Meaning of Integration

E.Q. What is an integral? How can we approximate area under a curve?

Day	Unit Dates	Topics	Assignments
1	Friday, October 23rd	**Keeper 6.1 - Antiderivatives**	Antiderivatives (packet p. 1 - 2)
2	Monday, October 26th	**Keeper 6.2 - Riemann Sums**	**Skills Check 6.1 A (Forms)** Riemann Sums (packet p. 3 - 5)
3	Tuesday, October 27th	**Keeper 6.3 - Riemann Sums to an Integral**	**Skills Check 6.1 B (Forms)** Riemann Sums to an Integral (packet p. 6 - 7)
4	Wednesday, October 28th	**Optional Q & A Session at 10am Review Keeper 6.1 – 6.2 Antiderivatives and Riemann Sums**	**Take Home Skills Check 6.2 (AP Classroom)** Get Caught up on all Keeper Notes and Homework
5	Thursday, October 29th	**Keeper 6.4 - Fundamental Theorem of Calculus Part 2**	**Skills Check 6.1 C (Forms)** The FTC Part 2 (packet p. 8 – 9)
6	Friday, October 30th	**Keeper 6.4 - Fundamental Theorem of Calculus Part 1**	**Skills Check 6.4 – FTC Part 2 (AP Classroom)** The Fundamental Theorem of Calculus Part 1 (packet p. 10 – 13)
7	Monday, November 2nd	**Keeper 6.5 – Total or Net Change**	**Skills Check 6.4 – FTC Part 1 (AP Classroom)** Definite Integrals and Rates of Change (packet p. 14)
8	Wednesday, November 4th	**Optional Q & A Session at 10am Unit 6 Additional Review**	Complete Homework Packet and Catch up on all Keeper Notes Study for Unit 6 Test Homework Packet Due in CTLS
9	Thursday, November 5th	**Test - The Meaning of Integration**	Good Luck ☺

Antiderivatives

Find the general antiderivative of each function.

