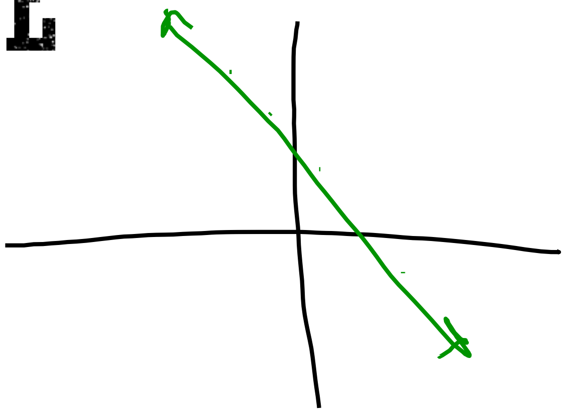
- Slope of a tangent line
- Rate of change at 1 point
- Instantaneous rate of change
- Denoted by y', $f'(x)$, or $\frac{dy}{dx}$ "deriv. of y with respect to x"

DEFINITION OF A DERIVATIVE

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

FIND THE DERIVATIVE USING THE DEFINITION

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$



1. $f(x) = -3x + 2$

$$f'(x) = \lim_{h\to 0} \frac{\overbrace{-3(x+h)+2}^{f(x+h)} - \overbrace{(-3x+2)}^{f(x)}}{h}$$

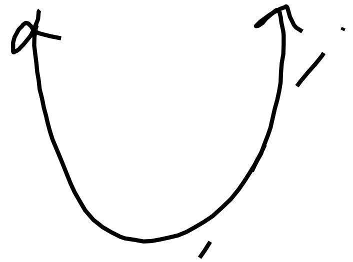
$$= \lim_{h\to 0} \frac{\cancel{-3x} - 3h + \cancel{2} \cancel{+3x} - \cancel{2}}{h} = \lim_{h\to 0} \frac{-3\cancel{h}}{\cancel{h}}$$

$$f'(x) = \lim_{h\to 0} -3 \qquad \boxed{f'(x) = -3}$$

Linear equations have a constant rate of change which is why the deriv. is the slope of the prob.

FIND THE DERIVATIVE

$(x+h)^2 = (x+h)(x+h)$ $\quad x^2+2xh+h^2$



2. $y = 3x^2 - 2x + 4$

$$y' = \lim_{h \to 0} \frac{3(x+h)^2 - 2(x+h) + 4 - (3x^2 - 2x + 4)}{h}$$

$$y' = \lim_{h \to 0} \frac{3(x^2+2xh+h^2) - 2x - 2h + 4 - 3x^2 + 2x - 4}{h}$$

$$y' = \lim_{h \to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 \cancel{- 2x} - 2h \cancel{+ 4} \cancel{- 3x^2} \cancel{+ 2x} \cancel{- 4}}{h}$$

$$y' = \lim_{h \to 0} \frac{\cancel{h}(6x + 3h - 2)}{\cancel{h}} \rightarrow \lim_{h \to 0} 6x + 3(0) - 2$$

$$y' = 6x - 2$$

FIND THE DERIVATIVE

3. $y = \dfrac{1}{x+1}$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{\frac{(x+1)}{(x+1)}\frac{1}{(x+h+1)} + \frac{-1}{(x+1)}\frac{(x+h+1)}{(x+h+1)}}{h}$$

$$\lim_{h \to 0} \frac{\frac{\cancel{x}+\cancel{1}}{(x+h+1)(x+1)} + \frac{\cancel{-x}-h\cancel{-1}}{(x+h+1)(x+1)}}{h} = \lim_{h \to 0} \frac{\frac{-h}{(x+h+1)(x+1)}}{h}$$

change to multiply by reciprocal

$$\lim_{h \to 0} \frac{-\cancel{h}}{(x+h+1)(x+1)} \cdot \frac{1}{\cancel{h}} \rightarrow \frac{dy}{dx} = \lim_{h \to 0} \frac{-1}{(x+0+1)(x+1)}$$

$$\frac{dy}{dx} = \frac{-1}{(x+1)^2}$$

FIND THE DERIVATIVE

4. $f(x) = 3\sqrt{x-2}$

*multiply by the conjugate

$$f'(x) = \lim_{h \to 0} \left(\frac{3\sqrt{x+h-2} - 3\sqrt{x-2}}{h}\right) \frac{(3\sqrt{x+h-2} + 3\sqrt{x-2})}{(3\sqrt{x+h-2} + 3\sqrt{x-2})}$$

$$f'(x) = \lim_{h \to 0} \frac{9(x+h-2) - 9(x-2)}{h(3\sqrt{x+h-2} + 3\sqrt{x-2})} = \lim_{h \to 0} \frac{\cancel{9x} + 9\cancel{h} \cancel{-18} \cancel{-9x} \cancel{+18}}{\cancel{h}(3\sqrt{x+h-2} + 3\sqrt{x-2})}$$

$$f'(x) = \lim_{h \to 0} \frac{9}{3\sqrt{x+0-2} + 3\sqrt{x-2}} \rightarrow f'(x) = \frac{9}{6\sqrt{x-2}} \text{ or } \frac{3}{2\sqrt{x-2}}$$

$3\sqrt{x-2} + 3\sqrt{x-2}$