Unit 2 Limits

- Notes and some practice are included
- Homework will be assigned on a daily basis

Topics Covered:

- Limits from Graphs
- Graphs from Limits
- One-Sided Limits & Continuity
- Creative Factoring
- Algebraic Limits
- Intermediate Value Theorem
- Asymptotes, End Behavior & Infinite Limits

Quiz is ______________________

Test is ______________________

Name: Bonanni

Limits from Table of Values

x	-0.3	-0.2	-0.1	0	0.1	0.2	0.3
f(x)	1.971	1.987	1.997	undefined	1.997	1.987	1.971
g(x)	2.018	2.008	2.002	2	2.002	2.008	2.018
h(x)	1	1	1	2	2	2	2

Find the following:

(a) $\lim_{x\to 0} f(x)$ = 2

(b) $\lim_{x\to 0} g(x)$ = 2

(c) $\lim_{x\to 0} h(x)$ DNE

x	2.75	2.9	2.99	2.999	3	3.001	3.01	3.1	3.25
f(x)	5.313	5.710	5.970	5.997	6	6.003	6.030	6.310	6.813
g(x)	1.99499	1.99950	1.99995	1.99999	und	2.00005	2.00050	2.00499	2.01
h(x)	1.99499	1.99950	1.99995	1.99999	2	6.003	6.030	6.310	6.813

Find the following:

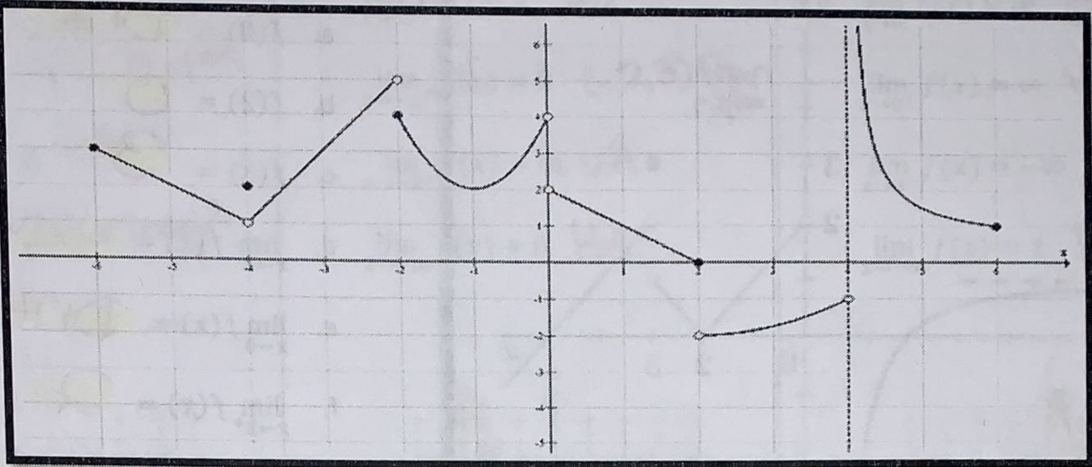
(a) $\lim_{x\to 3} f(x)$ = 6

(b) $\lim_{x\to 3} g(x)$ = 2

(c) $\lim_{x\to 3} h(x)$ DNE

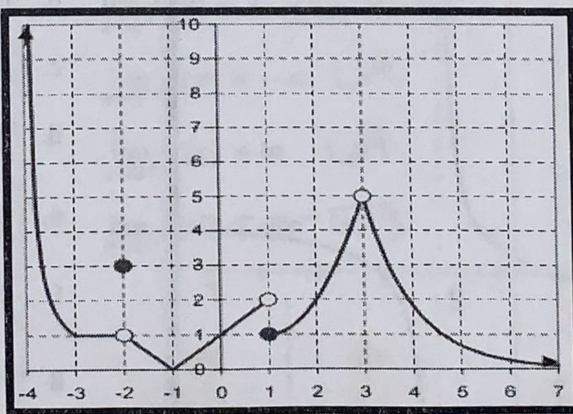
Finding Limits from a Graph

1. Use the graph to evaluate the limits below



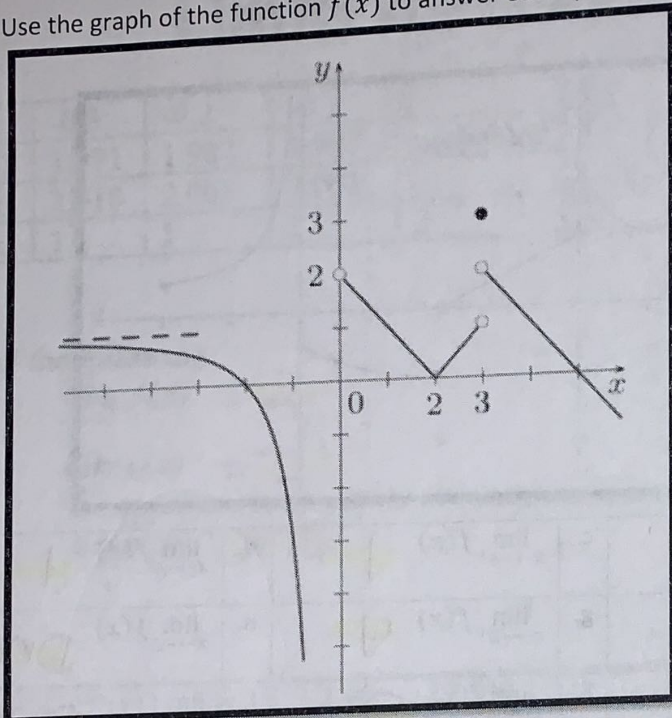
a.	$f(-4)$ 2	b.	$\lim_{x\to-4^-} f(x)$ 1	c.	$\lim_{x\to-4^+} f(x)$ 1	d.	$\lim_{x\to-4} f(x)$ 1
e.	$f(-2)$ 4	f.	$\lim_{x\to-2^-} f(x)$ 5	g.	$\lim_{x\to-2^+} f(x)$ 4	h.	$\lim_{x\to-2} f(x)$ DNE
i.	$f(0)$ DNE	j.	$\lim_{x\to0^-} f(x)$ 4	k.	$\lim_{x\to0^+} f(x)$ 2	l.	$\lim_{x\to0} f(x)$ DNE
m.	$f(2)$ 0	n.	$\lim_{x\to2^-} f(x)$ 0	o.	$\lim_{x\to2^+} f(x)$ -2	p.	$\lim_{x\to2} f(x)$ DNE
q.	$f(4)$ DNE	r.	$\lim_{x\to4^-} f(x)$ -1	s.	$\lim_{x\to4^+} f(x)$ ∞	t.	$\lim_{x\to4} f(x)$ DNE

2. Use the graph to evaluate the expressions below.



a.	$f(-2)$ $=3$	b.	$\lim_{x\to-2^+} f(x)$ $=1$	c.	$\lim_{x\to-2} f(x)$ $=1$
d.	$\lim_{x\to-1^+} f(x)$ $=0$	e.	$\lim_{x\to-1^-} f(x)$ $=0$	f.	$\lim_{x\to-1} f(x)$ $=0$
g.	$\lim_{x\to1^+} f(x)$ $=1$	h.	$\lim_{x\to1^-} f(x)$ $=2$	i.	$\lim_{x\to1} f(x)$ DNE
j.	$f(3)$ DNE	k.	$\lim_{x\to3^+} f(x)$ $=5$	l.	$\lim_{x\to3^-} f(x)$ $=5$
m.	$\lim_{x\to3} f(x)$ $=5$	n.	$\lim_{x\to-4^+} f(x)$ $=\infty$	o.	$\lim_{x\to\infty} f(x)$ $=0$
p.	$f(1)$ $=1$	q.	$\lim_{x\to-3} f(x)$ $=1$	r.	$f(-4)$ DNE

3. Use the graph of the function $f(x)$ to answer each question. Use $\infty, -\infty$, or DNE where appropriate.



a. $f(0) =$ DNE

b. $f(2) =$ 0

c. $f(3) =$ 3

d. $\lim_{x\to 0^-} f(x) = -\infty$

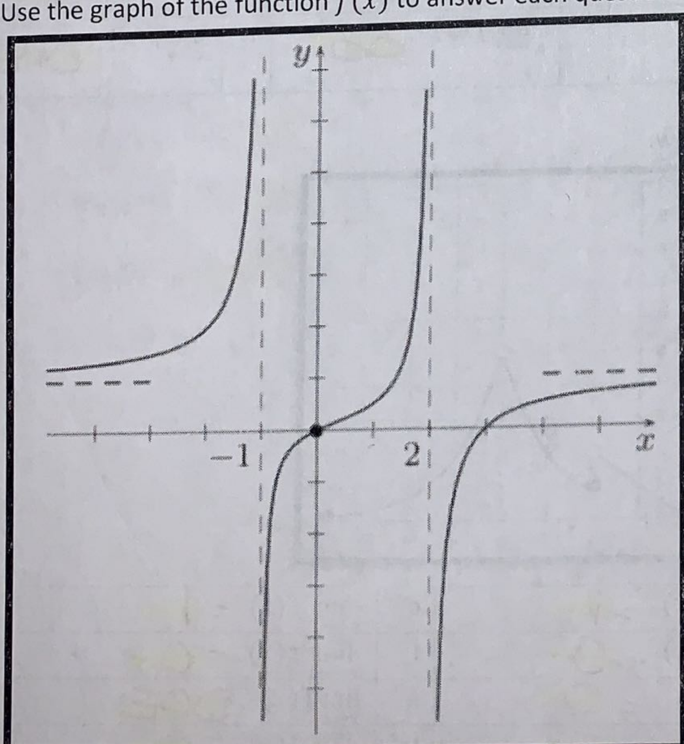
e. $\lim_{x\to 0} f(x) =$ DNE

f. $\lim_{x\to 3^+} f(x) =$ 2

g. $\lim_{x\to 3} f(x) =$ DNE

h. $\lim_{x\to -\infty} f(x) =$ 1

4. Use the graph of the function $f(x)$ to answer each question. Use $\infty, -\infty$, or DNE where appropriate.



a. $f(0) =$ 0

b. $f(2) =$ DNE

c. $f(3) =$ 0

d. $\lim_{x\to -1} f(x) =$ DNE

e. $\lim_{x\to 0} f(x) =$ 0

f. $\lim_{x\to 2^+} f(x) = -\infty$

g. $\lim_{x\to \infty} f(x) =$ 1

Graphs from Limit Worksheet

Draw a graph of a function with the give limits.

