# Meaning of Integration

*E.Q. What is an integral? How can we approximate area under a curve?*

| Day | Unit Dates | Topics | Assignments |
|---|---|---|---|
| 1 | Friday, October 23rd | **Keeper 6.1 - Antiderivatives** | Antiderivatives (packet p. 1 - 2) |
| 2 | Monday, October 26th | **Keeper 6.2 - Riemann Sums** | **Skills Check 6.1 A (Forms)**<br><br>Riemann Sums (packet p. 3 - 5) |
| 3 | Tuesday, October 27th | **Keeper 6.3 - Riemann Sums to an Integral** | **Skills Check 6.1 B (Forms)**<br><br>Riemann Sums to an Integral (packet p. 6 - 7) |
| 4 | Wednesday, October 28th | **Optional Q & A Session at 10am Review Keeper 6.1 – 6.2 Antiderivatives and Riemann Sums** | **Take Home Skills Check 6.2 (AP Classroom)**<br><br>Get Caught up on all Keeper Notes and Homework |
| 5 | Thursday, October 29th | **Keeper 6.4 - Fundamental Theorem of Calculus Part 2** | **Skills Check 6.1 C (Forms)**<br><br>The FTC Part 2 (packet p. 8 – 9) |
| 6 | Friday, October 30th | **Keeper 6.4 - Fundamental Theorem of Calculus Part 1** | **Skills Check 6.4 – FTC Part 2 (AP Classroom)**<br><br>The Fundamental Theorem of Calculus Part 1 (packet p. 10 – 13) |
| 7 | Monday, November 2nd | **Keeper 6.5 – Total or Net Change** | **Skills Check 6.4 – FTC Part 1 (AP Classroom)**<br><br>Definite Integrals and Rates of Change (packet p. 14) |
| 8 | Wednesday, November 4th | **Optional Q & A Session at 10am**<br>**Unit 6 Additional Review** | Complete Homework Packet and Catch up on all Keeper Notes<br><br>Study for Unit 6 Test<br><br>Homework Packet Due in CTLS |
| 9 | Thursday, November 5th | **Test - The Meaning of Integration** | Good Luck 😊 |

# Antiderivatives

Find the general antiderivative of each function.

1. $f(x) = 6x^2 - 8x + 3$

2. $f(x) = 1 - x^3 + 5x^5 - 3x^7$

3. $f(x) = \sqrt{x} + \sqrt[3]{x}$

4. $f(x) = \frac{3}{x^2} + \frac{5}{x}$

5. $f(x) = \frac{x^3 + 2x^2}{\sqrt{x}}$

6. $f(x) = \sqrt[3]{x^2} - \sqrt{x}$

7. $f(x) = 3\cos x - 4\sin x$

8. $f(x) = 4\sqrt{x} + e^x - \sec x \tan x$

9. $f(x) = \frac{x^2 + x + 1}{x}$

10. $f(x) = 6x^2 - 7\sec^2 x$

Find $f(x)$

11. $f'(x) = 1 - 6x; f(0) = 8$

12. $f'(x) = 3\sqrt{x} - \frac{1}{\sqrt{x}}; f(1) = 2$

13. $f'(x) = 3\cos x + 5\sin x\,; f(0) = 4$

14. $f''(x) = x; f(0) = -3, f'(0) = 2$

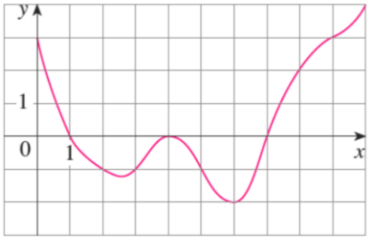
15. $f''(x) = x^2 + 3\cos x\,; f(0) = 2, f'(0) = 3$