1. $f(x) = 6x^2 - 8x + 3$

$\int 6x^2 - 8x + 3 \, dx$

$\frac{6x^3}{3} - \frac{8x^2}{2} + 3x + C$

$2x^3 - 4x^2 + 3x + C$

2. $f(x) = 1 - x^3 + 5x^5 - 3x^7$

$\int 1 - x^3 + 5x^5 - 3x^7 \, dx$

$x - \frac{x^4}{4} + \frac{5x^6}{6} - \frac{3x^8}{8} + C$

3. $f(x) = \sqrt{x} + \sqrt[3]{x}$

$\int x^{1/2} + x^{1/3} \, dx$

$\frac{2x^{3/2}}{3} + \frac{3x^{4/3}}{4} + C$

4. $f(x) = \frac{3}{x^2} + \frac{5}{x}$

$\int 3x^{-2} + 5x^{-1} \, dx$

$\frac{3x^{-1}}{-1} + 5\ln|x| + C$

$-\frac{3}{x} + 5\ln|x| + C$

5. $f(x) = \frac{x^3 + 2x^2}{\sqrt{x}} = \frac{x^3 + 2x^2}{x^{1/2}}$

$\int x^{5/2} + 2x^{3/2} + C$

$\frac{2x^{7/2}}{7} + 2 \cdot \frac{2x^{5/2}}{5} + C$

$\frac{2}{7}x^{7/2} + \frac{4}{5}x^{5/2} + C$

6. $f(x) = \sqrt[3]{x^2} - \sqrt{x}$

$\int x^{2/3} - x^{1/2} \, dx$

$\frac{3x^{5/3}}{5} - \frac{2x^{3/2}}{3} + C$

7. $f(x) = 3\cos x - 4\sin x$

$\int 3\cos x - 4\sin x$

$3\sin x + 4\cos x + C$

8. $f(x) = 4\sqrt{x} + e^x - \sec x \tan x$

$\int 4x^{1/2} + e^x - \sec x \tan x \, dx$

$\frac{2 \cdot 4x^{3/2}}{3} + e^x - \sec x + C$

$\frac{8}{3}x^{3/2} + e^x - \sec x + C$

9. $f(x) = \frac{x^2 + x + 1}{x}$

$f(x) = x + 1 + \frac{1}{x}$

$\int x + 1 + \frac{1}{x} \, dx$

$\frac{x^2}{2} + x + \ln|x| + C$

10. $f(x) = 6x^2 - 7\sec^2 x$

$\int 6x^2 - 7\sec^2 x \, dx$

$6\int x^2 dx - 7\int \sec^2 x \, dx$

$\frac{6x^3}{3} - 7\tan x + C$

$2x^3 - 7\tan x + C$

Find $f(x)$

11. $f'(x) = 1 - 6x; f(0) = 8$

$\int 1 - 6x \, dx$

$f(x) = x - 3x^2 + C$

$8 = 0 - 3(0)^2 + C$

$C = 8$

$f(x) = x - 3x^2 + 8$

12. $f'(x) = 3\sqrt{x} - \frac{1}{\sqrt{x}}; f(1) = 2$

$\int 3x^{1/2} - x^{-1/2} \, dx$

$\frac{2}{\cancel{3}} \cdot \cancel{3}x^{3/2} - 2x^{1/2} + C$

$2 = 2(1)^{3/2} - 2(1)^{1/2} + C$

$2 = 2 - 2 + C$

$C = 2$

$f(x) = 2x^{3/2} - 2x^{1/2} + 2$

13. $f'(x) = 3\cos x + 5\sin x; f(0) = 4$

$f(x) = 3\sin x - 5\cos x + C$

$4 = 3\sin 0 - 5\cos 0 + C$

$4 = 3(0) - 5(1) + C$

$C = 9$

$f(x) = 3\sin x - 5\cos x + 9$

14. $f''(x) = x; f(0) = -3, f'(0) = 2$

$\int x \, dx$

$f'(x) = \frac{x^2}{2} + C$

$2 = \frac{0^2}{2} + C$

$C = 2$

$f'(x) = \frac{x^2}{2} + 2$

$\int \frac{x^2}{2} + 2 \, dx$

$f(x) = \frac{x^3}{6} + 2x + C$

$-3 = \frac{0^3}{6} + 2(0) + C$

$C = -3$

$f(x) = \frac{x^3}{6} + 2x - 3$

15. $f''(x) = x^2 + 3\cos x; f(0) = 2, f'(0) = 3$

$f'(x) = \frac{x^3}{3} + 3\sin x + C$

$3 = \frac{0^3}{3} + 3\sin 0 + C$

$C = 3$

$f'(x) = \frac{x^3}{3} + 3\sin x + 3$

$f(x) = \frac{x^4}{12} - 3\cos x + 3x + C$

$2 = \frac{(0)^4}{12} - 3\cos(0) + 3(0) + C$

$2 = 0 - 3 + 0 + C$

$C = 5$

$f(x) = \frac{x^4}{12} - 3\cos x + 3x + 5$

16. $f''(x) = 12x^2 - 6x + 2; f(0) = 1, f'(2) = 11$