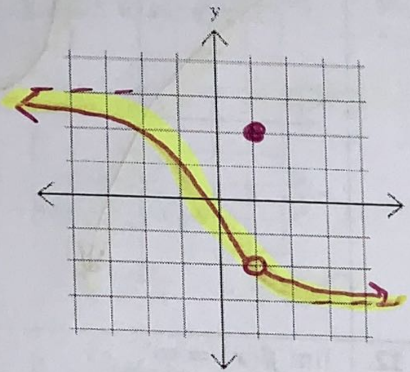
1.

$\lim_{x\to\infty} f(x) = -3$ HA

$\lim_{x\to 1} f(x) = -2$ (1,-2) open

$\lim_{x\to -\infty} f(x) = 2$ HA

$f(1) = 2$ (1,2) closed



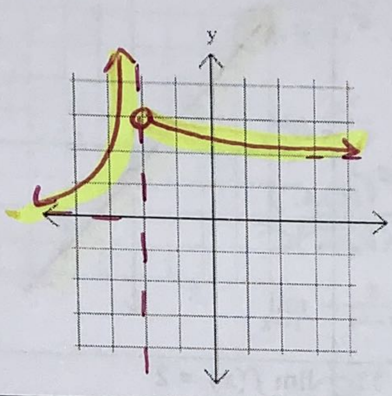
2.

$\lim_{x\to\infty} f(x) = 2$ HA

$\lim_{x\to -2^+} f(x) = 3$ (-2,3) from right

$\lim_{x\to -2^-} f(x) = \infty$ VA

$\lim_{x\to -\infty} f(x) = 0$ HA



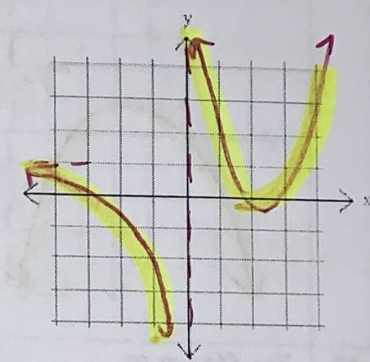
3.

$\lim_{x\to\infty} f(x) = \infty$ EB

$\lim_{x\to 0^+} f(x) = \infty$ VA

$\lim_{x\to 0^-} f(x) = -\infty$ VA

$\lim_{x\to -\infty} f(x) = 1$ HA



4.

$\lim_{x\to -\infty} f(x) = -\infty$ EB

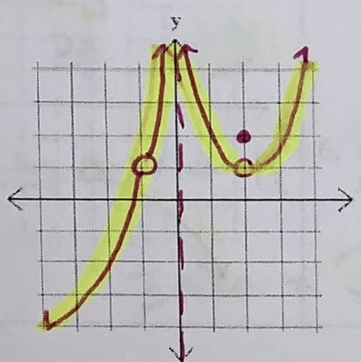
$\lim_{x\to -1} f(x) = 1$ (-1,1) both sides

$\lim_{x\to 0} f(x) = \infty$ VA both sides

$\lim_{x\to 2} f(x) = 1$ (2,1) both sides

$f(2) = 2$ (2,2) closed

$\lim_{x\to\infty} f(x) = \infty$ EB



5.

$\lim_{x\to -\infty} f(x) = -\infty$ EB

$\lim_{x\to -2^-} f(x) = \infty$ UA

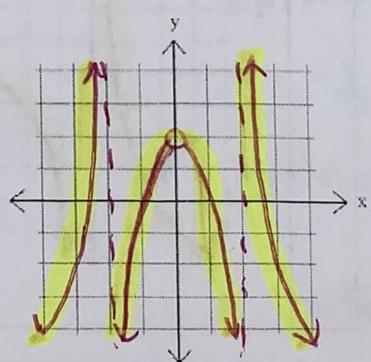
$\lim_{x\to -2^+} f(x) = -\infty$ UA

$\lim_{x\to 0} f(x) = 2$ (0,2) both

$\lim_{x\to 2^-} f(x) = -\infty$ UA

$\lim_{x\to 2^+} f(x) = \infty$ UA

$\lim_{x\to\infty} f(x) = -\infty$ EB



6.

$\lim_{x\to -\infty} f(x) = -2$ HA

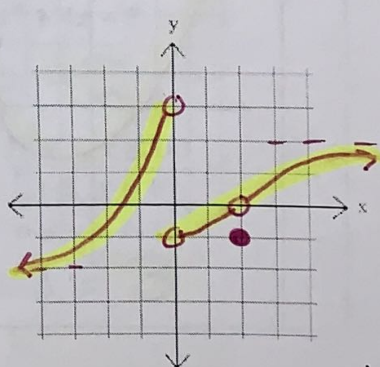
$\lim_{x\to 0^-} f(x) = 3$ (0,3) left

$\lim_{x\to 0^+} f(x) = -1$ (0,-1) right

$\lim_{x\to 2} f(x) = 0$ (2,0) both

$\lim_{x\to\infty} f(x) = 2$ HA

$f(2) = -1$ (2,-1) closed

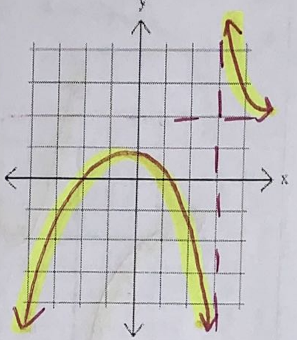


7. $\lim_{x\to\infty} f(x) = 2$

$\lim_{x\to 3^+} f(x) = \infty$

$\lim_{x\to 3^-} f(x) = -\infty$

$\lim_{x\to -\infty} f(x) = -\infty$



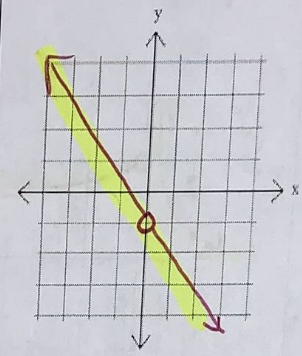
8. $\lim_{x\to\infty} f(x) = -\infty$

$\lim_{x\to -\infty} f(x) = \infty$

$\lim_{x\to 0^-} f(x) = -1$

$\lim_{x\to 0^+} f(x) = -1$

bothsides (0,-1)

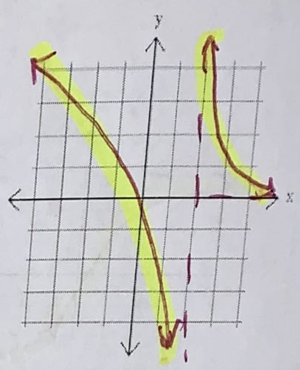


9. $\lim_{x\to\infty} f(x) = 0$

$\lim_{x\to 2^+} f(x) = \infty$

$\lim_{x\to 2^-} f(x) = -\infty$

$\lim_{x\to -\infty} f(x) = \infty$



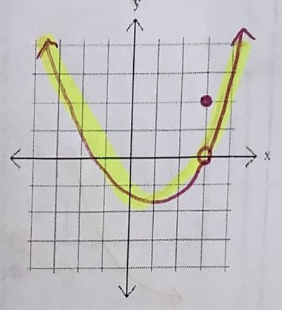
10. $\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to 3^+} f(x) = 0$ (3,0)

$\lim_{x\to 3^-} f(x) = 0$

$\lim_{x\to -\infty} f(x) = \infty$

$f(3) = 2$ (3,2) closed

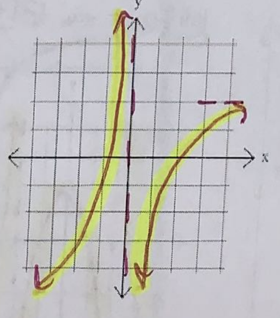


11. $\lim_{x\to\infty} f(x) = 2$

$\lim_{x\to 0^+} f(x) = -\infty$

$\lim_{x\to 0^-} f(x) = \infty$

$\lim_{x\to -\infty} f(x) = -\infty$

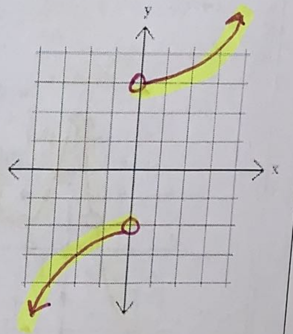


12. $\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to 0^+} f(x) = 3$

$\lim_{x\to 0^-} f(x) = -2$

$\lim_{x\to -\infty} f(x) = -\infty$



One-sided Limits Worksheet

Evaluate each limit.

1. $\lim_{x\to 2^+} \frac{x}{x-2}$ $= \infty$

$\frac{2.001}{2.001-2}$

2. $\lim_{x\to 3^+} \frac{x+1}{x^2-6x+9}$ $= \infty$

$\frac{3.001+1}{(3.001)^2-6(3.001)+9}$ $\frac{+}{+}$

3. $\lim_{x\to -3^-} \frac{x+2}{x^2+6x+9}$ $= -\infty$

$\frac{-3.01+2}{(-3.01)^2+6(-3.01)+9} = \frac{-}{+}$

4. $\lim_{x\to -2^+} \frac{x-2}{x^2+4x+4}$ $= -\infty$

$\frac{-1.99-2}{(-1.99)^2 \ldots} = \frac{-}{+}$

5. $\lim_{x\to -3^-} \frac{x^2}{3x+9}$ $= -\infty$

$\frac{(-3.01)^2}{3(-3.01)+9} = \frac{+}{-}$

6. $\lim_{x\to 2^+} \frac{x^2}{2x-4}$ $= \infty$

$\frac{(2.01)^2}{2(2.01)-4} = \frac{+}{+}$

7. $\lim_{x\to -2^+} \frac{1}{x^2-4}$ $= -\infty$

$\frac{1}{(1.99)^2-4} = \frac{+}{-}$

8. $\lim_{x\to 1^-} -\frac{2}{x^2-1}$ $= \infty$

$\frac{-2}{(.999)^2-1} = \frac{-}{-}$

9. $\lim_{x\to 3^-} f(x), f(x) = \begin{cases} -x+4, & x<3 \\ \frac{x}{2}+1, & x\ge 3 \end{cases}$

$-3+4 = 1$

10. $\lim_{x\to -1^+} f(x), f(x) = \begin{cases} x+3, & x\le -1 \\ -x-1, & x>-1 \end{cases}$

$-(-1)-1 = 0$

11. $\lim_{x\to -2^-} f(x), f(x) = \begin{cases} -x^2-8x-17, & x\le -2 \\ 2x-1, & x>-2 \end{cases}$

$-(-2)^2-8(-2)-17 = -5$

12. $\lim_{x\to 1^-} (|x-1|-2)$

$|1-1|-2 = -2$

13. $\lim_{x\to 0^+} \frac{2x}{|x|}$

$\frac{2x}{x} = 2$

14. $\lim_{x\to 1^-} f(x), f(x) = \begin{cases} -\frac{x}{2}-\frac{3}{2}, & x\le 1 \\ -x^2+4x-5, & x>1 \end{cases}$

$-\frac{1}{2}-\frac{3}{2} = -2$

15. $\lim_{x\to -3^-} f(x), f(x) = \begin{cases} x+6, & x<-3 \\ 3, & x\ge -3 \end{cases}$

$-3+6 = 3$

16. $\lim_{x\to 0^-} f(x), f(x) = \begin{cases} -2x+3, & x\le 0 \\ -\frac{x}{2}+3, & x>0 \end{cases}$

$-2(0)+3 = 3$

Continuity Worksheet

Determine if each function is continuous. If the function is not continuous, find the x-axis location of and classify each discontinuity.