16. $f''(x) = 12x^2 - 6x + 2; f(0) = 1, f'(2) = 11$

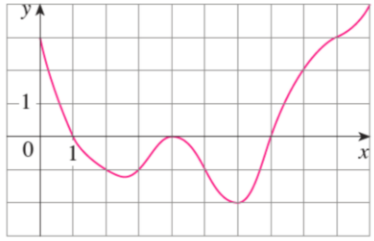
# Riemann Sums

1. The graph of a function $f$ is given. Estimate $\int_0^{10} f(x)\ dx$ using ten subintervals with

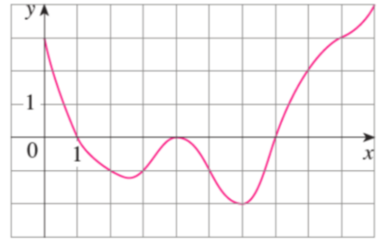
a. Right Endpoints




b. Left Endpoints

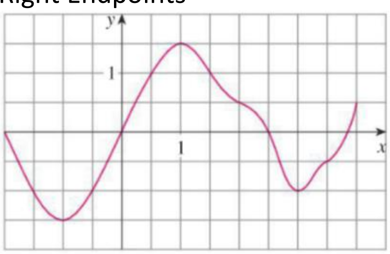



c. Midpoints

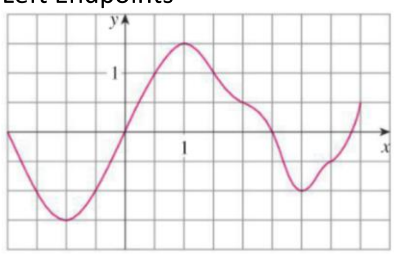



2. The graph of $g$ is shown. Estimate $\int_{-2}^{4} g(x)\ dx$ with six subintervals using

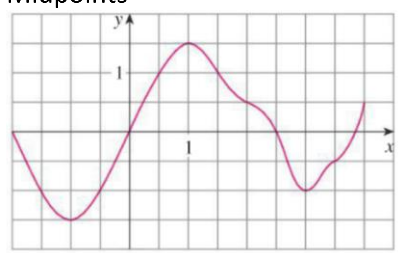
a. Right Endpoints




b. Left Endpoints




c. Midpoints




3. The table gives the values of a function obtained from an experiment. Use them to estimate $\int_3^9 f(x)\ dx$ using three equal subintervals with

a. Right Endpoints

| x | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|
| f(x) | -3.4 | -2.1 | -.6 | .3 | .9 | 1.4 | 1.8 |

b. Left Endpoints

| x | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|
| f(x) | -3.4 | -2.1 | -.6 | .3 | .9 | 1.4 | 1.8 |

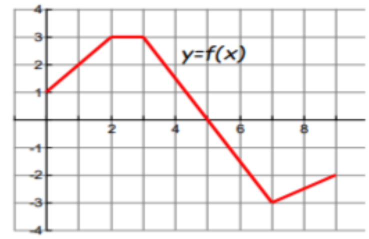
c. Midpoints

| x | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|
| f(x) | -3.4 | -2.1 | -.6 | .3 | .9 | 1.4 | 1.8 |

