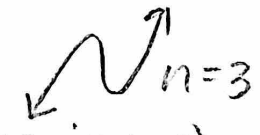
Degree

The degree, *n*, of a polynomial function, can tell us a lot of helpful information:

- *n* = the maximum number of zeros, or x-intercepts
- *n* = the maximum number of directions in which the graph will travel
- (*n* -1) = the maximum number of turns/extrema (minimums / maximums)
  "valleys" "mountains"
- End Behavior:
  - if the degree is even, the ends of the graph will go in same directions
  - if the degree is odd, the ends of the graph will go in opposite directions



Maximum and Minimum Values

These are the y - coordinates of the turning points of the graph.

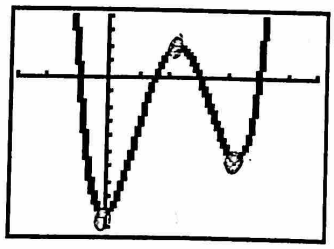
- Absolute maximum is the highest point on the graph
- Absolute minimum is the lowest point on the graph
- Relative maximum is found at the top of a peak, and is higher than any point nearby.
- Relative minimum is found at the bottom of a valley, and is lower than any point nearby.

Maximum and minimum values are called extrema.

Ex 1: Determine the least possible degree of the function shown.

To find the least possible degree, count the number of extrema, and add 1.

a.



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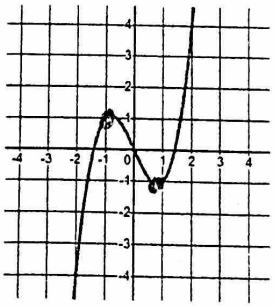
b.



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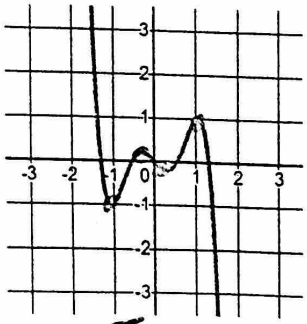
c.



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Ex 2: Determine the maximum number of extrema.

To find the maximum number of extrema, take the degree and subtract 1.

a. $f(x) = 2x^{3} - 3x^2 + 5$ (degree)

3-1=2

2 is the most number of extrema for this function

b. $y = -3x^{4} + 2x^2 - 1$

4-1=3

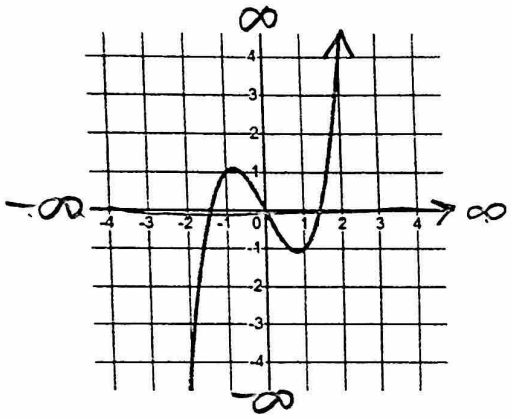
c. $g(x) = x^{5} + 3x^4 - x^3 - 3x^2$

5-1=4

End Behavior

Describes whether the y-values of a function increase or decrease as the x-values approach positive infinity on the right, and as the x-values approach negative infinity on the left.

Think: As x goes to the RIGHT (toward positive infinity), does the end of the graph go up or down?	**Say:** "As x approaches infinity, f of x approaches _____" ∞ or −∞	**Write:** As x → ∞, f(x) → _____ ∞ or −∞ up down
Think: As x goes to the LEFT (toward negative infinity), does the end of the graph go up or down?	**Say:** "As x approaches negative infinity, f of x approaches _____" ∞ or −∞	**Write:** As x → −∞, f(x) → _____ ∞ or −∞ up down



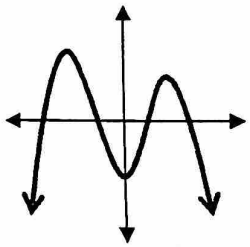
(right) $x \to +\infty$ $f(x) \to$ ∞ (up)

$x \to -\infty$ $f(x) \to$ −∞ (down)

(left)

Ex 3: Describe the end behavior of each graph.