1. $f(x) = -\frac{x}{2x^2+2x+1}$

$2x^2+2x+1=0$

$\frac{-2\pm\sqrt{4-4(2)(1)}}{2(2)}$

imag. sol.

Continuous

2. $f(x) = \frac{x}{x^2+6x+9}$

$x^2+6x+9=0$

$(x+3)(x+3)=0$

$x=-3$ V.A.

Infinite discontinuity at $x=-3$

3. $f(x) = \frac{x^2+4x+3}{x+3} = \frac{\cancel{(x+3)}(x+1)}{\cancel{x+3}}$ hole

Removable discontinuity at $x=-3$

4. $f(x) = \frac{x}{x^2-4x} = \frac{\cancel{x}}{\cancel{x}(x-4)} = \frac{1}{x-4}$

Removable disc. at $x=0$

Infinite disc. at $x=4$

5. $f(x) = \begin{cases} x+4, & x \le -2 \\ -2x-11, & x > -2 \end{cases}$

$\lim_{x\to -2^-} f(x) = 2$

$\lim_{x\to -2^+} f(x) = -7$

Jump discontinuity at $x=-2$

6. $f(x) = \frac{x+7}{x^2+3x}$ $\quad \frac{x+7}{x(x+3)}$

Infinite discontinuity at $x=0$ & $x=-3$

Find the intervals on which each function is continuous.

7. $f(x) = \begin{cases} x, & x \ne 4 \\ 2, & x = 4 \end{cases}$

$(-\infty,4)\cup(4,\infty)$

8. $f(x) = \begin{cases} -2, & x < 3 \\ -2x+6, & x \ge 3 \end{cases}$ open / closed

$(-\infty,3)\cup[3,\infty)$

9. $f(x) = \frac{(x-1)}{x^2-4x+3} = \frac{\cancel{x-1}}{\cancel{(x-1)}(x-3)} = \frac{1}{x-3}$

$(-\infty,1)\cup(1,3)\cup(3,\infty)$

10. $f(x) = \frac{x^2}{2} + 4x + 10$

$(-\infty,\infty)$

11. $f(x) = -x^2 - 4x + 2$

$(-\infty,\infty)$

12. $f(x) = -\frac{x-2}{x^2-3x+2} = -\frac{\cancel{x-2}}{\cancel{(x-2)}(x-1)} = \frac{-1}{x-1}$

$(-\infty,1)\cup(1,2)\cup(2,\infty)$

13. $f(x) = -\frac{x-1}{x^2-x} = -\frac{\cancel{x-1}}{x\cancel{(x-1)}}$

$(-\infty,0)\cup(0,1)\cup(1,\infty)$

14. $f(x) = \frac{x}{x^2-6x+9} = \frac{x}{(x-3)^2}$

$(-\infty,3)\cup(3,\infty)$

15. Critical Thinking: Write a function that has an infinite discontinuity at $x = 100$

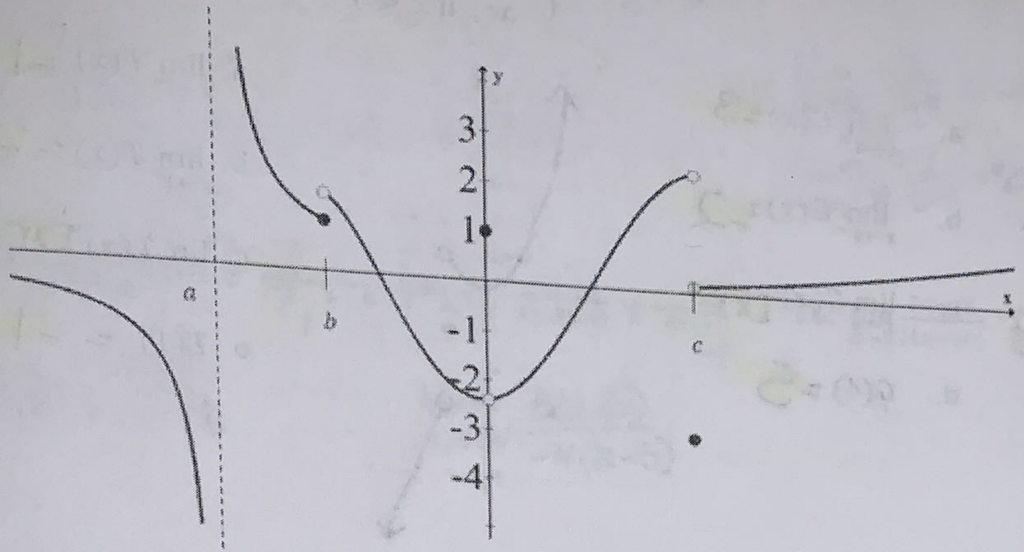
$f(x) = \frac{1}{x-100}$

16 Critical Thinking: Write a function that is continuous over $(-\infty, 0), (0,1), and\ (1, \infty)$ and discontinuous everywhere else.

$f(x) = \frac{1}{x(x-1)}$

Quiz Review

1. Using the graph of $f(x)$ below, find the limits.



a. $\lim_{x\to a^-} f(x) = -\infty$

b. $\lim_{x\to -\infty} f(x) = 0$

c. $\lim_{x\to a} f(x)$ DNE

d. $\lim_{x\to 0} f(x) \approx -2.5$

e. $\lim_{x\to b^+} f(x) \approx 1.5$

f. $\lim_{x\to b} f(x)$ DNE

g. $\lim_{x\to c} f(x)$ DNE

h. $\lim_{x\to c^+} f(x) \approx \frac{1}{4}$

2. Using the graph of $f(x)$ above, list any discontinuities and the type of discontinuity.

x = a Infinite x = 0 Removable

x = b Jump x = c Jump

3. Use the following information to sketch a graph.

$\lim_{x\to -\infty} f(x) = \infty$

$\lim_{x\to -1^-} f(x) = 4$

$\lim_{x\to -1^+} f(x) = -\infty$

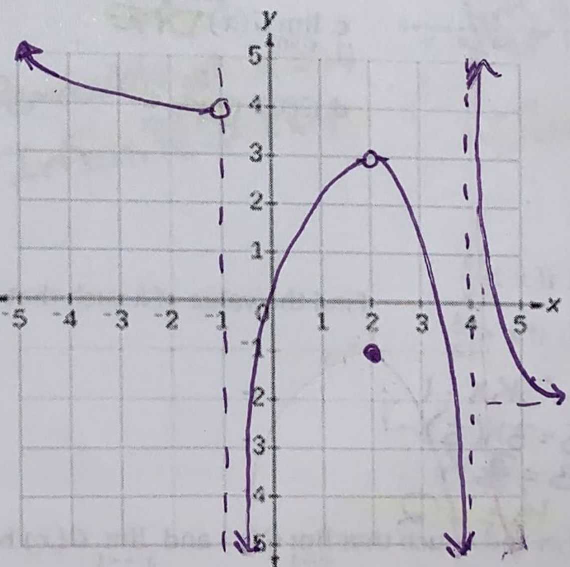
$\lim_{x\to 2} f(x) = 3$

$\lim_{x\to 4^-} f(x) = -\infty$

$\lim_{x\to 4^+} f(x) = \infty$

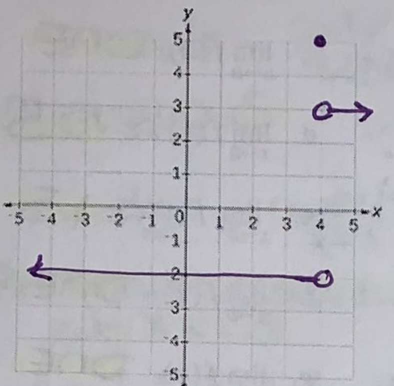
$\lim_{x\to \infty} f(x) = -2$

$f(2) = -1$



Draw a sketch. Find the indicated limit if it exists. If the limit does not exist, explain why.

4. $G(x) = \begin{cases} 3, & \text{if } x > 4 \\ 5, & \text{if } x = 4 \\ -2, & \text{if } x < 4 \end{cases}$



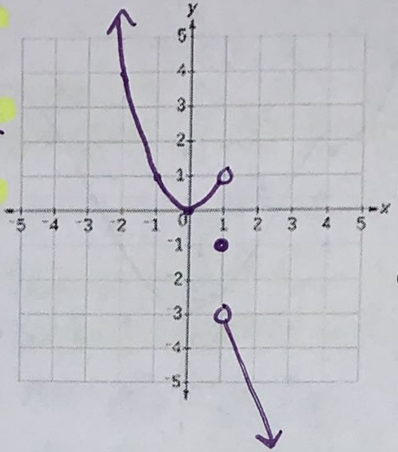
a. $\lim_{x \to 4^+} G(x) = 3$

b. $\lim_{x \to 4^-} G(x) = -2$

c. $\lim_{x \to 4} G(x)$ DNE

d. $G(4) = 5$

5. $T(x) = \begin{cases} 3 - 6x, & \text{if } x > 1 \\ -1, & \text{if } x = 1 \\ x^2, & \text{if } x < 1 \end{cases}$



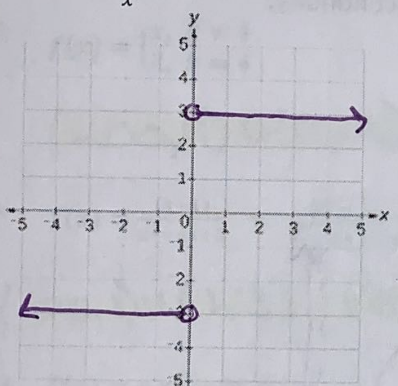
a. $\lim_{x \to 1^-} T(x) = 1$

b. $\lim_{x \to 1^+} T(x) = -3$

c. $\lim_{x \to 1} T(x)$ DNE

d. $T(1) = -1$

6. $G(x) = \frac{|3x|}{x}$



a. $\lim_{x \to 0^+} G(x) = 3$

b. $\lim_{x \to 0^-} G(x) = -3$

c. $\lim_{x \to 0} G(x)$ DNE

d. $G(0)$ DNE

7. Find the limits without sketching the graph:

$T(x) = \begin{cases} x^2 - 16, & \text{if } x < 3 \\ 5, & \text{if } x = 3 \\ 14 - x^2, & \text{if } x > 3 \end{cases}$

a. $\lim_{x \to 3^+} F(x) = 5$

b. $\lim_{x \to 3^-} F(x) = -7$

c. $\lim_{x \to 3} F(x)$ DNE

d. $F(3) = 5$

8. $F(x) = \begin{cases} 2x - 5, & \text{if } x > \frac{1}{2} \\ 3kx - 1, & \text{if } x < \frac{1}{2} \end{cases}$

Find the value of k such that $\lim_{x \to \frac{1}{2}} F(x)$ exists.