$f'(x) = 4x^3 - 3x^2 + 2x + C$

$11 = 4(2)^3 - 3(2)^2 + 2(2) + C$

$11 = 32 - 12 + 4 + C$

$C = -13$

$f'(x) = 4x^3 - 3x^2 + 2x - 13$

$f(x) = x^4 - x^3 + x^2 - 13x + C$

$1 = 0 - 0 + 0 - 0 + C$

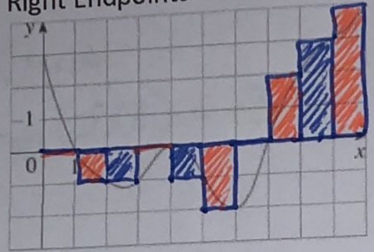
$C = 1$

$f(x) = x^4 - x^3 + x^2 - 13x + 1$

Riemann Sums

1. The graph of a function f is given. Estimate $\int_0^{10} f(x)\,dx$ using ten subintervals with

a. Right Endpoints

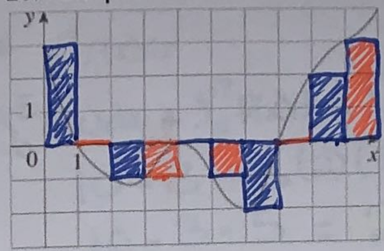


$\int_0^{10} f(x)\,dx \approx$

$1(0-1-1+0-1-2+0+2+3+4)$

≈ 4

b. Left Endpoints

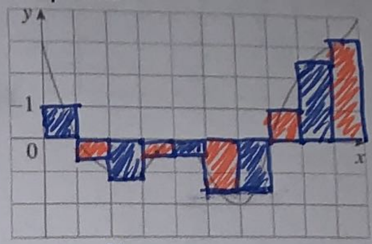


$\int_0^{10} f(x)\,dx \approx$

$1(3+0-1-1+0-1-2+0+2+3)$

≈ 3

c. Midpoints



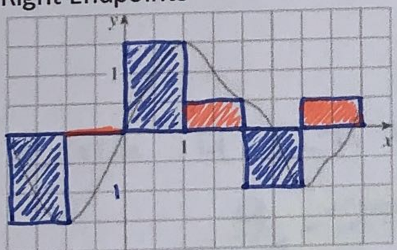
$\int_0^{10} f(x)\,dx \approx$

$1(1-.5-1.2-.5-.5-1.6-1.6+1+2.5+3.1)$

≈ 1.7

2. The graph of g is shown. Estimate $\int_{-2}^{4} g(x)\,dx$ with six subintervals using

a. Right Endpoints

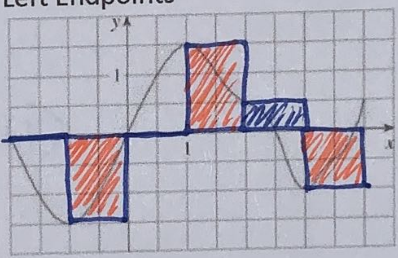


$\int_{-2}^{4} g(x)\,dx \approx$

$1(-1.5+0+1.5+.5-1+.5)$

≈ 0

b. Left Endpoints

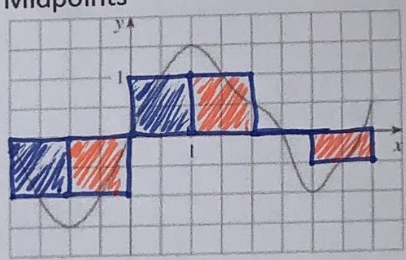


$\int_{-2}^{4} g(x)\,dx$

$1(0-1.5+0+1.5+.5-1)$

$\approx -.5$

c. Midpoints



$\int_{-2}^{4} g(x)\,dx$

$1(-1-1+1+1+0-.5)$

$\approx -.5$

3. The table gives the values of a function obtained from an experiment. Use them to estimate $\int_3^9 f(x)\,dx$ using three equal subintervals with

a. Right Endpoints

x	3	4	5	6	7	8	9
f(x)	-3.4	-2.1	-.6	.3	.9	1.4	1.8
			R		R		R

$\frac{b-a}{n}$

$\frac{9-3}{3}=2$

$\int_3^9 f(x)\,dx \approx$

$2(-.6+.9+1.8)$

≈ 4.2

b. Left Endpoints

x	3	4	5	6	7	8	9
f(x)	-3.4	-2.1	-.6	.3	.9	1.4	1.8
	L		L		L		

$\int_3^9 f(x)\,dx \approx$

$2(-3.4-.6+.9)$

≈ -6.2

c. Midpoints

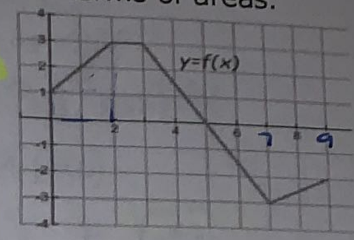
x	3	4	5	6	7	8	9
f(x)	-3.4	-2.1	-.6	.3	.9	1.4	1.8
		m		m		m	

$\int_3^9 f(x)\,dx \approx$

$2(-2.1+.3+1.4)$

$\approx -.8$

4. The graph of f is shown. Evaluate each integral by interpreting it in terms of areas.



a. $\int_0^2 f(x)dx$ Trapezoid

$\frac{1}{2}(2)(1+3)=4$

b. $\int_0^5 f(x)\,dx$

$4+(1)(3)+\frac{1}{2}(3)(2)=10$

c. $\int_5^7 f(x)dx$

$-\frac{1}{2}(2)(3)=-3$

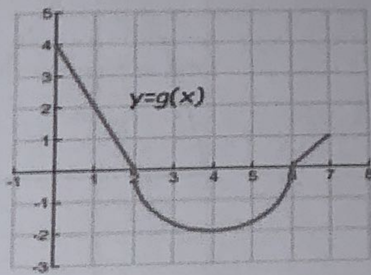
d. $\int_0^9 f(x)dx$

$\int_0^5 f(x)dx+\int_5^7 f(x)dx+\int_7^9 f(x)dx$

$10-3-\frac{1}{2}(2)(3+2)$

$10-3-5=2$

5. The graph of g consists of two straight lines and a semi-circle. Use it to evaluate each integral.



a. $\int_0^2 g(x)dx = \frac{1}{2}(2)(4)=4$

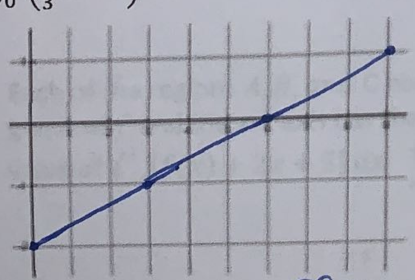
b. $\int_2^6 g(x)\,dx = -\frac{1}{2}\pi(2)^2=-2\pi$

c. $\int_0^7 g(x)\,dx$ $4-2\pi+\frac{1}{2}(1)(1)$

$\frac{9}{2}-2\pi$

Evaluate the integral by interpreting it in terms of areas.