4. The graph of $f$ is shown. Evaluate each integral by interpreting it in terms of areas.

a. $\int_0^2 f(x)dx$

b. $\int_0^5 f(x)\,dx$

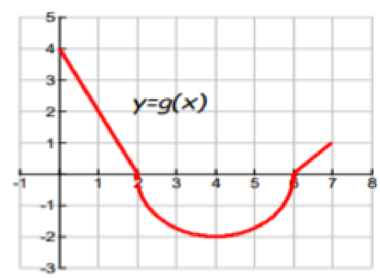



c. $\int_5^7 f(x)dx$

d. $\int_0^9 f(x)dx$

5. The graph of $g$ consists of two straight lines and a semi-circle. Use it to evaluate each integral.

a. $\int_0^2 g(x)dx$

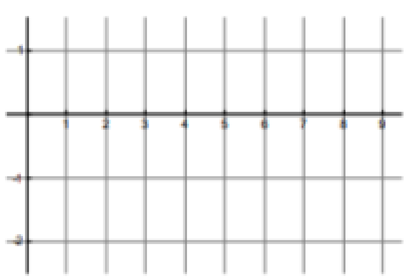



b. $\int_2^6 g(x)\,dx$

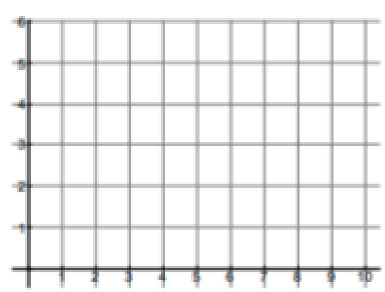
c. $\int_0^7 g(x)\,dx$

Evaluate the integral by interpreting it in terms of areas.

6. $\int_0^9 \left(\frac{1}{3}x - 2\right)\,dx$



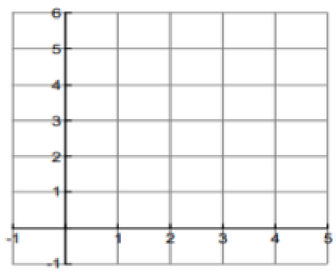
7. $\int_0^{10} |x-5|\,dx$



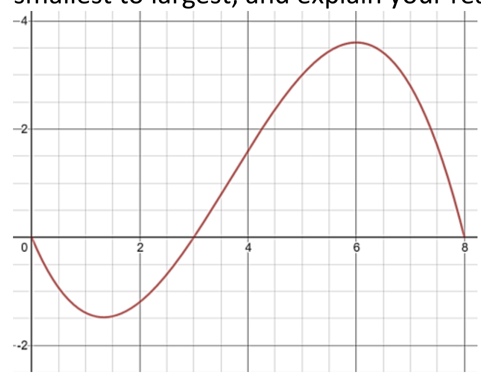
8. If $\int_1^5 f(x)dx = 12$ and $\int_4^5 f(x)dx = 3.6$, find $\int_1^4 f(x)dx$

9. If $\int_0^9 f(x)dx = 37$ and $\int_0^9 g(x)dx = 16$, find $\int_0^9 [2f(x) + 3g(x)]dx$

10. Find $\int_0^5 f(x)$ if $f(x) = \begin{cases} 3 & for\ x < 3 \\ x & for\ x \geq 3 \end{cases}$




11. For the function $f$ whose graph is shown, list the following quantities in increasing order from smallest to largest, and explain your reasoning.




a. $\int_0^8 f(x)dx$

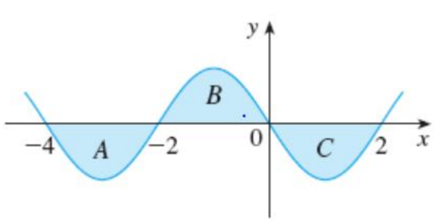
b. $\int_0^3 f(x)dx$

c. $\int_3^8 f(x)dx$

d. $\int_4^8 f(x)dx$

e. $f'(1)$

12. Each of the regions $A, B, and$ C bounded by the graph of $f$ and the $x$ −axis has the area 3. Find the value of $\int_{-4}^{2} [f(x) + 2x + 5]\, dx$



# Riemann Sums to an Integral

Each expression below is a right Riemann Sum approximation for an integral.

In each problem, state what integral the sum is approximating.

1. $\frac{1}{4}\left[\sin\left(\frac{9}{4}\right)+\sin\left(\frac{10}{4}\right)+\sin\left(\frac{11}{4}\right)+\sin(3)\right]$

2. $\frac{1}{5}\left[\ln\left(\frac{11}{5}\right)+\ln\left(\frac{12}{5}\right)+\ln\left(\frac{13}{5}\right)+\ln\left(\frac{14}{5}\right)+\ln(3)\right]$