$2x - 5 = 3Kx - 1$

$2(\frac{1}{2}) - 5 = 3K(\frac{1}{2}) - 1$

$-3 = \frac{3}{2}K$

$K = -2$

9. Find the values of m and k such that $\lim_{x \to 1} G(x)$ and $\lim_{x \to -1} G(x)$ both exist.

$G(x) = \begin{cases} 3x^2 - kx + m, & \text{if } x \geq 1 \\ mx - 2k, & \text{if } -1 < x < 1 \\ -3m + 4x^2k, & \text{if } x \leq -1 \end{cases}$

$3x^2 - Kx + m = mx - 2K$

$3(1)^2 - K(1) + m = m(1) - 2K$

$K = -3$

$mx - 2K = -3m + 4x^2K$

$m(-1) - 2(-3) = -3m + 4(-1)^2(-3)$

$-m + 6 = -3m - 12$

$2m = -18$

$m = -9$

ind the one-sided limits.

10. $\lim_{x\to 2^-} \frac{3}{x-2}$ $= -\infty$

$\frac{3}{1.99-2}$

11. $\lim_{x\to -3^+} \frac{5}{x+3}$ $= \infty$

$\frac{5}{-2.99+3}$

12. $\lim_{x\to 2} \frac{-7}{2-x}$ DNE

$\lim_{x\to 2^-} \frac{-7}{2-x} = -\infty$

$\lim_{x\to 2^+} \frac{-7}{2-x} = +\infty$

13. $\lim_{x\to 2^-} \frac{|x-2|}{x-2}$ $= -1$

$\frac{-1(x-2)}{x-2}$

14. $\lim_{x\to 5^+} \frac{3x-15}{|4x-20|}$ $= \frac{3}{4}$

$\lim_{x\to 5^+} \frac{3(x-5)}{4(x-5)}$

15. $\lim_{x\to 5^-} \frac{3x-15}{|4x-20|}$ $= -\frac{3}{4}$

$\lim_{x\to 5^-} \frac{3(x-5)}{-4(x-5)}$

16. $\lim_{x\to 5} \frac{3x-15}{|4x-20|}$ DNE

17. $\lim_{x\to 5^+} \frac{|x-4|}{x^2-3x+2}$

$\frac{|5-4|}{(5)^2-3(5)+2} = \frac{1}{12}$

At what values are the following functions discontinuous? State the type of discontinuity.

18. $f(x) = \frac{x}{x^2-25}$

$f(x) = \frac{x}{(x+5)(x-5)}$

Infinite discontinuity at $x=-5, 5$

19. $f(x) = \frac{x+4}{x^2-16}$

$f(x) = \frac{\cancel{x+4}}{(\cancel{x+4})(x-4)}$

$f(x) = \frac{1}{x-4}$

Removable at $x=-4$

Infinite at $x=4$

20. $f(x) = \begin{cases} 4x-5, & \text{if } x > 2 \\ 3x-1, & \text{if } x < 2 \end{cases}$

$\lim_{x\to 2^+} f(x) = 3$

$\lim_{x\to 2^-} f(x) = 5$

Jump at $x=2$

Creative Factoring & Other Interesting Algebra

Difference of Squares

Example: $x-16=\left(\sqrt{x}+4\right)\left(\sqrt{x}-4\right)$

1. $x-9$

$(\sqrt{x}+3)(\sqrt{x}-3)$

2. x^2-5

$(x+\sqrt{5})(\sqrt{x}-5)$

3. $x^{16}-1$

$(x^8+1)(x^8-1)$

$(x^8+1)(x^4+1)(x^4-1)$

$(x^8+1)(x^4+1)(x^2+1)(x^2-1)$

$(x^8+1)(x^4+1)(x^2+1)(x+1)(x-1)$

4. $(x+5)^2-25$

$(x+5+5)(x+5-5)$

$x(x+10)$

5. $9y-a^4$

$(3\sqrt{y}+a^2)(3\sqrt{y}-a^2)$

Sums or Differences of Cubes "SOAP"

Example: $a^3+b^3=(a+b)\left(a^2-ab+b^2\right)$

Example: $a^3-b^3=(a-b)\left(a^2+ab+b^2\right)$

6. $64a^3+125b^3$

$(4a+5b)(16a^2-20ab+25b^2)$

7. $64a^3x^3-125$

$(4ax-5)(16a^2x^2+20ax+25)$

8. $(x+1)^3+64$

$(x+1+4)((x+1)^2-4(x+1)+16)$

$(x+5)((x+1)^2-4x+12)$

9. $8c^3-(a+b)^3$

$(2c-(a+b))(4c^2+2c(a+b)+(a+b)^2)$

$(2c-a-b)(4c^2+2ac+2bc+a^2+2ab+b^2)$

Factor: x^6-y^6:

10. as a difference of **squares**

$(x^3+y^3)(x^3-y^3)$

$(x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)$

11. as a difference of **cubes**

$(x^2-y^2)(x^4+x^2y^2+y^4)$

$(x+y)(x-y)(x^4+x^2y^2+y^4)$

Compare #10 & #11. Which way will allow you to factor completely most easily? differece of cubes

Rationalize the Numerator

12. $\dfrac{\left(\sqrt{x+2}-\sqrt{2}\right)}{x}\cdot\dfrac{(\sqrt{x+2}+\sqrt{2})}{(\sqrt{x+2}+\sqrt{2})}$

$\dfrac{x+2-2}{x(\sqrt{x+2}+\sqrt{2})}=\dfrac{x}{x(\sqrt{x+2}+\sqrt{2})}$

$=\dfrac{1}{\sqrt{x+2}+\sqrt{2}}$

13. $\dfrac{\left(\sqrt{x+3}+\sqrt{3}\right)}{x}\cdot\dfrac{(\sqrt{x+3}-\sqrt{3})}{(\sqrt{x+3}-\sqrt{3})}$

$\dfrac{x+3-3}{x(\sqrt{x+3}-\sqrt{3})}=\dfrac{1}{\sqrt{x+3}-\sqrt{3}}$

Algebraic Limits Worksheet

1. $\lim_{x\to3} x^2+2x-7$

$(3)^2+2(3)-7=8$

2. $\lim_{x\to-1}\dfrac{\frac{1}{x}+1}{x+1} = \lim_{x\to-1}\dfrac{\frac{1+x}{x}}{x+1}$

$\lim_{x\to-1}\frac{1+x}{x}\cdot\frac{1}{x+1}=\lim_{x\to-1}\frac{1}{x}=\frac{1}{-1}$

$=-1$

3. $\lim_{x\to1}\dfrac{(4x^4-5x^2+1)}{x^2+2x-3}$

$\lim_{x\to1}\frac{(4x^2-1)(x^2-1)}{(x^2+3)(x-1)}$

$\lim_{x\to1}\frac{(4x^2-1)(x+1)\cancel{(x-1)}}{(\cancel{x}+3)\cancel{(x-1)}}=\frac{(3)(2)}{4}=\frac{3}{2}$

4. $\lim_{x\to0}\dfrac{(x+1)^2-1}{x}$

$\lim_{x\to0}\frac{(x+1+1)(x+1-1)}{x}$

$\lim_{x\to0}x+2=0+2=2$

5. $\lim_{x\to1}\dfrac{x^2-2x-15}{x-5}$

$\lim_{x\to1}\frac{(x-5)(x+3)}{x-5}$

$\lim_{x\to1}x+3=1+3=4$

6. $\lim_{x\to-3}\dfrac{2x^2+2x-12}{x^2+4x+3}$

$\lim_{x\to-3}\frac{2(x+3)(x-2)}{(x+3)(x+1)}$

$\lim_{x\to-3}\frac{2(x-2)}{x+1}=\frac{2(-5)}{-2}=5$

7. $\lim_{x\to2}\dfrac{(2x+1)^2-25}{x-2}$

$\lim_{x\to2}\frac{(2x+1+5)(2x+1-5)}{x-2}$

$\lim_{x\to2}\frac{(2x+6)2\cancel{(x-2)}}{\cancel{x-2}}=20$

8. $\lim_{x\to2}\dfrac{(3x-2)^2-(x+2)^2}{x-2}$

$\lim_{x\to2}\frac{(3x-2+x+2)(3x-2-x-2)}{x-2}$

$\lim_{x\to2}\frac{4x(2x-4)}{x-2}=\lim_{x\to2}8x=$

16

9. $\lim_{x\to1}\dfrac{\frac{2x}{x+1}-1}{x-1}=\lim_{x\to1}\dfrac{\frac{2x-x-1}{x+1}}{x-1}$

$\lim_{x\to1}\frac{x-1}{(x+1)(x-1)}=\lim_{x\to1}\frac{1}{x+1}$

$=\frac{1}{2}$

10. $\lim_{x\to2}\dfrac{\frac{2}{x^2}-\frac{1}{2}}{x-2}=\lim_{x\to2}\dfrac{\frac{4-x^2}{2x^2}}{x-2}$

$\lim_{x\to2}\frac{-1(x+2)\cancel{(x-2)}}{2x^2\cancel{(x-2)}}=\lim_{x\to2}\frac{-x-2}{2x^2}$

$=\frac{-4}{8}=-\frac{1}{2}$

11. $\lim_{x\to2}\dfrac{x^4-2x^2-8}{x^2-x-6}$

$\lim_{x\to2}\frac{(x^2-4)(x^2+2)}{(x-3)(x+2)}\cancel{\frac{(x+2)(x-2)(x^2+2)}{(x-3)(x+2)}}$