6. $\int_0^9 \left(\frac{1}{3}x-2\right)\,dx$



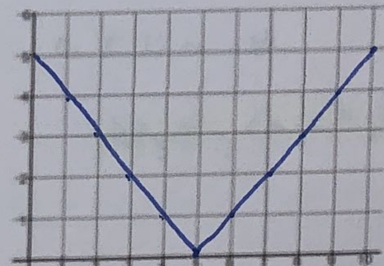
$\int_0^6 (\frac{1}{3}x-2)dx+\int_6^9(\frac{1}{3}x-2)dx$

$-\frac{1}{2}(2)(6)+\frac{1}{2}(3)(1)$

$-6+\frac{3}{2}$

$\frac{-9}{2}$

7. $\int_0^{10} |x-5|\,dx$



$\int_0^5 |x-5|dx+\int_5^{10}|x-5|dx$

$\frac{1}{2}(5)(5)+\frac{1}{2}(5)(5)$

$\frac{25}{2}+\frac{25}{2}=\frac{50}{2}$

25

8. If $\int_1^5 f(x)dx = 12$ and $\int_4^5 f(x)dx = 3.6$, find $\int_1^4 f(x)dx$

$\int_1^5 f(x)dx - \int_4^5 f(x)dx$

$12 - 3.6$

$\int_1^4 f(x) = 8.4$

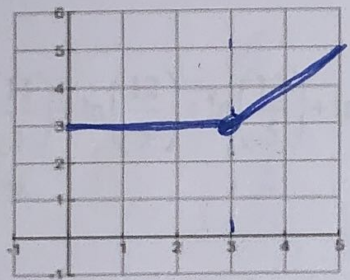
9. If $\int_0^9 f(x)dx = 37$ and $\int_0^9 g(x)dx = 16$, find $\int_0^9 [2f(x) + 3g(x)]dx$

$2\int_0^9 f(x)dx + 3\int_0^9 g(x)dx$

$2(37) + 3(16)$

$74 + 48 = 122$

10. Find $\int_0^5 f(x)$ if $f(x) = \begin{cases} 3 \ for\ x < 3 \\ x \ for\ x \geq 3 \end{cases}$



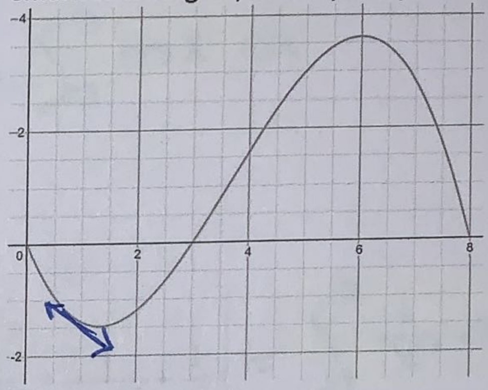
$\int_0^3 f(x)dx + \int_3^5 f(x)dx$

$3(3) + \frac{1}{2}(2)(3+5)$

$9 + 8$

17

11. For the function f whose graph is shown, list the following quantities in increasing order from smallest to largest, and explain your reasoning.



a. $\int_0^8 f(x)dx$ = +#

b. $\int_0^3 f(x)dx$ = -#

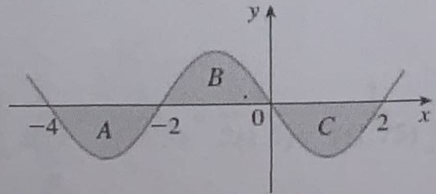
c. $\int_3^8 f(x)dx$ = +# biggest

d. $\int_4^8 f(x)dx$ = +#, b/n c+a

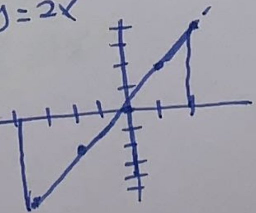
e. $f'(1)$ = -#

b, e, a, d, c

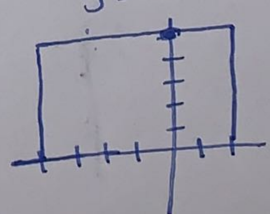
12. Each of the regions $A, B,$ and C bounded by the graph of f and the x –axis has the area 3. Find the value of $\int_{-4}^{2} [f(x) + 2x + 5]\, dx$ $\int f(x) = -3$



y=2x



y = 5



$\int_{-4}^{2} f(x)dx + \int_{-4}^{2} 2x\, dx + \int_{-4}^{2} 5dx$

$-3 - \frac{1}{2}(4)(8) + \frac{1}{2}(2)(4) + 6(5)$

$-3 - 16 + 4 + 30$

15

Riemann Sums to an Integral

Each expression below is a right Riemann Sum approximation for an integral.

In each problem, state what integral the sum is approximating.

