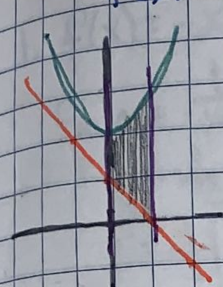
# 8.1 Area Between Two Curves



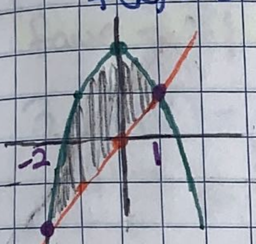
$A = \int_a^b [f(x) - g(x)]\,dx$ Top − Bottom or Right − Left

1. Find the area of the region bounded by the graphs
$f(x) = x^2 + 2$  $g(x) = -x$  $x = 0$  $x = 1$



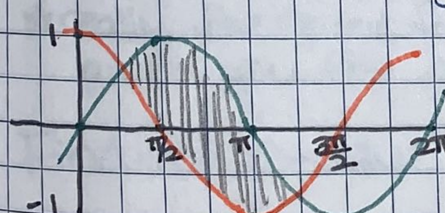
top − bottom
$\int_0^1 x^2 + 2 - (-x)\,dx$
$\int_0^1 x^2 + x + 2\,dx \rightarrow \frac{x^3}{3} + \frac{x^2}{2} + 2x \Big|_0^1$
$\left(\frac{1^3}{3} + \frac{1^2}{2} + 2(1)\right) - \left(\frac{0^3}{3} + \frac{0^2}{2} + 2(0)\right) = \frac{1}{3} + \frac{1}{2} + 2$
$A = \frac{17}{6}$

2. Find the area of the region bounded by the graphs
$f(x) = 2 - x^2$  $g(x) = x$



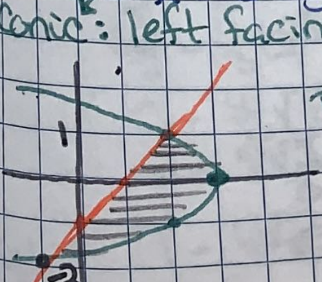

Top − Bottom
$\int_{-2}^1 2 - x^2 - x\,dx$
$2x - \frac{x^3}{3} - \frac{x^2}{2} \Big|_{-2}^1 = \left(2 - \frac{1}{3} - \frac{1}{2}\right) - \left(-4 + \frac{8}{3} - 2\right) =$
$2 - \frac{1}{3} - \frac{1}{2} + 4 - \frac{8}{3} + 2 = 8 - 3 - \frac{1}{2} = A = \frac{9}{2}$

3. Find the area of the region bounded by the graphs
$f(x) = \sin x$  $g(x) = \cos x$  for one of the repeated regions.




Top-Bottom
$\int_{\pi/4}^{5\pi/4} \sin x - \cos x\,dx = -\cos x - \sin x \Big|_{\pi/4}^{5\pi/4}$
$-\cos\left(\frac{5\pi}{4}\right) - \sin\left(\frac{5\pi}{4}\right) + \cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{4}\right)$
$\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \frac{4\sqrt{2}}{2}$  $A = 2\sqrt{2}$

4. Find the area of the region bounded by the graphs
$x = 3 - y^2$  $x = y + 1$
Conic: left facing parabola  $y = x - 1$




Right − Left ★need y bounds + write integral in terms of y when right − left
$\int_{-2}^1 3 - y^2 - (y+1)\,dy$
$\int_{-2}^1 2 - y^2 - y\,dy$
$2y - \frac{y^3}{3} - \frac{y^2}{2} \Big|_{-2}^1$
$\left(2 - \frac{1}{3} - \frac{1}{2}\right) - \left(-4 + \frac{8}{3} - 2\right) = 8 - 3 - \frac{1}{2} = A = \frac{9}{2}$

# 8.2 Average Function Value + Mean Value Theorem



$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$ ← Avg Function Value for Integrals

Find the average value of the function over the given interval:

1. $f(x) = -x^2 + 2x + 1 \quad [1,4]$

$f_{ave} = \frac{1}{4-1}\int_1^4 -x^2 + 2x + 1\,dx$

$= \frac{1}{3}\left(\frac{-x^3}{3} + x^2 + x\right)\Big|_1^4$

$= \frac{1}{3}\left[\frac{-4^3}{3} + 4^2 + 4 - \left(-\frac{1^3}{3} + 1^2 + 1\right)\right]$

$= \frac{1}{3}\left[-\frac{64}{3} + 16 + 4 + \frac{1}{3} - 1 - 1\right]$

$= \frac{1}{3} \cdot -3$

$f_{ave} = -1$

2. $f(x) = -2e^{2x+4} \quad [-3,-2]$

$f_{ave} = \frac{1}{-2-(-3)}\int_{-3}^{-2} -2e^{2x+4}\,dx$

$= 1 \cdot \frac{-2e^{2x+4}}{2}\Big|_{-3}^{-2}$

$= -e^{2(-2)+4} + e^{2(-3)+4}$

$= -e^0 + e^{-2}$

$f_{ave} = \frac{1}{e^2} - 1$

3. $f(x) = \csc^2 x \quad [\pi/2, 3\pi/4]$

$f_{ave} = \frac{1}{3\pi/4 - \pi/2}\int_{\pi/2}^{3\pi/4} \csc^2 x\,dx$

$= \frac{4}{\pi} \cdot -\cot x\Big|_{\pi/2}^{3\pi/4} = \frac{4}{\pi}\left(-\cot\left(\frac{3\pi}{4}\right) + \cot\left(\frac{\pi}{2}\right)\right) = \frac{4}{\pi}(1+0)$ $\quad f_{ave} = \frac{4}{\pi}$

Mean Value Theorem for Integrals: $\int_a^b f(x)\,dx = f(c)(b-a)$

★ really the same formula as AFV but you are finding c-value that gives you the average value (or x value)

Find the c-value guaranteed by the MVT:

1. $f(x) = x - 2\sqrt{x} \quad [0,2]$

$\int_0^2 x - 2\sqrt{x}\,dx = (c - 2\sqrt{c})(2-0)$

$\frac{x^2}{2} - \frac{4}{3}x^{3/2}\Big|_0^2 = 2c - 4\sqrt{c}$

$\frac{2^2}{2} - \frac{4}{3}(2)^{3/2} = 2c - 4\sqrt{c}$

$2 - \frac{4}{3}\sqrt{8} = 2c - 4\sqrt{c}$

$2 - \frac{8\sqrt{2}}{3} = 2c - 4\sqrt{c}$

$1 - \frac{4\sqrt{2}}{3} = c - 2\sqrt{c}$

Solve in calc!

$c = 0.438, 1.791$ both within $[0,2]$

menu 3, 1

$(1 - \frac{4\sqrt{2}}{3} = c - 2\sqrt{c}, c)$

2. $f(x) = \frac{9}{x^3} \quad [1,3]$

$\int_1^3 \frac{9}{x^3}\,dx = \frac{9}{c^3}(3-1)$

$\frac{9}{-2x^2}\Big|_1^3 = \frac{18}{c^3}$

$-\frac{9}{2(3)^2} + \frac{9}{2(1)^2} = \frac{18}{c^3}$

$-\frac{9}{18} + \frac{9}{2} = \frac{18}{c^3}$

$4 = \frac{18}{c^3}$

$4c^3 = 18$

$c^3 = \frac{9}{2}$

$c = \sqrt[3]{\frac{9}{2}}$

# 8.3 Volumes of Solids with Known Cross Sections

Cross Sections of area $A(x)$ taken perpendicular to x-axis:

$$V = \int_a^b A(x)\,dx$$

Cross Sections of area $A(y)$ taken perpendicular to y-axis:

$$V = \int_a^b A(y)\,dy$$

Steps:

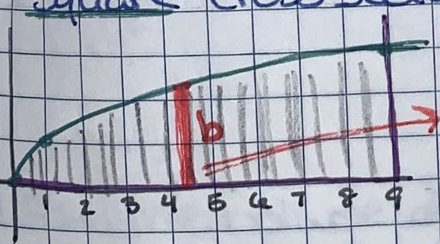
1. Draw the base on the xy-plane
2. Draw a representative cross section.
3. Find an area formula for the cross section $A(x)$ or $A(y)$
4. Set up integral with bounds
5. Integrate your area formula & use ~~F~~TC on your bounds.