$\frac{0}{-1}=0$

12. $\lim_{x\to1}\dfrac{x-1}{\sqrt{x}-1}$

$\lim_{x\to1}\frac{(\sqrt{x}+1)(\sqrt{x}-1)}{(\sqrt{x}-1)}$

$\lim_{x\to1}\sqrt{x}+1=2$

13. $\lim_{x\to0}\dfrac{x^2+7x+6}{x+3}$

$\frac{(0)^2+7(0)+6}{0+3}=\frac{6}{3}=2$

14. $\lim_{x\to-2}\dfrac{\frac{x}{x+4}+1}{x+2}$

$\lim_{x\to-2}\dfrac{\frac{x+x+4}{x+4}}{x+2}$

$\lim_{x\to-2}\frac{2(x+2)}{(x+4)(x+2)}$

$\lim_{x\to-2}\frac{2}{x+4}=\frac{2}{-2+4}=1$

15. $\lim_{x\to2}\dfrac{x^3+x^2-4x-4}{x^2+x-6}$

$\lim_{x\to2}\frac{x^2(x+1)-4(x+1)}{(x+3)(x-2)}$

$\lim_{x\to2}\frac{(x+2)(x-2)(x+1)}{(x+3)(x-2)}$

$\lim_{x\to2}\frac{(x+2)(x+1)}{x+3}=\frac{4(3)}{5}=12/5$

16. $\lim_{x\to2}\dfrac{x^2-2x-3}{x+5}$

$\frac{(2)^2-2(2)-3}{2+5}=\frac{-3}{7}$

17. $\lim_{x\to2}(x^2-x+1)$

$(2)^2-2+1=3$

18. $\lim_{x\to1}\dfrac{2x+1}{3x-2}$

$\frac{2(1)+1}{3(1)-2}=3$

19. $\lim_{x\to1}\sqrt{10x-1}$

$\sqrt{10(1)-1}=\sqrt{9}=3$

20. $\lim_{x\to1}\dfrac{x^2-x-2}{x-2}$

$\frac{(1)^2-1-2}{1-2}=\frac{-2}{-1}=2$

21. $\lim_{x\to4}\dfrac{\sqrt{x}-2}{x-4}$

$\lim_{x\to4}\frac{\sqrt{x}-2}{(\sqrt{x}+2)(\sqrt{x}-2)}$

$\lim_{x\to4}\frac{1}{\sqrt{x}+2}$

$\frac{1}{\sqrt{4}+2}=\frac{1}{4}$

22. $\lim_{x\to3}\frac{x^2-9}{x+3}$ $\frac{(3)^2-9}{3+3}=\frac{0}{6}=0$	23. $\lim_{x\to3}\frac{x^2-9}{2x^2+7x+3}$ $\frac{(3)^2-9}{2(3)^2+7(3)+3}=0$	24. $\lim_{x\to9}\frac{\sqrt{x}-3}{x-9}$ $\lim_{x\to9}\frac{\sqrt{x}-3}{(\sqrt{x}+3)(\sqrt{x}-3)}$ $\lim_{x\to9}\frac{1}{\sqrt{x}+3}=\frac{1}{\sqrt{9}+3}=\frac{1}{6}$
25. $\lim_{h\to0}\frac{(1+h)^2-1^2}{h}$ $\lim_{h\to0}\frac{(1+h-1)(1+h+1)}{h}$ $\lim_{h\to0}\frac{\cancel{h}(h+2)}{\cancel{h}}=0+2=2$	26. $\lim_{h\to0}\frac{(3+h)^2-3^2}{h}$ $\lim_{h\to0}\frac{(3+h-3)(3+h+3)}{h}$ $\lim_{h\to0}h+6=0+6=6$	27. $\lim_{h\to0}\frac{(x+h)^2-x^2}{h}$ $\lim_{h\to0}\frac{(\cancel{x}+h-\cancel{x})(x+h+x)}{\cancel{h}}$ $\lim_{h\to0}2x+h=2x+0=2x$
28. $\lim_{x\to3}(5x^2-6)$ $5(3)^2-6$ $45-6=39$	29. $\lim_{x\to-1}\frac{x-2}{x^2+4x-3}$ $\frac{-1-2}{(-1)^2+4(-1)-3}=\frac{-3}{-6}=\frac{1}{2}$	30. $\lim_{x\to4}\frac{x^2-16}{x-4}$ $\lim_{x\to4}\frac{(x+4)\cancel{(x-4)}}{\cancel{x-4}}$ $\lim_{x\to4}x+4=4+4=8$
31. $\lim_{x\to0}\frac{6x-9}{x^3-12x+3}$ $\frac{6(0)-9}{0^3-12(0)+3}=\frac{-9}{3}$ $=-3$	32. $\lim_{x\to2}\frac{x^2-4x+4}{x^2+x-6}$ $\lim_{x\to2}\frac{(x-2)(x-2)}{(x+3)(x-2)}$ $\lim_{x\to2}\frac{x-2}{x+3}$ $\frac{2-2}{2+3}=0$	33. $\lim_{x\to-2}\frac{x^3+8}{x+2}$ $\lim_{x\to-2}\frac{(x+2)(x^2-2x+4)}{(x+2)}$ $\lim_{x\to-2}x^2-2x+4$ $(-2)^2-2(-2)+4=12$

Given $\lim_{x\to a}f(x)=-3$, $\lim_{x\to a}g(x)=0$, *and* $\lim_{x\to a}h(x)=8$, find each limit if it exists.

34. $\lim_{x\to a}[f(x)+h(x)]$ $\lim_{x\to a}f(x)+\lim_{x\to a}h(x)$ $-3+8$ $=5$	35. $\lim_{x\to a}[f(x)]^2$ $(\lim_{x\to a}f(x))^2$ $(-3)^2=9$	36. $\lim_{x\to a}\sqrt[3]{h(x)}$ $\sqrt[3]{\lim_{x\to a}h(x)}$ $\sqrt[3]{8}=2$
37. $\lim_{x\to a}\frac{1}{f(x)}$ $\lim_{x\to a}f(x)^{-1}$ $(\lim_{x\to a}f(x))^{-1}=(-3)^{-1}=-\frac{1}{3}$	38. $\lim_{x\to a}\frac{g(x)}{h(x)}$ $\frac{\lim_{x\to a}g(x)}{\lim_{x\to a}h(x)}=\frac{0}{8}=0$	39. $\lim_{x\to a}\frac{h(x)}{g(x)}$ $\frac{\lim_{x\to a}h(x)}{\lim_{x\to a}g(x)}=\frac{8}{0}$ DNE
40. $\lim_{x\to a}\frac{2f(x)}{h(x)-f(x)}$ $\frac{2\lim_{x\to a}f(x)}{\lim_{x\to a}h(x)-\lim_{x\to a}f(x)}=\frac{2(-3)}{8-(-3)}$ $=-\frac{6}{11}$	41. $\lim_{x\to a}[f(x)h(x)]$ $\lim_{x\to a}f(x)\cdot\lim_{x\to a}h(x)$ $-3\cdot8$ -24	42. $\lim_{x\to a}\left[\frac{g(x)+h(x)}{f(x)}\right]$ $\frac{\lim_{x\to a}g(x)+\lim_{x\to a}h(x)}{\lim_{x\to a}f(x)}=\frac{0+8}{-3}$ $=-\frac{8}{3}$

Intermediate Value Theorem Worksheet

full verification of IVT

1. Verify the conditions of the IVT and find the guaranteed c value over [2,6] for
$$f(x) = x^2 + 2x - 11$$
when $f(c) = 4$.

Since $f(x)$ is continuous over $(2,6)$
$f(2)=-3$
$f(6)=37$
$\& -3<4<37$
$\therefore \exists c \in (2,6)$ s.t. $f(c)=4$

$f(c)=x^2+2x-11$
$4=x^2+2x-11$
$0=x^2+2x-15$
$0=(x+5)(x-3)$
$\cancel{x=-5}$ $x=3$
not in interval

2. Verify the conditions of the IVT and find the guaranteed c value over $[-1,3]$ for
$$f(x) = 2x^2 + x - 4$$
when $f(c) = 2$.

Since $f(x)$ is cont. over $(-1,3)$
$f(-1)=-3$ $f(3)=17$ $\& -3<2<17$
$\therefore \exists c \in (-1,3)$ s.t. $f(c)=2$
$2=2x^2+x-4$
$0=2x^2+x-6$
$0=(2x-3)(x+2)$ $x=3/2$

3. Use the IVT to show that
$$f(x) = x^3 - 3x^2 - 7x + 1$$
has a root in the interval (4,5)

Since $f(x)$ is continuous in $(4,5)$
$f(4)=-11$
$f(5)=16$
$\& -11<0<16$
$\therefore \exists c \in (4,5)$ s.t. $f(c)=0$

4. Use the IVT to show that
$$f(x) = x^4 + 3x^2 - 6$$
has a root in the interval (1,2) *and* (−2, −1)

Since $f(x)$ is continuous $(1,2)+(-2,1)$
$f(1)=-2$ $f(2)=22$ $\& -2<0<22$
$f(-2)=22$ $f(-1)=-2$ $-2<0<22$
$\therefore \exists c \in (1,2) \cup (-2,-1)$ s.t. $f(c)=0$

5. Verify the conditions of the IVT and find the guaranteed c value over [0,5] for
$$f(x) = x^2 + x - 1$$
when $f(c) = 11$.

Since $f(x)$ is cont. over $(0,5)$
$f(0)=-1$ $f(5)=29$
$-1<11<29$ $\therefore \exists c \in (0,5)$ s.t. $f(c)=11$
$11=x^2+x-1$
$0=x^2+x-12$
$0=(x+4)(x-3)$
$\cancel{x=-4}$ $x=3$

6. Verify the conditions of the IVT and find the guaranteed c value over [0,3] for
$$f(x) = x^2 - 6x + 8$$
when $f(c) = 0$.

Since $f(x)$ is cont. over $(0,3)$
$f(0)=8$ $f(3)=-1$ $\& -1<0<8$
$\therefore \exists c \in (0,3)$ s.t. $f(c)=0$
$0=x^2-6x+8$ $\cancel{x=4}$
$0=(x-4)(x-2)$ $x=2$

7. Verify the conditions of the IVT and find the guaranteed c value over [0,3] for
$$f(x) = x^3 - x^2 + x - 2$$
when $f(c) = 4$.

Since $f(x)$ is continuous in $(0,3)$
$f(0)=-2$ $f(3)=19$ $\& -2<4<19$
$\therefore \exists c \in (0,3)$ s.t. $f(c)=4$
$4=x^3-x^2+x-2$ $P/q=\pm1,\pm2,\pm3,\pm6$
$0=x^3-x^2+x-6$
$2 \,|\, 1 \;\; -1 \;\; 1 \;\; -6$
$\quad\;\; 2 \;\; 2 \;\; 6$
$\quad 1 \;\; 1 \;\; 3 : 0$ ✓
$x=2$

8. Verify the conditions of the IVT and find the guaranteed c value over $[\frac{5}{2}, 4]$ for
$$f(x) = \frac{x^2 + x}{x - 1}$$
when $f(c) = 6$.