3. $\frac{1}{8}\left[\frac{1}{\left(1\frac{1}{8}\right)^2}+\frac{1}{\left(1\frac{1}{4}\right)^2}+\frac{1}{\left(1\frac{3}{8}\right)^2}+\frac{1}{\left(1\frac{1}{2}\right)^2}+\frac{1}{\left(1\frac{5}{8}\right)^2}+\frac{1}{\left(1\frac{3}{4}\right)^2}+\frac{1}{\left(1\frac{7}{8}\right)^2}+\frac{1}{2^2}\right]$

4. $\frac{1}{3}\left[\sqrt[3]{\frac{4}{3}}+\sqrt[3]{\frac{5}{3}}+\sqrt[3]{2}\right]$

5. $\frac{1}{2}\left[3(3.5)^2+3(4)^2+3(4.5)^2+3(5)^2\right]$

The following is a left Riemann Sum approximation for some integral. What integral is it approximating?

6. $\frac{1}{4}\left[\frac{1}{2(1)}+\frac{1}{2(1.25)}+\frac{1}{2(1.5)}+\frac{1}{2(1.75)}\right]$

1. $$\lim_{N\to\infty} \sum_{i=1}^{N} \frac{i}{N^2}$$

2. $$\lim_{N\to\infty} \sum_{j=1}^{N} \frac{j^3}{N^4}$$

3. $$\lim_{N\to\infty} \sum_{i=1}^{N} \frac{i^2 - i + 1}{N^3}$$

4. $$\lim_{N\to\infty} \sum_{i=1}^{N} \left(\frac{i^3}{N^4} - \frac{20}{N}\right)$$

5. $$\lim_{N\to\infty} \frac{2}{N} \sum_{j=1}^{N} \sin\left(\frac{2j}{N}\right)$$

6. $$\lim_{N\to\infty} \frac{4}{N} \sum_{k=1}^{N} \left(3 + \frac{4k}{N}\right)$$

7. $$\lim_{N\to\infty} \frac{\pi}{N} \sum_{j=0}^{N-1} \sin\left(\frac{\pi}{2} + \frac{\pi j}{N}\right)$$

8. $$\lim_{N\to\infty} \frac{4}{N} \sum_{k=1}^{N} \frac{1}{\left(3 + \frac{4k}{N}\right)^2}$$

9. $$\lim_{N\to\infty} \frac{1^k + 2^k + \cdots + N^k}{N^{k+1}} \quad (k > 0)$$

# Fundamental Theorem of Calculus – Part 2

1. $\int_{-1}^{2}(x^3 - 2x)dx$

2. $\int_{1}^{4}(5 - 2t + 3t^2)dt$

3. $\int_{1}^{9}\sqrt{x}dx$

4. $\int_{\frac{\pi}{6}}^{\pi}\sin\theta\ d\theta$

5. $\int_{0}^{1}(u + 2)(u - 3)du$

6. $\int_{0}^{4}(4 - t)\sqrt{t}dt$

7. $\int_{1}^{9}\frac{x-1}{\sqrt{x}}\ dx$

8. $\int_{0}^{\frac{\pi}{4}}\sec^2 t\ dt$

9. $\int_0^{\frac{\pi}{4}} \sec\theta \tan\theta \, d\theta$

10. $\int_1^2 (1+2y)^2 dy$

11. $\int_0^3 (2\sin x - e^x) dx$

12. $\int_1^2 \frac{v^3+3v^6}{v^4} dv$

13. $\int_0^1 (x^e + e^x) dx$

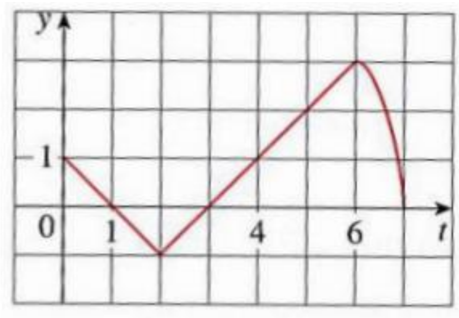
14. $\int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{8}{1+x^2} dx$