Important Area Formulas to Know!

| Square | Semicircle | Rectangle | Isos. Right Triangle w/ base as leg |
|---|---|---|---|
| $A = b^2$ | $A = \frac{1}{2}\pi\left(\frac{b}{2}\right)^2$, $A = \frac{\pi}{8}b^2$ | $A = bh$ | $A = \frac{1}{2}b^2$ |
| | | | **Isos. Right Triangle w/ base as hyp.** |
| | | | $A = \frac{1}{4}b^2$ |

1. Find the volume of the solid whose base is the region bounded by $y=\sqrt{x}$, $x=9$, + x-axis. Bases of cross sections are $\perp$ to the x-axis.

Square Cross sections:





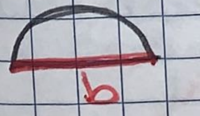
$A = b^2$

top - bottom $\int_0^9 (\sqrt{x} - 0)^2\,dx$

$\int_0^9 x\,dx = \frac{x^2}{2}\Big|_0^9 = \frac{9^2}{2} - \frac{0^2}{2}$

$V = \frac{81}{2}$

Semicircle cross sections:



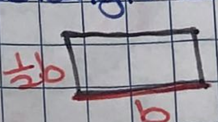

$A = \frac{\pi}{8}b^2$

$V = \frac{\pi}{8}\int_0^9 (\sqrt{x})^2\,dx$

$= \frac{\pi}{8}\int_0^9 x\,dx = \frac{\pi}{8}\left(\frac{x^2}{2}\Big|_0^9\right) = \frac{\pi}{8}\left(\frac{81}{2}\right)$

$V = \frac{81\pi}{16}$

Rectangle cross sections: $h = \frac{1}{2}b$




$A = bh$

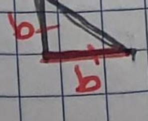
$A = b\left(\frac{1}{2}b\right)$

$V = \int_0^9 \frac{1}{2}\sqrt{x}\sqrt{x}\,dx$

$= \frac{1}{2}\int_0^9 x\,dx$

$= \frac{1}{2}\cdot\frac{x^2}{2}\Big|_0^9 = \frac{81}{4} - \frac{0}{4}$ $V = \frac{81}{4}$

Isosceles Right Triangle w/ base as leg:

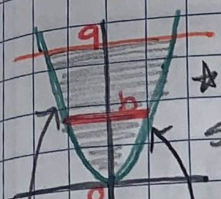



$A = \frac{1}{2}b^2$

$V = \frac{1}{2}\int_0^9 \sqrt{x}^2\,dx$

$= \frac{1}{2}\cdot\frac{x^2}{2}\Big|_0^9 = \frac{81}{4} - \frac{0}{4}$ $V = \frac{81}{4}$

2. Find the volume of the solid whose base is the region bounded by $y = x^2$ & $y = 9$. Bases of cross-sections are $\perp$ to y-axis.
Square cross sections:




$x = -\sqrt{y}$ $\quad$ $x = +\sqrt{y}$

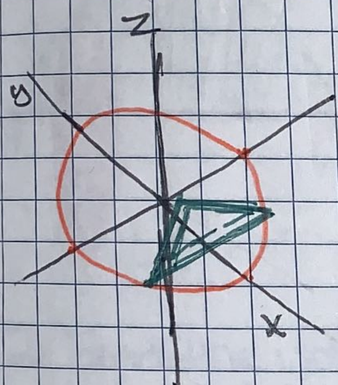
*not to scale

★ bc cross sections are $\perp$ to y-axis, functions + bounds need to be in terms of y!