Since $f(x)$ is cont. in $(5/2,4)$
$f(5/2)=\frac{\frac{25}{4}+\frac{5}{2}}{\frac{5}{2}-1}=\frac{35}{4}\cdot\frac{2}{3}=\frac{35}{6}$
$f(4)=20/3$ $\&$ $\frac{35}{6}<6<\frac{20}{3}$ $\therefore \exists c \in (\frac{5}{2},4)$ s.t. $f(c)=6$

$\frac{6}{1}=\frac{x^2+x}{x-1}$
$x^2+x=6x-6$
$x^2-5x+6=0$
$(x-3)(x-2)=0$
$x=3$ $\cancel{x=2}$

9. Use the IVT to show that
$$f(x) = x^3 + x - 1$$
has a root in the interval [0,1]

Since $f(x)$ is continuous in $(0,1)$
$f(0)=-1$
$f(1)=1$
$\& -1<0<1$
$\therefore \exists c \in (0,1)$ s.t. $f(c)=0$

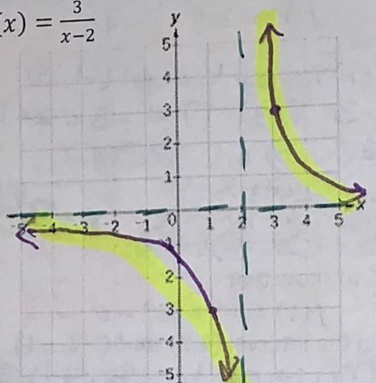
10. Use the IVT to show that
$$f(x) = x^3 + 3x - 2$$
has a root in the interval [0,1]

Since $f(x)$ is continuous in $(0,1)$
$f(0)=-2$
$f(1)=2$
$\& -2<0<2$
$\therefore \exists c \in (0,1)$ s.t. $f(c)=0$

Vertical and Horizontal Asymptotes Worksheet

State the vertical, horizontal, or slant asymptotes for the following. Sketch the graph and find the end behavior.

1. $f(x) = \frac{3}{x-2}$



Vertical Asymptote: $x = 2$

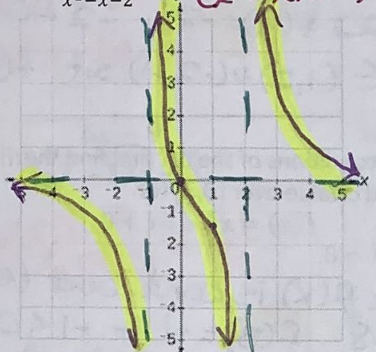
Horizontal Asymptote: $y = 0$

Slant Asymptote: none

End Behavior: $\lim_{x\to\infty} f(x) = 0$

$\lim_{x\to-\infty} f(x) = 0$

2. $f(x) = \frac{3x}{x^2-x-2} = \frac{3x}{(x-2)(x+1)}$



Vertical Asymptote: $x = 2, x = -1$

Horizontal Asymptote: $y = 0$

Slant Asymptote: none

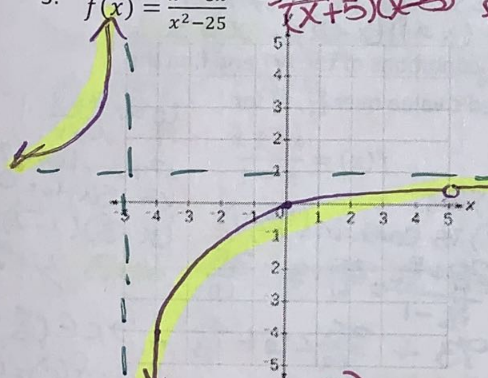
End Behavior: $\lim_{x\to\infty} f(x) = 0$

$\lim_{x\to-\infty} f(x) = 0$

3. $f(x) = \frac{x^2-5x}{x^2-25} = \frac{x(\cancel{x-5})}{(x+5)(\cancel{x-5})}$

hole at $(5, \frac{1}{2})$

$f(x) = \frac{x}{x+5}$



Vertical Asymptote: $x = -5$

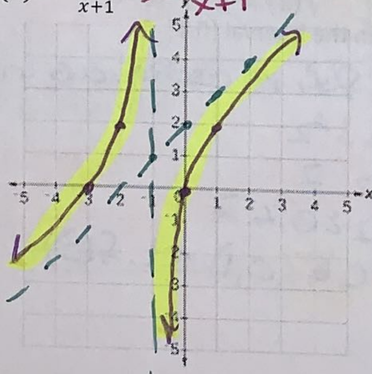
Horizontal Asymptote: $y = 1$

Slant Asymptote: none

End Behavior: $\lim_{x\to\infty} f(x) = 1$

$\lim_{x\to-\infty} f(x) = 1$

4. $f(x) = \frac{x^2+3x}{x+1} = \frac{x(x+3)}{x+1}$



Vertical Asymptote: $x = -1$

Horizontal Asymptote: none

Slant Asymptote: $y = x + 2$

$$\begin{array}{r|rrr} -1 & 1 & 3 & 0 \\ & \downarrow & -1 & -2 \\ \hline & 1 & 2 & -2 \end{array}$$

$x + 2$

End Behavior: $\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to-\infty} f(x) = -\infty$

Infinite Limits Worksheet

Find the Limit.

1. $\lim_{x\to\infty} 3$ $\leftrightarrow$ horizontal line 3 a limit of a constant is a constant	2. $\lim_{x\to-\infty} 3$ 3	3. $\lim_{x\to-\infty}(-3)$ -3
4. $\lim_{x\to\infty}(-2x)$ $-\infty$ or $-2(\infty) = -\infty$	5. $\lim_{x\to\infty}(3-x)$ $-\infty$ or $3-\infty = -\infty$	6. $\lim_{x\to\infty}\sqrt{x}$ $\sqrt{\infty} = \infty$ ∞
7. $\lim_{x\to-\infty}(4-x)$ ∞ or $4--\infty = \infty$	8. $\lim_{x\to\infty}\frac{8}{5-3x}$ HA: $y=0$ 0	9. $\lim_{x\to\infty}\frac{1}{x-12}$ HA: $y=0$ 0
10. $\lim_{x\to-\infty}\frac{3}{x+4}$ HA: $y=0$ 0	11. $\lim_{x\to\infty}(1+2x-3x^5)$ odd - $-\infty$ or $-3(\infty)^5 = -\infty$	12. $\lim_{x\to\infty}(2x^3-110x+5)$ ∞
13. $\lim_{x\to\infty}\frac{(3+2x^2)}{4+5x}$ no HA behaves like $\lim_{x\to\infty}\frac{2x^2}{5x} = \frac{2(\infty)^2}{5(\infty)}$ $\frac{+}{+}$ ∞	14. $\lim_{x\to\infty}\frac{x^2+x}{3-x}$ $\frac{\infty^2}{-\infty} = \frac{+}{-} = -\infty$	15. $\lim_{x\to\infty}\frac{x+4}{x^2-2x+5}$ HA: $y=0$ 0
16. $\lim_{x\to-\infty}-\frac{x-2}{x^2+2x+1}$ HA $y=0$ 0	17. $\lim_{x\to\infty}\frac{7-6x^5}{x+3}$ behaves like $\lim_{x\to\infty}\frac{-6x^5}{x} = \frac{-}{+} = -$ $-\infty$	18. $\lim_{x\to\infty}\frac{6-x^3}{7x^3+3}$ HA: $y=\frac{-1}{7}$ $\frac{-1}{7}$
19. $\lim_{x\to\infty}\frac{1}{x^2+1}$ HA: $y=0$ 0	20. $\lim_{x\to\infty}\frac{x^4+x^2}{x^4+1}$ HA: $y=1$ 1	21. $\lim_{x\to\infty}\frac{1+x^2}{2-x^2}$ HA: $y=-1$ -1
22. $\lim_{x\to\infty}\frac{2x^2}{x^2+1}$ HA: $y=2$ 2	23. $\lim_{x\to-\infty}\frac{x+4}{3x^2-5}$ HA: $y=0$ 0	24. $\lim_{x\to\infty}\frac{3x^3+25x^2-x+1}{4x^3-7x^2+2x+2}$ HA $y=3/4$ 3/4

Limits Review 1

The limit of a constant is a constant.

1. $\lim_{x\to e}\sqrt{7}$ $= \sqrt{7}$

2. $\lim_{x\to\sqrt{5}}\pi$ $= \pi$

Direct Substitution – ALWAYS try direct substitution first!

3. $\lim_{x\to5}(2x^2 - x + 3)$

$2(5)^2-5+3$

48

4. $\lim_{y\to2^-}\frac{y^2-3y+2}{y+1}$

$\frac{(2)^2-3(2)+2}{2+1} = 0$

5. $\lim_{x\to4}\frac{|5-3x|}{2x+1}$

$\frac{|5-3(4)|}{2(4)+1} = \frac{7}{9}$

6. $\lim_{x\to4}\cos\left(\frac{3\pi}{x}\right)$

$\lim_{x\to4}\cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$

If substitution results in $\frac{0}{0}$, try to factor, reduce & substitute again.

7. $\lim_{x\to1}\frac{x^4-1}{x-1}$

$\lim_{x\to1}\frac{(x^2+1)(x+1)\cancel{(x-1)}}{\cancel{x-1}}$

$(1^2+1)(1+1) = 4$

8. $\lim_{x\to1}\frac{x-1}{(x^3-x^2)+(x-1)}$

$\lim_{x\to1}\frac{x-1}{x^2(x-1)+1(x-1)}$

$\lim_{x\to1}\frac{\cancel{x-1}}{\cancel{(x-1)}(x^2+1)}$

$\lim_{x\to1}\frac{1}{x^2+1}$

$\frac{1}{1^2+1} = \frac{1}{2}$

9. $\lim_{x\to9}\frac{\sqrt{x}-3}{x-9}$

$\lim_{x\to9}\frac{\sqrt{x}-3}{(\sqrt{x}+3)(\sqrt{x}-3)}$

$\lim_{x\to9}\frac{1}{\sqrt{x}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{6}$

10. $\lim_{x\to-2}\frac{x+2}{x^3+8}$

$\lim_{x\to-2}\frac{x+2}{(x+2)(x^2-2x+4)}$

$\lim_{x\to-2}\frac{1}{x^2-2x+4} = \frac{1}{4+4+4} = \frac{1}{12}$

If substitution results in $\frac{0}{0}$, try to multiply by the conjugate.