15. $\int_{-1}^1 e^{u+1} du$

16. $\int_0^{\pi} f(x)dx$ were $f(x) = \begin{cases} \sin x, & 0 \le x < \frac{\pi}{2} \\ \cos x, & \frac{\pi}{2} \le x < \pi \end{cases}$

# The Fundamental Theorem of Calculus Part 1

1. Let $g(x) = \int_0^x f(t)\,dt$, where $f$ is the function whose graph is shown.




a. Evaluate

$x = 0$ $\quad$ $x = 1$

$x = 2$ $\quad$ $x = 3$

$x = 4$ $\quad$ $x = 5$

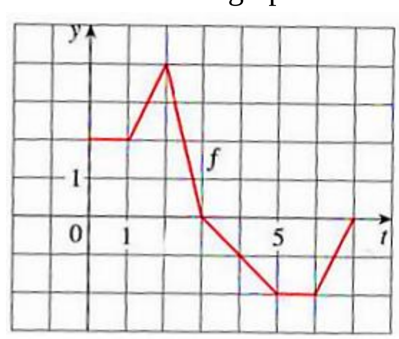
$x = 6$

b. Estimate $g(7)$

c. Where does $g$ have a maximum value? Where does it have a minimum value?

d. Sketch a graph of $g$.

2. Let $g(x) = \int_0^x f(t)\,dt$, where $f$ is the function whose graph is shown.




a. Evaluate

$g(0)$ $\quad$ $g(1)$

$g(2)$ $\quad$ $g(3)$

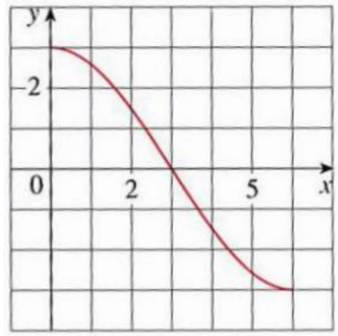
$g(6)$

b. On what intervals is $g$ increasing?

c. Where does $g$ have a maximum value?

d. Sketch a graph of $g$.

3. Let $g(x) = \int_0^x f(t)\, dt$, where $f$ is the function whose graph is shown.




a. Evaluate

$g(0)$

$g(6)$

b. Evaluate

$x = 1$ $x = 2$

$x = 3$ $x = 4$

$x = 5$

c. On what interval is $g$ increasing?

d. Where does $g$ have a maximum value?

e. Sketch a rough graph of $g$.

Use the 1st Fundamental Theorem of Calculus to find the derivative of the functions.

4. $g(x) = \int_1^x \frac{1}{t^3+1} dt$

5. $g(x) = \int_3^x e^{t^2-t} dt$

6. $g(s) = \int_5^s (t - t^2)^8 dt$

7. $g(r) = \int_0^r \sqrt{x^2 + 4} dx$

8. $G(x) = \int_x^1 \cos\sqrt{t}\,dt$

9. $h(x) = \int_1^{e^x} \ln t \; dt$

10. $h(x) = \int_1^{\sqrt{x}} \frac{z^2}{z^4+1} dz$

11. $y = \int_0^{\tan x} \sqrt{t + \sqrt{t}} \; dt$

12. $y = \int_0^{x^4} \cos^2\theta \; d\theta$

13. $y = \int_{1-3x}^{1} \frac{u^3}{1+u^2} \; du$

14. $y = \int_{\sin x}^{1} \sqrt{(1+t^2)}dt$

15. $F(x) = \int_x^{\pi} \sqrt{1+\sec t}\,dt$

16. If $f(x) = \int_0^x (1 - t^2)\, e^{t^2} dt$, on what interval is $f$ increasing?

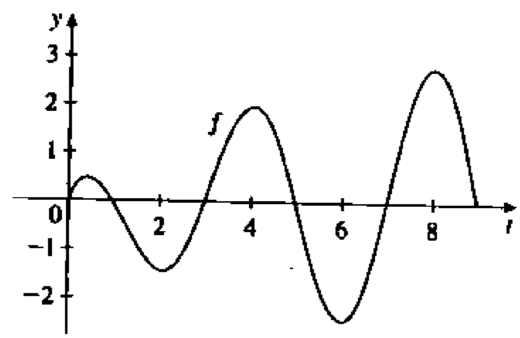
17. On what interval is the curve $y = \int_0^x \frac{t^2}{t^2+t+2}\, dt$ concave down?