1. $\frac{1}{4}\left[\sin\left(\frac{9}{4}\right)+\sin\left(\frac{10}{4}\right)+\sin\left(\frac{11}{4}\right)+\sin(\overset{b}{3})\right] = \int_2^3 \sin x \, dx$

$a = \frac{8}{4} = 2$

2. $\frac{1}{5}\left[\ln\left(\frac{11}{5}\right)+\ln\left(\frac{12}{5}\right)+\ln\left(\frac{13}{5}\right)+\ln\left(\frac{14}{5}\right)+\ln(3)\right] = \int_2^3 \ln x \, dx$

$\frac{10}{5} = 2$

3. $\frac{1}{8}\left[\frac{1}{\left(1\frac{1}{8}\right)^2}+\frac{1}{\left(1\frac{1}{4}\right)^2}+\frac{1}{\left(1\frac{3}{8}\right)^2}+\frac{1}{\left(1\frac{1}{2}\right)^2}+\frac{1}{\left(1\frac{5}{8}\right)^2}+\frac{1}{\left(1\frac{3}{4}\right)^2}+\frac{1}{\left(1\frac{7}{8}\right)^2}+\frac{1}{2^2}\right] = \int_1^2 \frac{1}{x^2} \, dx$

4. $\frac{1}{3}\left[\sqrt[3]{\frac{4}{3}}+\sqrt[3]{\frac{5}{3}}+\sqrt[3]{2}\right] = \int_1^2 \sqrt{x} \, dx$

5. $\frac{1}{2}\left[3(3.5)^2+3(4)^2+3(4.5)^2+3(5)^2\right] = \int_3^5 3x^2 \, dx$

The following is a left Riemann Sum approximation for some integral. What integral is it approximating?

6. $\frac{1}{4}\left[\frac{1}{2(\underset{a}{1})}+\frac{1}{2(1.25)}+\frac{1}{2(1.5)}+\frac{1}{2(1.75)}\right] = \int_1^2 \frac{1}{2x} \, dx$

1. $\lim_{N\to\infty} \sum_{i=1}^{N} \frac{i}{N^2}$ $a=0$ $\Delta x=\frac{1}{n}$ $f(x)=x$

$\frac{b-a}{n}=\frac{1}{n}$

$b-0=1$

$b=1$

$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i}{n}\cdot\frac{1}{n}$

$\int_0^1 x\,dx$

2. $\lim_{N\to\infty} \sum_{j=1}^{N} \frac{j^3}{N^4}$ $a=0$ $\Delta x=\frac{1}{n}$ $f(x)=x^3$

$b=1$

$\lim_{n\to\infty} \sum_{j=1}^{n} \frac{1}{n}\left(\frac{j}{n}\right)^3$

$\int_0^1 x^3\,dx$

3. $\lim_{N\to\infty} \sum_{i=1}^{N} \frac{i^2-i+1}{N^3}$ $a=0$ $\Delta x=\frac{1}{n}$ $f(x)=x^2$

$b=1$

$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{(i-1)^2}{n^2}\cdot\frac{1}{n}$

$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i-1}{n}\right)^2\cdot\frac{1}{n}$

$\int_0^1 x^2\,dx$

4. $\lim_{N\to\infty} \sum_{i=1}^{N} \left(\frac{i^3}{N^4}-\frac{20}{N}\right)$ $a=0$ $\Delta x=\frac{1}{n}$ $f(x)=x^3-20$

$b=1$

$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left(\frac{i^3}{n^3}-20\right)$

$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left[\left(\frac{i}{n}\right)^3-20\right]$

$\int_0^1 x^3-20\,dx$

5. $\lim_{N\to\infty} \frac{2}{N}\sum_{j=1}^{N} \sin\left(\frac{2j}{N}\right)$ $a=0$ $\Delta x=\frac{2}{n}$ $f(x)=\sin x$

$\frac{b-a}{n}=\frac{2}{n}$

$b-0=2$

$b=2$

$\int_0^2 \sin x\,dx$

6. $\lim_{N\to\infty} \frac{4}{N}\sum_{k=1}^{N} \left(3+\frac{4k}{N}\right)$ $a=3$ $\Delta x=\frac{4}{n}$ $f(x)=x$

$\frac{b-a}{n}=\frac{4}{n}$

$b-3=4$

$b=7$

$\int_3^7 x\,dx$

7. $\lim_{N\to\infty} \frac{\pi}{N}\sum_{j=0}^{N-1} \sin\left(\frac{\pi}{2}+\frac{\pi j}{N}\right)$ $a=\frac{\pi}{2}$ $\Delta x=\frac{\pi}{n}$ $f(x)=\sin x$

$\frac{b-a}{n}=\frac{\pi}{n}$

$b-\frac{\pi}{2}=\pi$

$b=\frac{3\pi}{2}$

$\int_{\pi/2}^{3\pi/2} \sin x\,dx$

8. $\lim_{N\to\infty} \frac{4}{N}\sum_{k=1}^{N} \frac{1}{\left(3+\frac{4k}{N}\right)^2}$ $a=3$ $\Delta x=\frac{4}{n}$ $f(x)=\frac{1}{x^2}$