11. $\lim_{x\to2}\frac{\sqrt{5x+6}-4}{x-2}$

$\lim_{x\to2}\frac{(\sqrt{5x+6}-4)(\sqrt{5x+6}+4)}{(x-2)(\sqrt{5x+6}+4)}$

$\lim_{x\to2}\frac{5x+6-16}{(x-2)(\sqrt{5x+6}+4)}$

$\lim_{x\to2}\frac{5}{\sqrt{5x+6}+4} = \frac{5}{\sqrt{16}+4} = \frac{5}{8}$

12. $\lim_{x\to4}\frac{3-\sqrt{x+5}}{x-4}$

$\lim_{x\to4}\frac{(3-\sqrt{x+5})(3+\sqrt{x+5})}{(x-4)(3+\sqrt{x+5})}$

$\lim_{x\to4}\frac{9-x-5}{(x-4)(3+\sqrt{x+5})}$

$\lim_{x\to4}\frac{-1\cancel{(x-4)}}{\cancel{(x-4)}(3+\sqrt{x+5})} = \frac{-1}{3+\sqrt{9}} = \frac{-1}{6}$

13. $\lim_{x\to0}\frac{\sqrt{x+3}-\sqrt{3}}{x}\cdot\frac{(\sqrt{x+3}+\sqrt{3})}{(\sqrt{x+3}+\sqrt{3})}$

$\lim_{x\to0}\frac{x+3-3}{x(\sqrt{x+3}+\sqrt{3})}$

$\lim_{x\to0}\frac{1}{\sqrt{x+3}+\sqrt{3}} = \frac{1}{\sqrt{3}+\sqrt{3}}$

$\frac{1}{2\sqrt{3}}$

If substitution results in $\frac{0}{0}$ with complex fractions, try to clear the "little denominators".

14. $\lim_{h\to0}\frac{\frac{1}{2+h}-\frac{1}{2}}{h}$

$\lim_{h\to0}\frac{\frac{2-2-h}{2(2+h)}}{h}$

$\lim_{h\to0}\frac{-h}{2h(2+h)}$

$\lim_{h\to0}\frac{-1}{2(2+h)} = \frac{-1}{2(2+0)} = \frac{-1}{4}$

15. $\lim_{x\to10}\frac{\frac{x}{5}-2}{x-10}$

$\lim_{x\to10}\frac{\frac{x-10}{5}}{x-10}$

$\lim_{x\to10}\frac{x-10}{5}\cdot\frac{1}{x-10}$

$\lim_{x\to10}\frac{1}{5} = \frac{1}{5}$

16. $\lim_{h\to-2}\frac{(h+5)^{-1}-3^{-1}}{h+2} = \lim_{h\to-2}\frac{\frac{1}{h+5}-\frac{1}{3}}{h+2}$

$\lim_{h\to-2}\frac{3-h-5}{3(h+5)(h+2)}$

$\lim_{h\to-2}\frac{-1(h+2)}{3(h+5)(h+2)}$

$\lim_{h\to-2}\frac{-1}{3(h+5)} = \frac{-1}{3(-2+5)} = \frac{-1}{9}$

Write the absolute value.

17. $\lim_{x\to5^-} \frac{|2x-10|}{3x-15}$

behaves like $\lim_{x\to5^-} \frac{2|x-5|}{3(x-5)}$

$\frac{-2(x-5)}{3(x-5)} = -\frac{2}{3}$

18. $\lim_{x\to7^-} \frac{3x-21}{|7-x|}$

$\lim_{x\to7^-} \frac{3(x-7)}{|-1(x-7)|}$

$\lim_{x\to7^-} \frac{3(x-7)}{|x-7|} = \frac{3(x-7)}{-1(x-7)} = -3$

19. $\lim_{x\to2} \frac{|x-2|}{x-2}$

$\lim_{x\to2^+} \frac{x-2}{x-2}$ $\lim_{x\to2^-} \frac{-1(x-2)}{x-2}$

$\lim_{x\to2^+} = 1$

$\lim_{x\to2^-} = -1$

DNE

One-sided limits when you get $\frac{\#}{0}$, do you get ∞ or $-\infty$? Reason it out!

20. $\lim_{x\to3^-} \frac{5}{x-3} = -\infty$

$\frac{5}{2.999-3} = \frac{+}{-}$

21. $\lim_{x\to3^+} \frac{-4}{x-3}$ $-\infty$

$\frac{-4}{3.001-3} = \frac{-}{+}$

22. $\lim_{x\to6^+} \frac{x+6}{x^2-36} = \frac{x+6}{(x+6)(x-6)}$

$\lim_{x\to6^+} \frac{1}{x-6}$

$\frac{1}{6.001-6} = \infty$

Limits to infinity. You can do a **behaves like** only in limits to infinity. You can also divide by the highest power in the denominator, simplify, and then find the limit.

$\lim_{x\to\pm\infty}$ (polynomial) – Use end behavior rules.

23. $\lim_{x\to\infty} (3x^2 - 4x + 2)$ even +

∞

24. $\lim_{x\to-\infty} (5x^3 - 2x^2 + 1)$ odd +

$-\infty$

$$\lim_{x\to\pm\infty} \frac{\text{degree smaller}}{\text{DEGREE LARGER}} = 0$$

25. $\lim_{x\to\infty} \frac{3x-5}{x^2+1} = 0$

26. $\lim_{x\to-\infty} \frac{4x^2-3x}{6x^5-3x+1} = 0$

$$\lim_{x\to\pm\infty} \frac{\text{degree =}}{\text{degree =}} = \text{ratio of the leading coefficients}$$

27. $\lim_{x\to\infty} \frac{5x^2-11}{x^2+1}$

5

28. $\lim_{x\to-\infty} \frac{4x^2-5x+2}{3x^2+1}$

4/3

29. $\lim_{x\to\infty} \frac{2x^2+1}{(2-x)(2+x)}$ $\frac{2x^2+1}{4-x^2}$

-2

$$\lim_{x\to\pm\infty} \frac{\text{DEGREE LARGER}}{\text{degree smaller}} = \infty \text{ or } -\infty$$

30. $\lim_{x\to\infty} \frac{7-6x^5}{x+3}$

$\frac{7-6(\infty)^5}{\infty+3} = \frac{-}{+}$

$-\infty$

31. $\lim_{x\to-\infty} \frac{7-6x^5}{x+3}$

$\frac{7-6(-\infty)^5}{-\infty+3} = \frac{+}{-}$

$-\infty$

32. $\lim_{x\to\infty} \frac{5+x^3-3x^4}{2x-1}$

$\frac{-3(\infty)^4}{2(\infty)} = \frac{-}{+}$

$-\infty$

33. $\lim_{x\to-\infty} \frac{5+x^3-3x^4}{2x-1}$

$\frac{-3(\infty)^4}{2(\infty)} = \frac{-}{+}$

$-\infty$

$\lim_{x\to\pm\infty}$ involving square roots: Use the **behaves like** method & remember the $\sqrt{x^2} = |x|$!

34. $\lim_{x\to\infty} \frac{\sqrt{4x^2-2}}{x+1}$

$\frac{\sqrt{4x^2}}{x} = \frac{|2x|}{x} = 2$

35. $\lim_{x\to-\infty} \frac{\sqrt{4x^2-2}}{x+1}$

$\frac{|2x|}{x} = \frac{-2x}{x} = -2$

36. $\lim_{x\to\infty} \frac{2-x}{\sqrt{7+9x^2}}$

$\frac{-x}{\sqrt{9x^2}} = \frac{-x}{|3x|} = \frac{-1}{3}$

37. $\lim_{x\to-\infty} \frac{2-x}{\sqrt{7+9x^2}}$

$\frac{-x}{-3x} = \frac{1}{3}$

Write the equations of the vertical and horizontal asymptotes.

38. $y = \frac{2x^2-5x-3}{x^2-2x-3}$

$\frac{(2x+1)\cancel{(x-3)}}{(x+1)\cancel{(x-3)}}$ hole at $x=3$

HA: $y=2$

VA: $x=-1$

39. $y = \frac{3-x}{9-x^2}$

$\frac{\cancel{(3-x)}}{\cancel{(3-x)}(3+x)} = \frac{1}{3+x}$ hole at $x=3$

HA: $y=0$

VA: $x=-3$

Continuity: Limit from right = limit from left = value of $f(x)$ at the point.

Is $f(x)$ continuous? Why?

40. $f(x) = \begin{cases} -5-x, & x > -1 \\ 6x+2, & x \le -1 \end{cases}$

continuous

bc

$\lim_{x\to-1^-} f(x) = \lim_{x\to-1^+} f(x)$

$= \lim_{x\to-1} f(x) = f(-1)$

$\lim_{x\to-1^-} 6(-1)+2 = -4$

$\lim_{x\to-1^+} -5-(-1) = -4$

$f(-1) = -4$

41. $f(x) = \frac{|x+2|}{x+2}$

no; Jump discontinuity at $x=-2$

$\lim_{x\to-2^-} f(x) = -1$

$\lim_{x\to-2^+} f(x) = 1$

$\lim_{x\to-2^-} f(x) \ne \lim_{x\to-2^+} f(x)$

Intermediate Value Theorem.

42. Verify the conditions of the Intermediate Value Theorem, and **find** c guaranteed by the theorem when $f(x) = x^2 - 6x + 7$ over the interval $[0,3]$ and $f(c) = -1$.

Since $f(x)$ is continuous over $(0,3)$, $f(0)=7$, $f(3)=-2$ & $-2<x<7$

$\therefore \exists c \in (0,3)$ s.t. $f(c) = -1$

$-1 = x^2 - 6x + 7$

$0 = x^2 - 6x + 8$

$0 = (x-2)(x-4)$

$x = 2$

Finding values that make a function continuous.

43. Find the value of a that would make the function continuous.

$$f(x) = \begin{cases} 3xa+5 & \text{if } x \le -1 \\ -2x+5a & \text{if } x > -1 \end{cases}$$

$3xa + 5 = -2x + 5a$

$3(-1)a + 5 = -2(-1) + 5a$

$-8a = -3$

$a = 3/8$

44. Find the value of *m* and *n* that would make the function continuous.

$$g(x) = \begin{cases} 3mx - 4n & \text{if } x \le -1 \\ 4 + nx - mx^2 & \text{if } -1 < x < 2 \\ x^2 - mx + 7n & \text{if } x \ge 2 \end{cases}$$

$3mx - 4n = 4 + nx - mx^2$

$3m(-1) - 4n = 4 + n(-1) - m(-1)^2$

$-3m - 4n = 4 - n - m$

$-2m = 3n + 4$

$4 + nx - mx^2 = x^2 - mx + 7n$

$4 + n(2) - m(2)^2 = (2)^2 - m(2) + 7n$

$4 + 2n - 4m = 4 - 2m + 7n$

$-2m = 5n$

$3n + 4 = 5n$

$-2n = -4$

$n = 2$

$-2m = 5n$

$-2m = 5(2)$

$m = -5$

Limits Review 2

Evaluate the following limits.