18. If $f(x) = \int_0^{\sin x} \sqrt{1 + t^2} dt$ and $g(y) = \int_3^y f(x) dx$, find $g''\left(\frac{\pi}{6}\right)$

19. If $f(1) = 12, f'$ is continuous, and $\int_1^4 f'(x) dx = 17$, what is the value of $f(4)$?

20. Let $g(x) = \int_0^x f(t) dt$, where $f$ is the function whose graph is shown.

a. At what values of $x$ does the local maximum and minimum of $g$ occur?




b. Where does $g$ attain its absolute maximum value?

c. On what intervals is $g$ concave downward?

d. Sketch the graph of $g$.

# Definite Integrals and Rate of Change

1. If $w'(t)$ is the rate of growth of a child in pounds per year, what does $\int_5^{10} w'(t)\ dt$ represent?

2. If oil leaks from a tank at a rate of $r(t)$ gallons per minute at time $t$, what does $\int_0^{120} r(t)dt$ represent?

3. A honeybee population starts with 100 bees and increases at a rate of $n'(t)$ bees per week. What does $100 + \int_0^{15} n'(t)dt$ represent?

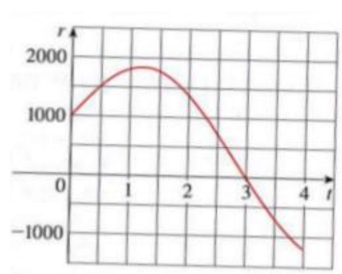
4. The linear density of a rod of length 4m is given by $p(x) = 9 + 2\sqrt{x}$ measured in kilograms per meter, where $x$ is measured in meters from one end of the rod. Find the total mass of the rod.

5. Water flows from the bottom of a storage tank at a rate of $r(t) = 200 - 4t$ liters per minute, where $0 \leq t \leq 50$. Find the amount of water that flows from the tank during the first 10 minutes.

6. The velocity of a car was read from its speedometer at 10-second intervals and recorded in the table. Use the Midpoint Rule to estimate the distance traveled by the car.

| t(s) | v(mi/h) | t(s) | v(mi/h) |
|---|---|---|---|
| 0 | 0 | 60 | 56 |
| 10 | 38 | 70 | 53 |
| 20 | 52 | 80 | 50 |
| 30 | 58 | 90 | 47 |
| 40 | 55 | 100 | 45 |
| 50 | 51 | | |

7. Water flows into and out of a storage tank. A graph of the rate of change $r(t)$ of the volume of water in the tank, in liters per day, is shown. If the amount of water in the tank at time $t = 0$ is 25,000 L, use the Midpoint Rule to estimate the amount in the tank 4 days later.



# Practice Test - Meaning of Integration

Calculator <u>not allowed</u> on this section.

1. What is the meaning of $\int_3^7 m(t)dt$, if $m$(t) is the rate of traffic flow (number of cars per hour passing an observation point along a highway), and t is measured in hours from
8:00 am on December 20, 2005?

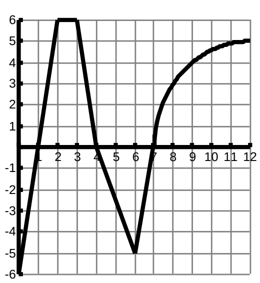
3. A particle moves along a straight line with acceleration $a(t) = 5 + 4t - 6t^2$. The velocity at $t = 1$ second is 3 m/sec. Its position at time $t = 0$ is 10 meters. Find both the velocity function and the position function.