$\frac{b-a}{n}=\frac{4}{n}$

$b-3=4$

$b=7$

$\int_3^7 \frac{1}{x^2}\,dx$

9. $\lim_{N\to\infty} \frac{1^k+2^k+\cdots+N^k}{N^{k+1}}$ $(k>0)$ $a=0$ $\Delta x=\frac{1}{n}$ $f(x)=x^k$

$b=1$

$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^k}{n^k\cdot n}$

$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^k\cdot\frac{1}{n}$

$\int_0^1 x^k\,dx$

Fundamental Theorem of Calculus – Part 2

1. $\int_{-1}^{2}(x^3 - 2x)dx$

$\frac{x^4}{4} - x^2 \Big|_{-1}^{2}$

$\left(\frac{2^4}{4} - 2^2\right) - \left(\frac{(-1)^4}{4} - (-1)^2\right)$

$(4-4) - \left(\frac{1}{4} - 1\right) = \frac{3}{4}$

2. $\int_{1}^{4}(5 - 2t + 3t^2)dt$

$5t - t^2 + t^3 \Big|_{1}^{4}$

$(5 \cdot 4 - (4)^2 + (4)^3) - (5 \cdot 1 - (1)^2 + (1)^3)$

$20 - 16 + 64 - 5 + 1 - 1$

63

3. $\int_{1}^{9}\sqrt{x}dx$

$\frac{2x^{3/2}}{3}\Big|_{1}^{9}$

$\frac{2}{3}(9)^{3/2} - \frac{2}{3}(1)^{3/2}$

$18 - \frac{2}{3} = \frac{52}{3}$

4. $\int_{\frac{\pi}{6}}^{\pi} \sin\theta \, d\theta$

$-\cos\theta \Big|_{\pi/6}^{\pi}$

$-\cos\pi + \cos\pi/6$

$1 + \frac{\sqrt{3}}{2} = \frac{2+\sqrt{3}}{2}$

5. $\int_{0}^{1}(u + 2)(u - 3)du$

$\int_0^1 u^2 - u - 6 \, du$

$\frac{u^3}{3} - \frac{u^2}{2} - 6u \Big|_0^1$

$\left(\frac{1}{3} - \frac{1}{2} - 6\right) - (0 - 0 - 0)$

$-\frac{37}{6}$

6. $\int_{0}^{4}(4 - t)\sqrt{t}dt$

$\int_0^4 4t^{1/2} - t^{3/2} dt$

$\frac{2}{3} \cdot 4t^{3/2} - \frac{2t^{5/2}}{5}\Big|_0^4$

$\left(\frac{8}{3}\sqrt{4}^3 - \frac{2}{5}\sqrt{4}^5\right) - (0 - 0)$

$\frac{64}{3} - \frac{64}{5} = \frac{128}{15}$

7. $\int_{1}^{9}\frac{x-1}{\sqrt{x}} dx$ $\quad \frac{x}{x^{1/2}} - \frac{1}{x^{1/2}}$

$\int_1^9 x^{1/2} - x^{-1/2} dx$

$\frac{2x^{3/2}}{3} - 2x^{1/2}\Big|_1^9$

$\left(\frac{2}{3}\sqrt{9}^3 - 2\sqrt{9}\right) - \left(\frac{2}{3}\sqrt{1}^3 - 2\sqrt{1}\right)$

