Integration Rules Additional Review

Integrate:

1. $\int x^3 \sin(5x)\, dx$

A T

u	dv
$+x^3$	$\sin(5x)$
$-3x^2$	$-\frac{1}{5}\cos(5x)$
$+6x$	$-\frac{1}{25}\sin(5x)$
-6	$\frac{1}{125}\cos(5x)$
$+0$	$\frac{1}{625}\sin(5x)$

$-\frac{1}{5}x^3\cos(5x)+\frac{3}{25}x^2\sin(5x)+\frac{6}{125}\cos(5x)-\frac{6}{625}\sin(5x)+C$

2. $\int \tan^4 z \sec^2 z\, dz$

$u=\tan z$

$du=\sec^2 z\, dz$

$\int u^4\, du$

$\frac{u^5}{5}+C$

$\frac{\tan^5 z}{5}+C$

3. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$

$u=\sqrt{x}$

$du=\frac{1}{2\sqrt{x}}dx$

$2du=\frac{1}{\sqrt{x}}dx$

$2\int e^u\, du$

$2e^{\sqrt{x}}+C$

4. $\int \cot(5x-2)\, dx$

$\int \frac{\cos(5x-2)}{\sin(5x-2)}dx$

$u=\sin(5x-2)$

$du=-5\cos(5x-2)\,dx$

$\frac{du}{5}=\cos(5x-2)\,dx$

$\frac{1}{5}\int \frac{1}{u}\, du$

$\frac{1}{5}\ln|\sin(5x-2)|+C$

5. $\int 2x\,(x+2)^{\frac{1}{3}}\, dx$

$u=x+2$

$du=dx$

$x=u-2$

$2\int (u-2)u^{1/3}\, du$

$2\int u^{4/3}-2u^{1/3}\, du$

$\frac{6}{7}u^{7/3}-\frac{3}{2}u^{4/3}+C$

$\frac{6}{7}(x+2)^{7/3}-\frac{3}{2}(x+2)^{4/3}+C$

6. $\int 2\cos^5 x \sin^4 x\, dx$

$u=\sin x$

$du=\cos x\, dx$

$2\int \cos x\cdot\cos^4 x\cdot\sin^4 x\, dx$

$2\int \cos x\,(1-\sin^2 x)(1-\sin^2 x)\sin^4 x\, dx$

$2\int (1-u^2)(1-u^2)u^4\, du$

$2\int u^4-2u^6+u^8\, du$

$\frac{2u^5}{5}-\frac{2u^7}{7}+\frac{u^9}{9}+C$

$\frac{2\sin^5 x}{5}-\frac{4\sin^7 x}{7}+\frac{2\sin^9 x}{9}+C$

7. $\int \frac{2x^3-4x^2+2x-1}{x^2+1}\,dx$

$2x-4+\frac{3}{x^2+1}$

$x^2+0x+1 \overline{)2x^3-4x^2+2x-1}$

$-2x^3-0x^2-2x$

$-4x^2+0x-1$

$+4x^2+0x+4$

3

$\int 2x-4+\frac{3}{x^2+1}\,dx$

$x^2-4x+3\tan^{-1}x+C$

8. $\int \frac{3x+4}{x^2+5x+6}\,dx$

$(x+3)(x+2)$

$\frac{A}{x+3}+\frac{B}{x+2}$

$Ax+2A+Bx+3B=3x+4$

$-2(A+B=3)$

$2A+3B=4$

$B=-2$

$A=5$

$\int \frac{5}{x+3}+\frac{-2}{x+2}\,dx$

$5\ln|x+3|-2\ln|x+2|+C$

9. $\int \frac{5x}{\sqrt{1-9x^2}}\,dx$

$u=1-9x^2$

$du=-18x\,dx$

$\frac{du}{-18}=x\,dx$

$-\frac{5}{18}\int u^{-1/2}\,du$

$-\frac{5}{18}\cdot 2u^{1/2}+C$

$-\frac{5}{9}\sqrt{1-9x^2}+C$

10. $\int \frac{2}{1+4x^2}\,dx$

$2\int \frac{1}{1+(2x)^2}\,dx$

$u=2x$

$du=2\,dx$

$\frac{du}{2}=dx$

$\frac{1}{2}\cdot 2\int \frac{1}{1+u^2}\,du$

$\tan^{-1}(2x)+C$

11. $\int e^{3x}\sin x\,dx$

$u=\sin x \quad dv=e^{3x}$

$du=\cos x\,dx \quad v=\frac{e^{3x}}{3}$

$\frac{1}{3}e^{3x}\sin x-\frac{1}{3}\int e^{3x}\cos x\,dx$

$u=\cos x \quad dv=e^{3x}\,dx$

$du=-\sin x\,dx \quad v=\frac{e^{3x}}{3}$

$\frac{1}{3}e^{3x}\sin x-\frac{1}{3}\left(\frac{e^{3x}\cos x}{3}+\frac{1}{3}\int e^{3x}\sin x\,dx\right)$