1. $\lim\limits_{x\to 0} \frac{9-4x}{2x^3-4x^2+3}$

$\frac{9-4(0)}{2(0)^3-4(0)^2+3} = \frac{9}{3} = 3$

2. $\lim\limits_{x\to 2} \frac{2x^2+x-10}{x^2+x-6}$

$\lim\limits_{x\to 2} \frac{(2x+5)\cancel{(x-2)}}{(x+3)\cancel{(x-2)}} = \frac{9}{5}$

3. $\lim\limits_{x\to -3} \frac{x^3+27}{x+3}$

$\lim\limits_{x\to -3} \frac{\cancel{(x+3)}(x^2-3x+9)}{\cancel{(x+3)}}$

$(-3)^2-3(-3)+9 = 27$

4. $\lim\limits_{x\to 5} \frac{x}{x^2-25} = \frac{5}{0}$ V.A.

$\lim\limits_{x\to 5^-} \frac{x}{x^2-25} = -\infty$ so DNE

$\lim\limits_{x\to 5^+} \frac{x}{x^2-25} = \infty$

5. $\lim\limits_{x\to 0} \frac{x^3-8}{x^2-4}$

$\frac{(0)^3-8}{(0)^2-4} = \frac{-8}{-4} = 2$

6. $\lim\limits_{x\to 3} \frac{(3-x)^2}{x-3}$

$\lim\limits_{x\to 3} \frac{(3-x)^2}{-1(3-x)}$

$\lim\limits_{x\to 3} \frac{3-x}{-1} = \frac{3-3}{-1} = 0$

7. $\lim\limits_{\theta\to 0} \frac{1-\cos\theta}{2\sin^2\theta}$

$\lim\limits_{\theta\to 0} \frac{1-\cos\theta}{2(1-\cos^2\theta)}$

$\lim\limits_{\theta\to 0} \frac{\cancel{1-\cos\theta}}{2(1+\cos\theta)\cancel{(1-\cos\theta)}}$

$\lim\limits_{\theta\to 0} \frac{1}{2(1+\cos\theta)}$

$\frac{1}{2(1+1)} = \frac{1}{4}$

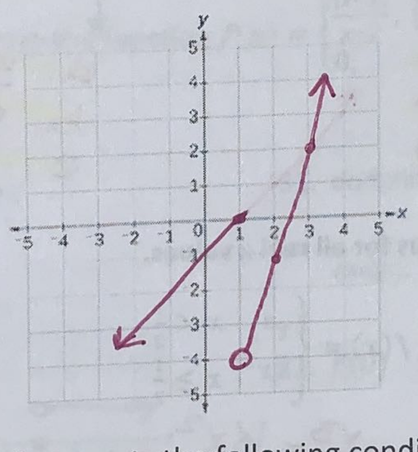
8. $\lim\limits_{x\to -1} \frac{x^4-1}{x+1}$

$\lim\limits_{x\to -1} \frac{(x^2-1)(x^2+1)}{x+1}$

$\lim\limits_{x\to -1} \frac{\cancel{(x+1)}(x-1)(x^2+1)}{\cancel{x+1}} = (-1-1)(1+1) = -4$

For problems #9-12, use the function $f(x) = \begin{cases} x-1, & x \le 1 \\ 3x-7, & x > 1 \end{cases}$

9. Graph the function



10. $\lim\limits_{x\to 1^-} f(x) = 1-1 = 0$

$x<1$

11. $\lim\limits_{x\to 1^+} f(x) = 3(1)-7 = -4$

$x>1$

12. $\lim\limits_{x\to 1} f(x)$ DNE

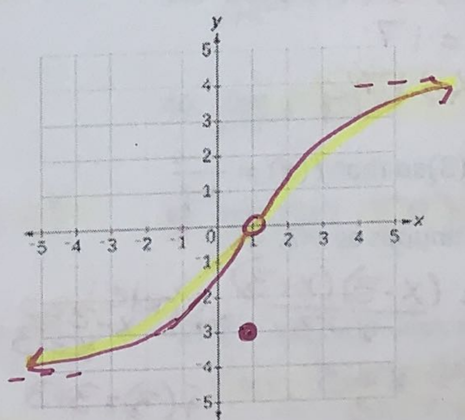
13. Draw a function that meets the following conditions. Is this function continuous? Explain.

$\lim\limits_{x\to\infty} f(x) = 4$

$\lim\limits_{x\to 1} f(x) = 0$

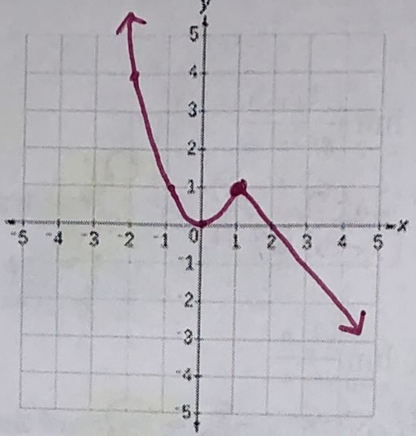
$\lim\limits_{x\to -\infty} f(x) = -4$

$f(-1) = -3$



14. Draw a function that meets the following conditions. Find the indicated limit if it exists. Is this function continuous? Explain.

$$f(x) = \begin{cases} x^2, & x < 1 \\ 2 - x, & x > 1 \\ 2, & x = 1 \end{cases}$$



15. $\lim_{x \to 1^+} f(x) = 2 - 1 = 1$
$x > 1$

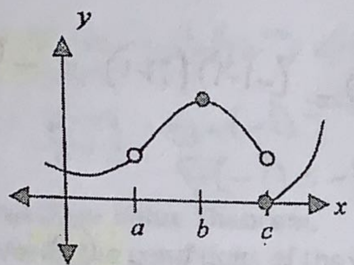
16. $\lim_{x \to 1^-} f(x) = (1)^2 = 1$
$x < 1$

17. $\lim_{x \to 1} f(x) = 1$

18. $f(1) = 2$

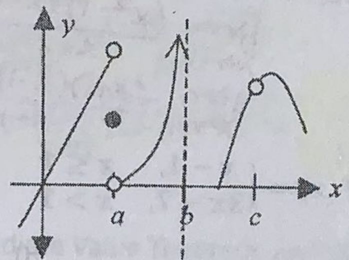
Indicate whether the function whose graph is given is continuous at each of the points a, b, and c.

19.



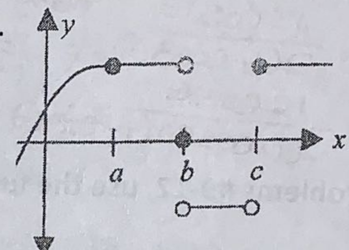
a. no
b. yes
c. no

20.



a. no
b. no
c. no

21.



a. yes
b. no
c. no

Find a value of k which will cause $f(x)$ to be continuous for all real x values.

22. $f(x) = \begin{cases} kx^2, & x < -3 \\ 5 - 4x, & x \geq -3 \end{cases}$

$Kx^2 = 5 - 4x$
$K(-3)^2 = 5 - 4(-3)$
$9K = 17$
$K = 17/9$

23. $f(x) = \begin{cases} x^3, & x < \frac{1}{2} \\ kx^2, & x \geq \frac{1}{2} \end{cases}$

$x^3 = Kx^2$
$(\frac{1}{2})^3 = K(\frac{1}{2})^2$
$\frac{1}{8} = \frac{1}{4}K$
$K = 1/2$

24. Define $f(3)$ so that $f(x) = \frac{x^2 - 9}{x - 3}$ is continuous at x=3.

$f(x) = \frac{(x-3)(x+3)}{x-3}$ → hole: $x - 3 = 0$, $x = 3$

$f(x) = x + 3$

$f(3) = 3 + 3$
$f(3) = 6$

25. Define $f(3)$ so that $f(x) = \frac{x^3 - 1}{x^2 - 1}$ is continuous at x=1.

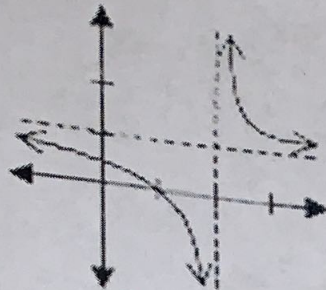
$f(x) = \frac{\cancel{(x-1)}(x^2 + x + 1)}{(x+1)\cancel{(x-1)}}$ hole: $x = 1$

$f(1) = \frac{(1)^2 + 1 + 1}{1 + 1}$

$f(1) = \frac{3}{2}$

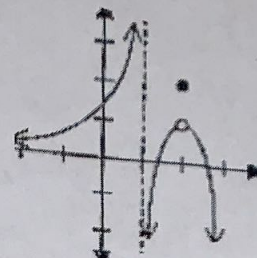
Use the graph to determine the intervals for which the function is continuous.

26.



$(-\infty, 2) \cup (2, \infty)$

27.



$(-\infty, 1) \cup (1, 2) \cup (2, \infty)$

At what values are the following functions discontinuous? Explain the type of discontinuity.

28. $f(x) = \frac{x+3}{x^2-3x-10}$

$f(x) = \frac{x+3}{(x-5)(x+2)}$ $\quad x \neq 5$, $x \neq -2$

Infinite discontinuity at $x=5$ & $x=-2$

29. $f(x) = \frac{x+2}{4-x^2}$

hole at $x=-2$

$f(x) = \frac{\cancel{x+2}}{(\cancel{2+x})(2-x)}$

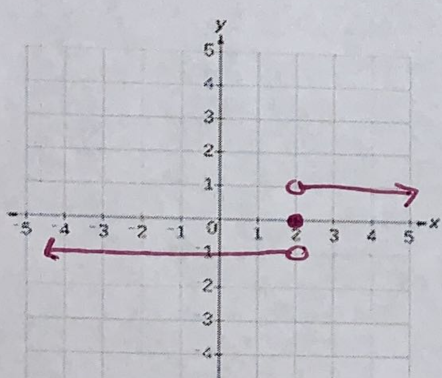
$f(x) = \frac{1}{2-x}$

$x=-2$ Removable disc.

$x=2$ Infinite disc.

For problems #30-42, use the function $f(x) = \begin{cases} \frac{|x-2|}{x-2}, & x \neq 2 \\ 0, & x = 2 \end{cases}$

30. Graph the function.



31. domain: $(-\infty, \infty)$

range: $\{-1, 0, 1\}$

32. $f(0) = -1$

33. $f(2) = 0$

34. $f(4) = 1$

35. $\lim_{x\to 0^+} f(x) = -1$

36. $\lim_{x\to 0^-} f(x) = -1$

37. $\lim_{x\to 0} f(x) = -1$

38. Is $f(x)$ continuous at x=0?

yes

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x) = \lim_{x\to 0} f(x) = f(0)$

39. $\lim_{x\to 2^-} f(x) = -1$

40. $\lim_{x\to 2^+} f(x) = 1$

41. $\lim_{x\to 2} f(x)$ DNE

42. Is $f(x)$ continuous at x=2?

no; Jump discontinuity at $x=2$

$\lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x)$