$y = x^2$
$x = \pm\sqrt{y}$
b = right − left
$b = \sqrt{y} - -\sqrt{y}$
$b = 2\sqrt{y}$

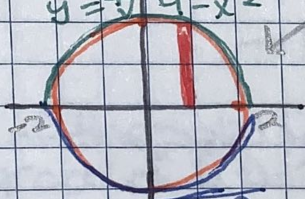
$\int_0^9 (2\sqrt{y})^2\, dy$
$4\int_0^9 y\, dy$
$4\frac{y^2}{2}\Big|_0^9 = 2y^2\Big|_0^9$
$2(9)^2 - 2(0)^2$
$V = 162$

3. A solid has a circular base of radius 2 in the xy plane. Cross sections $\perp$ to the x-axis are in the shape of isosceles right triangles with their hypotenuse as the base of the solid. Find the volume of the solid.




Circle: $x^2 + y^2 = 4$
$y^2 = 4 - x^2$
$y = \pm\sqrt{4 - x^2}$

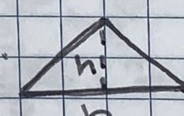
$y = +\sqrt{4 - x^2}$




$y = -\sqrt{4 - x^2}$

Top − Bottom
$b = \sqrt{4 - x^2} - -\sqrt{4 - x^2}$
$b = 2\sqrt{4 - x^2}$

C.S.




$A = \frac{1}{4} b^2$

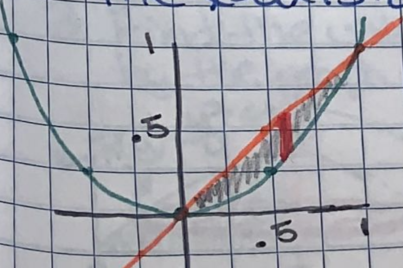
$\frac{1}{4}\int_{-2}^{2} (2\sqrt{4 - x^2})^2\, dx$

$\frac{4}{4}\int_{-2}^{2} 4(4 - x^2)\, dx = \int_{-2}^{2} 4 - x^2\, dx$

$= 4x - \frac{x^3}{3}\Big|_{-2}^{2} = 8 - \frac{8}{3} + 8 - \frac{8}{3}$

$V = \frac{32}{3}$

4. Find the volume of the solid whose base is the region bounded between the curves $y = x$ & $y = x^2$ & whose cross sections $\perp$ to the x-axis are squares.




Top − Bottom
$b = x - x^2$

$A = b^2$

$V = \int_0^1 (x - x^2)^2\, dx$
$= \int_0^1 x^2 - 2x^3 + x^4\, dx$
$= \frac{x^3}{3} - \frac{x^4}{2} + \frac{x^5}{5}\Big|_0^1$
$= \frac{1}{3} - \frac{1}{2} + \frac{1}{5}$

$V = \frac{1}{30}$

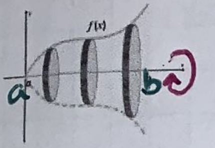
# 8.4 Volumes of Revolutions

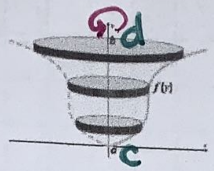


**THE DISK METHOD**

To find the volume of a solid of revolution with the disk method, use one of the following:

| Horizontal Axis of Revolution | Vertical Axis of Revolution |
|---|---|
| $\text{Volume} = V = \pi \int_a^b [R(x)]^2\,dx$ | $\text{Volume} = V = \pi \int_c^d [R(y)]^2\,dy$ |




Area of a disk is $\pi r^2$

over x → dx

over y → dy

1. Find the volume of the solid formed by revolving the region bounded by the graph of $f(x) = 16 - x^2$ & $y = 0$ about the x-axis.