$\int e^{3x}\sin x=\frac{1}{3}e^{3x}\sin x-\frac{1}{9}e^{3x}\cos x-\frac{1}{9}\int e^{3x}\sin x\,dx$

$\frac{10}{9}\int e^{3x}\sin x=\frac{1}{3}e^{3x}\sin x-\frac{1}{9}e^{3x}\cos x$

Evaluate: $\frac{3}{10}e^{3x}\sin x-\frac{1}{10}e^{3x}\cos x+C$

12. $\int_{-2}^{2} x(5+x^2)^2\,dx$

$u=5+x^2$

$du=2x\,dx$

$\frac{du}{2}=x\,dx$

$\frac{1}{2}\int_9^9 u^2\,du$

$\left.\frac{u^3}{6}\right|_9^9 = 0$

13. $\int_{\pi/6}^{\pi/4} \cos^2(3x)\,dx$

$\frac{1}{2}\int_{\pi/6}^{\pi/4} 1+\cos 6x\,dx$

$\left.\frac{1}{2}x+\frac{1}{12}\sin 6x\right|_{\pi/6}^{\pi/4}$

$\frac{\pi}{8}+\frac{1}{12}\sin\frac{3\pi}{2}-\frac{\pi}{12}-\frac{1}{12}\sin\pi$

$\frac{\pi}{8}-\frac{1}{12}-\frac{\pi}{12}-0$

$\frac{\pi}{24}-\frac{1}{12}$

14. $\int_0^{\ln 2} \frac{2e^x}{1-e^x}\,dx$

$u=1-e^x$

$du=-e^x\,dx$

$-du=e^x\,dx$

$-2\int_0^{-1} \frac{1}{u}\,du$

$u=1-e^{\ln 2}=-1$

$u=1-e^0=0$

$\left.-2\ln|u|\right|_0^{-1}$

$-2\ln|-1|=-2(0)=0$

15. – 17. No partial credit will be given on the multiple choice questions. Work carefully so you do not make careless errors.

15. $\int_e^{e^2} \frac{dx}{x \ln x}$

$\int_1^2 \frac{1}{u}\,du = \ln|u|\Big|_1^2$

$\ln 2 - \ln 1$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$u = \ln e^2 = 2$

$u = \ln e = 1$

a. ln2 b. ½ c. 1 d. 2 e. e

16. If the substitution $u = 25 - x^2$ is made, the integral $\int_0^3 x\sqrt{25 - x^2}\,dx =$

$u = 25 - x^2$

$du = -2x\,dx$

$\frac{du}{-2} = x\,dx$

$-\frac{1}{2}\int_{25}^{16} \sqrt{u}\,du$

$\frac{1}{2}\int_{16}^{25} \sqrt{u}\,du$

a. $\frac{1}{2}\int_0^3 \sqrt{u}\,du$ b. $\frac{1}{2}\int_{25}^{16} \sqrt{u}\,du$ c. $-\frac{1}{2}\int_0^3 \sqrt{u}\,du$

d. $\frac{1}{2}\int_{16}^{25} \sqrt{u}\,du$ e. $2\int_{16}^{25} \sqrt{u}\,du$

_____ 17. If $0 < k < \pi$, then $\int_0^k \sin(x)dx = \frac{1}{2}$ when $k =$

A) $\frac{\pi}{2}$ B) $\frac{\pi}{3}$ C) $\frac{\pi}{4}$

D) $\frac{\pi}{6}$ E) $\frac{\pi}{12}$

$-\cos x \Big|_0^K = \frac{1}{2}$

$-\cos K + \cos 0 = \frac{1}{2}$

$-\cos K + 1 = \frac{1}{2}$

$-\cos K = -\frac{1}{2}$

$\cos K = \frac{1}{2}$

$K = \cos^{-1}\left(\frac{1}{2}\right)$

$K = \frac{\pi}{3}$