$18 - 6 - \frac{2}{3} + 2 = \frac{40}{3}$

8. $\int_{0}^{\frac{\pi}{4}} \sec^2 t \, dt$

$\tan t \Big|_0^{\pi/4}$

$\tan\frac{\pi}{4} - \tan 0$

$1 - 0 = 1$

9. $\int_0^{\frac{\pi}{4}} \sec\theta \tan\theta \, d\theta$

$\sec\theta \Big|_0^{\pi/4}$

$\sec\frac{\pi}{4} - \sec 0$

$\sqrt{2} - 1$

10. $\int_1^2 (1+2y)^2 dy$

$\int_1^2 1 + 4y + 4y^2 \, dy$

$y + 2y^2 + \frac{4}{3}y^3 \Big|_1^2$

$(2 + 2(2)^2 + \frac{4}{3}(8)) - (1 + 2 + \frac{4}{3})$

$2 + 8 + \frac{32}{3} - 1 - 2 - \frac{4}{3}$

$49/3$

11. $\int_0^3 (2\sin x - e^x) dx$

$-2\cos x - e^x \Big|_0^3$

$(-2\cos 3 - e^3) - (-2\cos 0 - e^0)$

$-2\cos 3 - e^3 + 2 + 1$

$3 - 2\cos 3 - e^3$

12. $\int_1^2 \frac{v^3 + 3v^6}{v^4} dv$

$\int_1^2 v^{-1} + 3v^2 \, dv$

$\ln|v| + v^3 \Big|_1^2$

$\ln 2 + 8 - \ln 1 - 1$

$\ln 2 + 7$

13. $\int_0^1 (x^e + e^x) dx$

$\frac{x^{e+1}}{e+1} + e^x \Big|_0^1$

$\left(\frac{1^{e+1}}{e+1} + e\right) - \left(\frac{0^e}{e+1} + 1\right)$

$\frac{1}{e+1} + e - 1$

14. $\int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{8}{1+x^2} dx$

$8\tan^{-1}x \Big|_{1/\sqrt{3}}^{\sqrt{3}}$

$8\tan^{-1}\sqrt{3} - 8\tan^{-1}(1/\sqrt{3})$

$8\left(\frac{\pi}{3}\right) - 8\left(\frac{\pi}{6}\right)$

$\frac{8\pi}{3} - \frac{4\pi}{3} = \frac{4\pi}{3}$

15. $\int_{-1}^1 e^{u+1} du$

$e^{u+1} \Big|_{-1}^1$

$e^2 - e^0$

$e^2 - 1$

16. $\int_0^\pi f(x)dx$ were $f(x) = \begin{cases} \sin x, & 0 \le x < \frac{\pi}{2} \\ \cos x, & \frac{\pi}{2} \le x < \pi \end{cases}$

$\int_0^{\pi/2} \sin x \, dx + \int_{\pi/2}^{\pi} \cos x \, dx$

$-\cos x \Big|_0^{\pi/2} + \sin x \Big|_{\pi/2}^{\pi}$

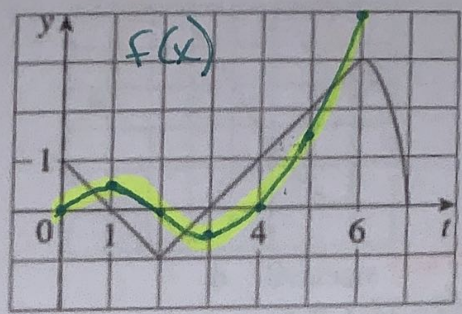
$-\cos\pi/2 + \cos 0 + \sin\pi - \sin\pi/2$

$0 + 1 + 0 - 1$

0

The Fundamental Theorem of Calculus Part 1

1. Let $g(x) = \int_0^x f(t)\,dt$, where f is the function whose graph is shown.



a. Evaluate

$x = 0$ $\int_0^0 f(t)\,dt = 0$ $x = 1$ $\int_0^1 f(t)\,dt = \frac{1}{2}$

$x = 2$ $\int_0^2 f(t)\,dt = 0$ $x = 3$ $\int_0^3 f(t)\,dt = -\frac{1}{2}$

$x = 4$ $\int_0^4 f(t)\,dt = 0$ $x = 5$ $\int_0^5 f(t)\,dt = \frac{3}{2}$

$x = 6$ $\int_0^6 f(t)\,dt = 4$

$\frac{1}{2}(2+3) = \frac{5}{2} + \frac{3}{2}$

b. Estimate $g(7)$

$\int_0^7 g(t) = 4 + 2.2 = 6.2$

c. Where does g have a maximum value? Where does it have a minimum value?

$g'(x) = f(x) = 0$

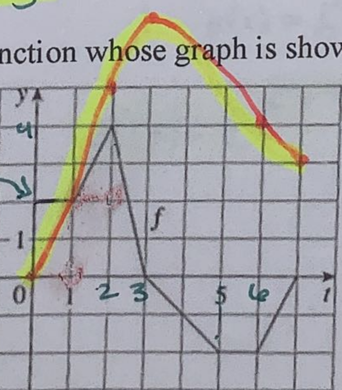
max at $x = 1$

min at $x = 3$

d. Sketch a graph of g.

used points in part a but this is just one possible graph

2. Let $g(x) = \int_0^x f(t)\,dt$, where f is the function whose graph is shown.



a. Evaluate

$g(0) = 0$ $g(1) = (1)(2) = 2$

$g(2) = 2 + \frac{1}{2}(1)(2+4) = 5$ $g(3)$ $5 + \frac{1}{2}(1)(4) = 7$

$g(6) = 7 - \frac{1}{2}(2)(2) - \frac{1}{2}(1)(2)$

$7 - 2 - 1 = 4$

b. On what intervals is g increasing?

$g'(x) = f(x)$

$g(x)$ is Increasing $(0,3)$ since $g'(x)$ is positive

c. Where does g have a maximum value?