$A = \pi r^2$

$$\pi \int_{-4}^{4} (\overset{\text{Top}}{16 - x^2} - \overset{\text{bottom}}{0})^2\,dx$$

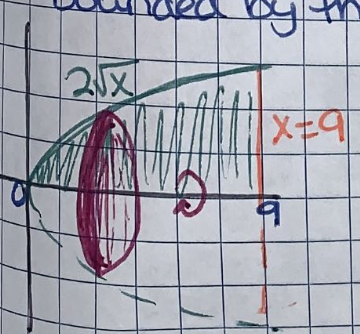
$$\pi \int_{-4}^{4} 256 - 32x^2 + x^4\,dx$$

$$\pi \left(256x - \frac{32x^3}{3} + \frac{x^5}{5} \Big|_{-4}^{4}\right)$$

$$\pi \left(256 \cdot 4 - \frac{32(4)^3}{3} + \frac{(4)^5}{5} - 256(-4) + \frac{32(-4)^3}{3} - \frac{(-4)^5}{5}\right)$$

$$\pi \left(2048 - \frac{4096}{3} + \frac{2048}{5}\right) = \frac{16{,}384\pi}{15}$$

2. Find the volume of the solid formed by revolving the region bounded by the graph of $f(x) = 2\sqrt{x}$, $y = 0$, & $x = 9$ about the x-axis.




Top - Bottom

$2\sqrt{x} - 0$

$R(x) = 2\sqrt{x}$

$$V = \pi \int_0^9 (2\sqrt{x} - 0)^2\,dx$$

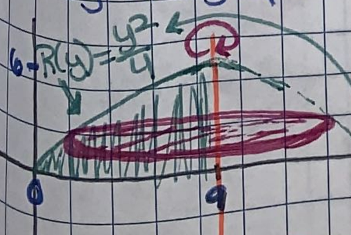
$$= \pi \int_0^9 4x\,dx$$

$$= 4\pi \int x\,dx$$

$$= 4\pi \cdot \frac{x^2}{2} \Big|_0^9$$

$$= 4\pi \left(\frac{81}{2} - 0\right) = 162\pi$$

3. Find the volume of the solid formed by revolving the region bounded by the graph of $f(x) = 2\sqrt{x}$, $y = 0$, & $x = 9$ about the line $x = 9$.




Right - Left

$R(y) = 9 - \frac{y^2}{4}$

$y = 2\sqrt{x}$

$\frac{y}{2} = \sqrt{x}$

$\left(\frac{y}{2}\right)^2 = x$

pt. of intersection

$\frac{y^2}{4} = 9$

$y^2 = 36$

$y = 6$

$$V = \pi \int_{0 \text{ bottom y bound}}^{6 \text{ top y bound}} \left(9 - \frac{y^2}{4}\right)^2 dy$$

$$= \pi \int_0^6 81 - \frac{9}{2}y^2 + \frac{y^4}{16}\,dy$$

$$= \pi \left(81x - \frac{3}{2}y^3 + \frac{y^5}{80} \Big|_0^6\right)$$

$$= \pi \left(81 \cdot 6 - \frac{3}{2}(6)^3 + \frac{(6)^5}{80}\right)$$

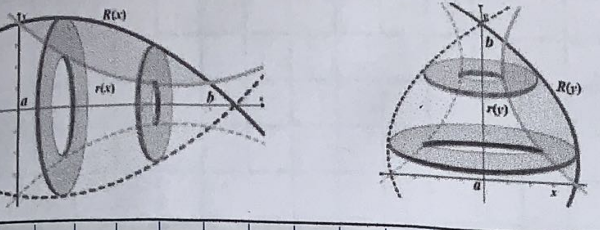
$$= \pi \left(486 - 324 + \frac{486}{5}\right)$$

$$V = \frac{1296\pi}{5}$$

## THE WASHER METHOD

Use the washer method for solids of revolution with holes.

$$V = \pi \int_a^b ([R(x)]^2 - [r(x)]^2)\, dx$$




$R(x)$ = outer radius

$r(x)$ = inner (hole) radius

1. Determine the volume of the solid by rotating the portion of the region bounded by $y=\sqrt[3]{x}$ & $y=\frac{x}{4}$ that lies in 1st quadrant about y-axis


