Integrated Integration

Evaluate the integral.

1. $\int \frac{4}{1+4x^2}dx$

$u=2x$

$du=2dx$

$\frac{du}{2}=dx$

$4\int \frac{1}{1+(2x)^2}dx$

$2\int \frac{1}{1+u^2}du$

$2\arctan 2x + C$

2. $\int \frac{4x}{1+4x^2}dx$

$u=1+4x^2$

$du=8x\,dx$

$\frac{du}{2}=4x\,dx$

$\frac{1}{2}\int \frac{1}{u}du$

$\frac{1}{2}\ln(1+4x^2)+C$

3. $\int \frac{3}{\sqrt{5x-2}}dx$

$3\int (5x-2)^{-1/2}dx$

$6(5x-2)^{1/2}+C$

4. $\int xe^x dx$

	u	dv
+	x	e^x
−	1	e^x
+	0	e^x

$xe^x - e^x + C$

5. $\int \frac{2x-11}{x^2-x-6}dx$

$(x-3)(x+2)$

$\frac{A}{x-3}+\frac{B}{x+2}$

$Ax+2A+Bx-3B=2x-11$

$3(A+B=2)$

$2A-3B=-11$

$5A=-5 \therefore A=-1,\ B=3$

$\int \frac{-1}{x-3}+\frac{3}{x+2}dx$

$-\ln|x-3|+3\ln|x+2|+C$

6. $\int \frac{x^3-3x+1}{x+2}dx$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -3 & 1 \\ & \downarrow & -2 & 4 & -2 \\ \hline & 1 & -2 & 1 & -1 \end{array}$$

$\int x^2-2x+1-\frac{1}{x+2}dx$

$\frac{x^3}{3}-x^2+x-\ln|x+2|+C$

7. $\int \frac{3x^2-x+4}{\sqrt[3]{x}}dx$

$\int 3x^{5/3}-x^{2/3}+4x^{-1/3}dx$

$\frac{9}{8}x^{8/3}-\frac{3}{5}x^{5/3}+6x^{2/3}+C$

8. $\int \frac{(\ln x)^2}{x}dx$

$u=\ln x$

$du=\frac{1}{x}dx$

$\int u^2 du$

$\frac{u^3}{3}+C$

$\frac{(\ln x)^3}{3}+C$

9. $\int \ln x\,dx$

$u=\ln x \quad dv=dx$

$du=\frac{1}{x}dx \quad v=x$

$x\ln x-\int dx$

$x\ln x - x + C$

10. $\int x^3 \sin(2x)dx$

	u	dv
+	x^3	$\sin 2x$
−	$3x^2$	$-\frac{1}{2}\cos 2x$
+	$6x$	$-\frac{1}{4}\sin 2x$
−	6	$\frac{1}{8}\cos 2x$
+	0	$\frac{1}{16}\sin 2x$

$-\frac{1}{2}x^3\cos 2x+\frac{3}{4}\sin 2x+\frac{3}{4}\cos 2x$

$-\frac{3}{8}\sin 2x + C$

11. $\int x(2-x)^5\,dx$ $\quad u = 2-x,\ x = 2-u$

$du = -dx$

$-du = dx$

$-\int (2-u)u^5\,du$

$\int -2u^5 + u^6\,du$

$-\frac{2u^6}{6} + \frac{u^7}{7} + C$

$-\frac{1}{3}(2-x)^6 + \frac{1}{7}(2-x)^7 + C$

12. $\int \frac{x^2}{x+1}\,dx$

$$\begin{array}{r|rrr} -1 & 1 & 0 & 0 \\ & \downarrow & -1 & 1 \\ \hline & 1 & -1 & 1 \end{array}$$