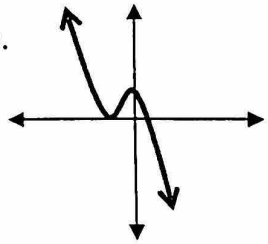
a.



$x \to +\infty$ $f(x) \to$ −∞

$x \to -\infty$ $f(x) \to$ −∞

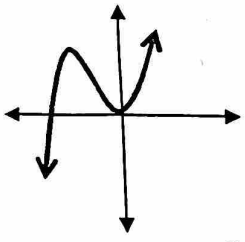
b.



$x \to +\infty$ $f(x) \to$ −∞

$x \to -\infty$ $f(x) \to$ ∞

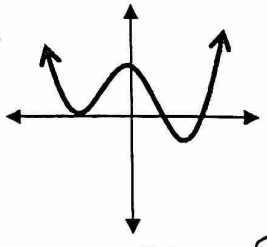
c.



$x \to +\infty$ $f(x) \to$ ∞

$x \to -\infty$ $f(x) \to$ −∞

d.



$x \to +\infty$ $f(x) \to$ ∞

$x \to -\infty$ $f(x) \to$ ∞

End behavior can also be determined by looking at the leading coefficient and degree of the function.

Leading Coefficient tells us what happens on the **RIGHT:**

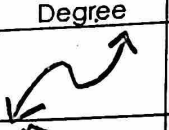
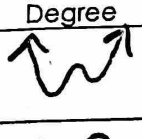
POSITIVE ∞

NEGATIVE $-\infty$

Degree tells us what happens on the **LEFT:** (same as the right, or opposite?)

EVEN Same

ODD Opposite

	ODD Degree	EVEN Degree
POSITIVE Leading Coefficient		
NEGATIVE Leading Coefficient		

Ex 4: Determine the end behavior of the function.

a. $f(x) = -2x^3 + x - 4$

LC − (right/down)

Degree ODD (opposite)

$x \to +\infty$ $f(x) \to -\infty$

$x \to -\infty$ $f(x) \to \infty$

b. $f(x) = x^4 + 2x^3 - x^2 - 1$

LC + (right/up)

Degree EVEN (same)

$x \to +\infty$ $f(x) \to \infty$

$x \to -\infty$ $f(x) \to \infty$

c. $f(x) = 6x^5 - 4x^3 - 9$

LC + (right/up)

Degree ODD (opposite)

$x \to +\infty$ $f(x) \to \infty$

$x \to -\infty$ $f(x) \to -\infty$

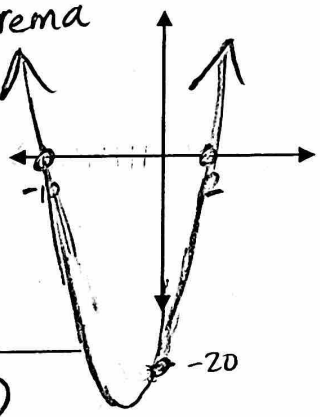
Putting it all together!

Ex 5: Given the polynomial and zeros, sketch a graph and determine the characteristics

a. $f(x) = x^2 + 8x - 20$

zeros: -10, 2

2 − 1 = 1 extrema



# of Zeros: 2

Y-Int: (0, −20)

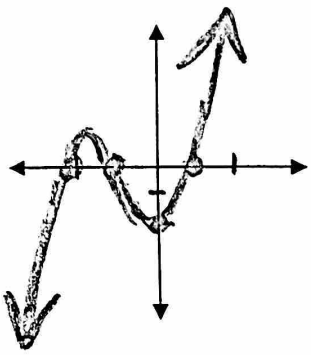
Max # of extrema: 1

$x \to +\infty$ $f(x) \to \infty$

$x \to -\infty$ $f(x) \to \infty$

b. $f(x) = x^3 + 2x^2 - x - 2$

zeros: -2, -1, 1



# of Zeros: 3

Y-Int: (0, −2)

Max # of extrema: 2 3 − 1 = 2

$x \to +\infty$ $f(x) \to \infty$

$x \to -\infty$ $f(x) \to -\infty$