$y=\frac{x}{4}$ $\quad$ $y=\sqrt[3]{x}$

$x=4y$ $\quad$ $x=y^3$

Pts. of Intersection: $y^3=4y$

$y(y^2-4)=0$

$y=0, 2, \cancel{-2}$ QI

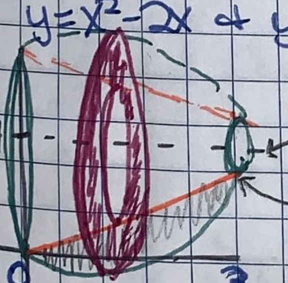
$$V=\pi\int_0^2 (\underbrace{4y-0}_{R(y)})^2-(\underbrace{y^3-0}_{r(y)})^2\,dy$$

$$=\pi\int_0^2 16y^2-y^6\,dy$$

$$=\pi\left(\frac{16y^3}{3}-\frac{y^7}{7}\Big|_0^2\right)$$

$$=\pi\left(\frac{16\cdot 2^3}{3}-\frac{2^7}{7}\right) \rightarrow V=\frac{512\pi}{21}$$

2. Determine the volume of the solid by rotating the region bounded by $y=x^2-2x$ & $y=x$ about the line $y=4$.




Top - Bottom dx

Intersect at: $x^2-2x=x$

$x^2-3x=0$

$x(x-3)=0$

$x=0, 3$

$$V=\pi\int_0^3 (\underbrace{4-(x^2-2x)}_{\text{Outer: Top-bottom}})^2-(\underbrace{4-x}_{\text{Inner: Top-Bottom}})^2\,dx$$

★ Calculator $\quad V=\frac{153\pi}{5}$

3. Determine the volume of the solid by rotating the region bounded by $y=2\sqrt{x-1}$ + $y=x-1$ about the line $x=-1$.


Right - Left

$R(y)=y+1-(-1)=y+2$

$r(y)=\frac{y^2}{4}-(-1)=\frac{y^2}{4}+2$

$$V=\pi\int_0^4 (y+2)^2-\left(\frac{y^2}{4}+2\right)^2 dy$$

★ calculator

$V=\frac{96\pi}{5}$

$y=2\sqrt{x-1}$ $\quad$ $y=x-1$

$\frac{y}{2}=\sqrt{x-1}$ $\quad$ $x=y+1$

$\left(\frac{y}{2}\right)^2+1=x$

$x=\frac{y^2}{4}+1$

# 8.5 Slope Fields



Slope fields (also called vector fields or direction fields) are a tool to graphically obtain the solutions to the antiderivative.

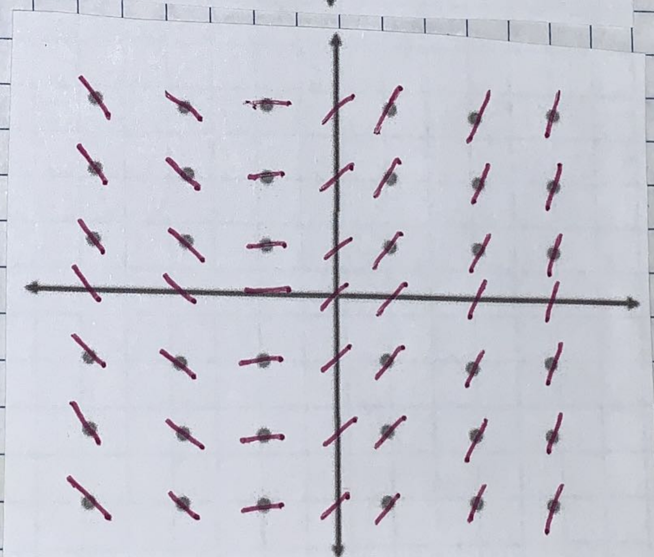
Steps for creating a slope field:
1. Create an xy table & include all values of x provided on the x-axis.
2. Plug the x-values into f' to obtain the slope at that value.
3. Draw a short line segment for each y-value of x that rep. slope.