4. If $\int_3^7 f(x)dx = 11$ and $\int_3^7 g(x) = -5$ and $\int_5^7 f(x)dx = 4$ , find the following. (2 points each)

$a)\ \int_3^7 [2f(x) + 3g(x)] =$ $b)\ \int_7^3 4g(x)dx =$ $c)\ \int_5^7 (f(x)+2)dx =$ $d)\ \int_3^5 f(x)dx$

5. The graph of the velocity (in m/min) of a bicycle as a function of time t (in min) is graphed below. The graph consists of line segments and a quarter of a circle. Use the graph of velocity to determine:

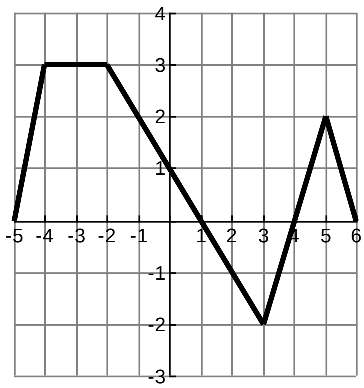
v(t)




a. the displacement of the bicycle during the first 12 minutes.

b. the total distance traveled by the bicycle during the first 12 minutes.

6. Let $g(x) = \int_{-1}^{x} f(t)dt$, where f is the function whose graph is shown below:




a) Find g(0) and g(−3).

b) Find g' (0) and g' (−3) .

c) Find g ″ (0) and g ″ (−3).

c) At what value(s) of *x* does g attain a local max and/or local min?

d) At what value(s) of *x* does *g* attain an absolute max and/or an absolute min?

7. Evaluate the definite integrals**.**

a. $\int_{2}^{e^2} \frac{3}{t} dt$

b. $\int_{0}^{\frac{\pi}{3}} \sec x \cdot \tan x \, dx$

c. $\int_{0}^{5} |4 - x| \, dx$

d. $\int_{8}^{27} \frac{4}{\sqrt[3]{x}} \, dx$

8. Use the Fundamental Theorem of Calculus to simplify:

a. if $f(x) = \int_1^{x^3} \frac{1}{1+t^2}\, dt$

Find $F(1) =$

$F\,'(x) =$

$F\,'(1) =$

9. Simplify.

a. $\frac{d}{dx}\int_{3x^2}^{4} \sin(t^2)\, dt$

b. $F(x) = \int_{x^2}^{\sec x} \sqrt{1+t}\, dt$, find $F'(x)$

c. $\int_1^x \frac{1}{1+t^2} dt$

10. Express the limit as a definite integral on the given interval and evaluate.

$$\lim_{n\to\infty}\sum_{i=1}^{n}[6(x_i^*)^2 - 3x_i^*]\Delta x \qquad [1, 2]$$

**Integral:**

**Value of Integral:**

11. Express the limit $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}\sin\left(\frac{k}{n}\right)$ as an integral.

**Practice Test - Meaning of Integration**
Calculator <u>allowed</u>

12. The penguin population on an island is modeled by a differentiable function P of time t, where P(t) is the number of penguins and t is measured in years, for $0 \le t \le 40$. There are 100,000 penguins on the island at time $t = 0$. The birth rate for the penguins on the island is modeled by $B(t) = 1000e^{0.06t}$ penguins per year. To the nearest whole number, find how many penguins are on the island after 40 years.

13. Use your calculator to find the value of the integral.

$$\int_{4}^{10} (\ln(x) + 5\sin(x) - 1)dx$$

14. The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a differentiable and strictly increasing function R of time t. A table of selected values of R(t), for the time interval $0 \le t \le 90$ minutes, are shown below. Approximate the value of $\int_0^{90} R(t)dt$ using a trapezoidal approximation with the five subintervals indicated by the data in the table.

| t (minutes) | R(t) (gal per min) |
|---|---|
| 0 | 15 |
| 10 | 25 |
| 40 | 50 |
| 50 | 55 |
| 70 | 65 |
| 90 | 70 |