$\int x - 1 + \frac{1}{x+1}\,dx$

$\frac{x^2}{2} - x + \ln|x+1| + C$

13. $\int \frac{1}{\sqrt{9-x^2}}\,dx$ $\quad u = \frac{x}{3}$

$du = \frac{1}{3}dx$

$3du = dx$

$\int \frac{1}{\sqrt{9(1-(\frac{x}{3})^2)}}\,dx$

$3 \cdot \frac{1}{3}\int \frac{1}{\sqrt{1-u^2}}\,du$

$\arcsin\left(\frac{x}{3}\right) + C$

14. $\int \frac{2x}{\sqrt{9-x^2}}\,dx$ $\quad u = 9-x^2$

$du = -2x\,dx$

$-du = 2x\,dx$

$-\int \frac{1}{\sqrt{u}}\,du$

$-\int u^{-1/2}\,du$

$-2u^{1/2} + C = -2\sqrt{9-x^2} + C$

15. $\int \sin^2 x\cos^3 x\,dx$

$\int \sin^2 x\cos x\cos^2 x\,dx$ $\quad u = \sin x$, $du = \cos x\,dx$

$\int \sin^2 x\cos x(1-\sin^2 x)\,dx$

$\int u^2(1-u^2)\,du$

$\int u^2 - u^4\,du$

$\frac{u^3}{3} - \frac{u^4}{4} + C = \frac{\sin^3 x}{3} - \frac{\sin^4 x}{4} + C$

16. $\int \tan^3 x\sec^2 x\,dx$ $\quad u = \tan x$

$du = \sec^2 x\,dx$

$\int u^3\,du$

$\frac{u^4}{4} + C$

$\frac{\tan^4 x}{4} + C$

17. $\int \sin^2 x\,dx$

$\frac{1}{2}\int 1 - \cos 2x\,dx$

$\frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) + C$

$\frac{1}{2}x - \frac{1}{4}\sin 2x + C$

18. $\int_0^1 \frac{x^3}{x+1}\,dx$

$$\begin{array}{r|rrrr} -1 & 1 & 0 & 0 & 0 \\ & \downarrow & -1 & 1 & -1 \\ \hline & 1 & -1 & 1 & -1 \end{array}$$

$\int_0^1 x^2 - x + 1 - \frac{1}{x+1}\,dx$

$\frac{x^3}{3} - \frac{x^2}{2} + x - \ln|x+1|\Big|_0^1$

$\left(\frac{1}{3} - \frac{1}{2} + 1 - \ln 2\right) - (0 - 0 + 0 - \ln 1)$

$\frac{5}{6} - \ln 2$

19. $\int e^{2x}\cos x\,dx$ $\quad u = \cos x \quad dv = e^{2x}dx$

$du = -\sin x\,dx \quad v = \frac{1}{2}e^{2x}$

$\frac{1}{2}e^{2x}\cos x + \frac{1}{2}\int e^{2x}\sin x\,dx$ $\quad u = \sin x \quad dv = e^{2x}dx$

$du = \cos x\,dx \quad v = \frac{1}{2}e^{2x}$

$\frac{1}{2}e^{2x}\cos x + \frac{1}{2}\left(\frac{1}{2}e^{2x}\sin x - \frac{1}{2}\int e^{2x}\cos x\,dx\right)$

$\int e^{2x}\cos x\,dx = \frac{1}{2}e^{2x}\cos x + \frac{1}{4}e^{2x}\sin x - \frac{1}{4}\int e^{2x}\cos x\,dx$

$\frac{5}{4}e^{2x}\cos x\,dx = \frac{1}{2}e^{2x}\cos x + \frac{1}{4}e^{2x}\sin x$

$= \frac{2}{5}e^{2x}\cos x + \frac{1}{5}e^{2x}\sin x + C$

20. $\int_0^{\pi/2} \sin^2 x\cos^2 x\,dx$

$\frac{1}{4}\int_0^{\pi/2} (1-\cos 2x)(1+\cos 2x)$

$\frac{1}{4}\int_0^{\pi/2} 1 - \cos^2(2x)\,dx$

$\frac{1}{4}\int_0^{\pi/2} 1 - \frac{1}{2} - \frac{1}{2}\cos 4x\,dx$

$\frac{1}{4}\left(\frac{1}{2}x - \frac{1}{8}\sin 4x\right)\Big|_0^{\pi/2}$

$\frac{\pi}{16} - \frac{1}{8}\sin 2\pi - 0 + \frac{1}{32}\sin 0 = \frac{\pi}{16}$