1. $f'(x) = 1$

| X | f'(x) |
|---|---|
| -3 | 1 |
| -2 | 1 |
| -1 | 1 |
| 0 | 1 |
| 1 | 1 |
| 2 | 1 |
| 3 | 1 |

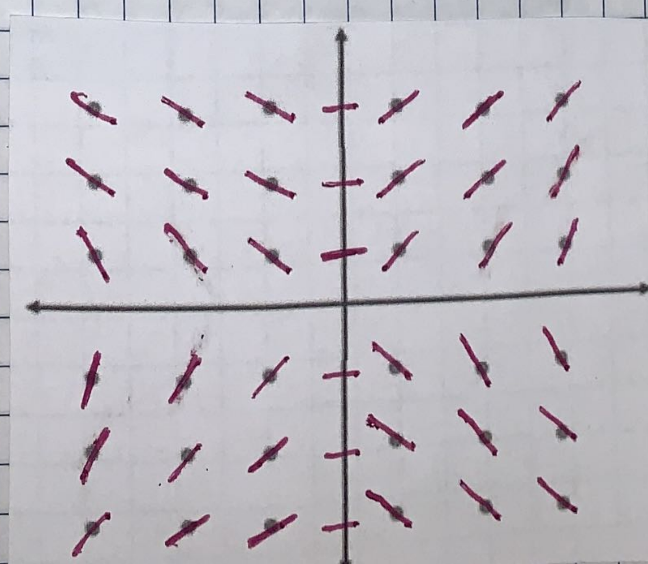
2. $f'(x) = x + 1$

| X | f'(x) |
|---|---|
| -3 | -2 |
| -2 | -1 |
| -1 | 0 |
| 0 | 1 |
| 1 | 2 |
| 2 | 3 |
| 3 | 4 |



3. $f'(x) = \frac{x}{y}$

| X | f'(x) |
|---|---|
| (0,0) | undef |
| (1,1) | 1 |
| (1,-1) | -1 |
| (-1,-1) | 1 |
| (-1,1) | -1 |
| (2,2) | 1 |
| (2,-2) | -1 |
| (-2,-2) | 1 |



# 8.6 Differential Equations



Solving a Differential Equation: Rewrite the equation so that each variable occurs on only one side of the equation.

1. $\frac{dy}{dx} = \frac{2x}{y}$

$\int 2x\,dx = \int y\,dy$

$x^2 + c = \frac{y^2}{2}$

$y^2 = 2x^2 + c$

$y = \pm\sqrt{2x^2 + c}$

2. $(1+x^2)\frac{dy}{dx} - 2xy = 0$

$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{2xy}{1+x^2} \cdot \frac{1}{y}$

$dx \cdot \frac{1}{y}\frac{dy}{dx} = \frac{2x}{1+x^2} \cdot dx$

$\int \frac{1}{y}\,dy = \int \frac{2x}{1+x^2}\,dx$ $\quad u = x^2$, $du = 2x\,dx$, $\frac{du}{2} = x\,dx$

$e^{\ln|y|} = e^{\ln|1+x^2| + c}$

$|y| = e^{\ln|1+x^2| + c}$

$|y| = e^c \cdot e^{\ln|1+x^2|}$

$|y| = c \cdot (1+x^2)$

so $y = \pm c(1+x^2)$

3. $\frac{dy}{dx} = x(1+y)$

$dy = x(1+y)\,dx$

$\int \frac{1}{1+y}\,dy = \int x\,dx$

$e^{\ln|1+y|} = e^{\frac{x^2}{2} + c}$

$e^{\ln|1+y|} = e^{\frac{x^2}{2} + c}$

$|1+y| = ce^{x^2/2}$

$1 + y = \pm ce^{x^2/2}$

$y = -1 \pm ce^{x^2/2}$

## Find the particular solution

4. $\frac{dy}{dx} = x^2 - 2x - 4$ ; $y = -6$ when $x = 3$

$\int dy = \int x^2 - 2x - 4\,dx$

$y = \frac{x^3}{3} - x^2 - 4x + c$

Plug in initial conditions as soon as you integrate + find c

$-6 = \frac{(3)^3}{3} - (3)^2 - 4(3) + c$

$-6 = -12 + c$

$c = 6$

$y = \frac{x^3}{3} - x^2 - 4x + 6$

5. $\frac{dx}{y} = \frac{4dy}{x}$ ; $y = -2$ when $x = 4$

$\int 4y\,dy = \int x\,dx$

$2y^2 = \frac{x^2}{2} + c$

$2(-2)^2 = \frac{(4)^2}{2} + c$

$8 = 8 + c$

$c = 0$

$2y^2 = \frac{x^2}{2} + 0$

$y^2 = \frac{x^2}{4}$

$y = \pm\frac{x}{2}$

make sure it satisfies initial condition

$y = \frac{x}{2}$ or $y = \frac{-x}{2}$

$-2 = \frac{4}{2}$ $\quad -2 = \frac{-4}{2}$

$-2 \neq 2$ $\quad -2 = -2$ ✓

$y = \frac{-x}{2}$

8.7 Exponential Growth and Decay 

Law of Natural Growth/Decay
If $y(t)$ uses the value of a quantity $y$ at time $t$ & if the rate of change of $y$ with respect to $t$ is proportional to its size $y(t)$ at any time, then

$$\frac{dy}{dt} = Ky$$

Theorem: The only solutions to the differential equation $\frac{dy}{dt} = Ky$ are the exponential functions $y(t) = y(0)e^{Kt}$

**EXAMPLE 1**

Use the fact that the world population was 2560 million in 1950 and 3040 million in 1960 to model the population of the world in the second half of the 20th century. (Assume that the growth rate is proportional to the population size.) What is the relative growth rate? Use the model to estimate the world population in 1993 and to predict the population in the year 2020.

Rel. Growth Rate

$3040 = 2560e^{10K}$

$\frac{19}{16} = e^{10K}$

$0.1719 = 10K$

$K = 0.0172$

Growth rate of 1.72% per year

Pop. in 1993 + 2020

$t = 43$

$P = 2560e^{0.0172(43)}$

$P = 5363$

$t = 70$

$P = 2560e^{0.0172(70)}$

$P = 8534$

**EXAMPLE 2**

The half-life of radium-226 is 1590 years.

a. A sample of radium-226 has a mass of 100 mg. Find a formula for the mass of the sample that remains after $t$ years.

$y(0) = 100$ $y(t) = 50$

$t = 1590$

see below

b. Find the mass after 1000 years correct to the nearest milligram.

$y = 100e^{-\frac{\ln(2)}{1590}\cdot 1000}$

$y = 65\text{mg}$

c. When will the mass be reduced to 30 mg

$t = 2762\text{yrs}$

a. $\frac{1}{2} = e^{1590K}$

$\ln\frac{1}{2} = 1590K$

$-\ln 2 = 1590K$

$K = -\frac{\ln 2}{1590}$

$y = 100e^{-\ln 2/1590 \cdot t}$

c. $30 = 100e^{-\frac{\ln 2}{1590}t}$

solve in calculator or with logs

**EXAMPLE 3**

At any time $t$, the rate of increase in the area of a bacteria is twice the area of the bacteria. If the initial area of the bacteria is 10, then what is the area at time $t$?

initial

$$\frac{dA}{dt} = 2A$$

$$\int \frac{1}{A}\,dA = \int 2\,dt$$

$$\ln A = 2t + C$$

$$A = e^{2t+C}$$

$$A = Ce^{2t}$$

$$A = 10e^{2t}$$

**EXAMPLE 4**

The number of bacteria in a culture is growing at a rate of $3000e^{\frac{2t}{5}}$ per unit of time $t$. At $t = 0$, the number of bacteria present was 7,500. find the number present at $t = 5$.

K

$$\frac{dB}{dt} = 3000e^{2t/5}$$

$$\int 3000e^{2t/5}\,dt = \int dB$$

$$\frac{5}{2}\cdot 3000e^{2t/5} = B$$

$$B = 7500e^{2t/5}$$

$$B = 7500e^{\frac{2}{5}(5)}$$

$$B = 7500e^{2}$$