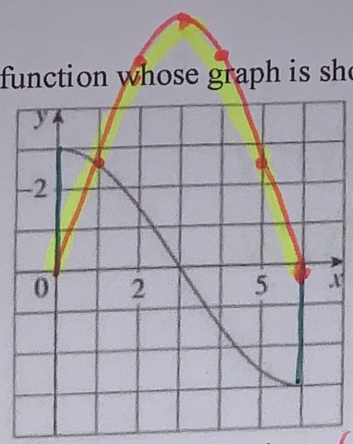
$g'(x)$: $+$ on $(0,3)$, $-$ on $(3,7)$

max at $x = 3$

d. Sketch a graph of g.

* used points in part a but it could be shifted up or down bc of $+C$

3. Let $g(x) = \int_0^x f(t)\,dt$, where f is the function whose graph is shown.



a. Evaluate
$g(0)$ $\int_0^0 f(t)dt = 0$

$g(6)$ $\int_0^6 f(t)dt = 0$

b. Evaluate (approximate)
$x = 1$ $1(2.8) \approx 2.8$ $x = 2$ $2.8 + 1.8 \approx 4.6$

$x = 3$ $4.6 + .8 \approx 5.4$ $x = 4$ $5.4 - .8 \approx 4.6$

$x = 5$ $4.6 - 2.1 \approx 2.5$

c. On what interval is g increasing?
$(0, 3)$

d. Where does g have a maximum value?
$x = 3$

$+$ $-$
3

e. Sketch a rough graph of g.
used pts from part b but it could be shifted up or down bc of $+C$

Use the 1st Fundamental Theorem of Calculus to find the derivative of the functions.

4. $g(x) = \int_1^x \frac{1}{t^3+1}\,dt$

$g'(x) = \frac{1}{x^3+1}$

5. $g(x) = \int_3^x e^{t^2-t}\,dt$

$g'(x) = e^{x^2-x}$

6. $g(s) = \int_5^s (t - t^2)^8\,dt$

$g'(s) = (s - s^2)^8$

7. $g(r) = \int_0^r \sqrt{x^2+4}\,dx$

$g'(r) = \sqrt{r^2+4}$

8. $G(x) = \int_x^1 \cos\sqrt{t}\,dt$

$G'(x) = -\cos\sqrt{x}$

9. $h(x) = \int_1^{e^x} \ln t\ dt$

$h'(x) = \ln e^x \cdot e^x$

$h'(x) = x \cdot e^x$

10. $h(x) = \int_1^{\sqrt{x}} \frac{z^2}{z^4+1} dz$

$h'(x) = \frac{(\sqrt{x})^2}{(\sqrt{x})^4+1} \cdot \frac{1}{2\sqrt{x}}$

$h'(x) = \frac{x}{(x^2+1) \cdot 2\sqrt{x}}$

$h'(x) = \frac{\sqrt{x}}{2(x^2+1)}$

11. $y = \int_0^{\tan x} \sqrt{t+\sqrt{t}}\ dt$

$y' = \sqrt{\tan x + \sqrt{\tan x}} \cdot \sec^2 x$

12. $y = \int_0^{x^4} \cos^2\theta\ d\theta$

$y' = \cos^2(x^4) \cdot 4x^3$

$y' = 4x^3 \cos^2 x^4$

13. $y = \int_{1-3x}^{1} \frac{u^3}{1+u^2} du$

$y' = -\frac{(1-3x)^3}{1+(1-3x)^2} \cdot -3$

$y' = \frac{3(1-3x)^3}{1+(1-3x)^2}$

14. $y = \int_{\sin x}^{1} \sqrt{(1+t^2)}dt$

$y' = -\sqrt{(1+\sin^2 x)} \cdot \cos x$

$y' = -\cos x \sqrt{1+\sin^2 x}$

15. $F(x) = \int_x^{\pi} \sqrt{1+\sec t}\,dt$

$F'(x) = -\sqrt{1+\sec x}$

16. If $f(x) = \int_0^x (1-t^2)\, e^{t^2} dt$, on what interval is f increasing?

$f'(x) = (1-x^2)e^{x^2} = 0$

$0 = 1 - x^2 \qquad e^{x^2} = 0$ DNE

$x = \pm 1$

$- \quad + \quad -$ (-1, 1)

Increasing: $(-1, 1)$

17. On what interval is the curve $y = \int_0^x \frac{t^2}{t^2+t+2}\, dt$ concave down?

$y' = \frac{x^2}{x^2+x+2}$

$y'' = \frac{2x(x^2+x+2) - x^2(2x+1)}{(x^2+x+2)^2} = 0$

$0 = 2x^3 + 2x^2 + 4x - 2x^3 - x^2$

$0 = x^2 + 4x$

$x = -4, 0$

$+ \quad - \quad +$ (-4, 0)

Conc ↓ $(-4, 0)$

18. If $f(x) = \int_0^{\sin x} \sqrt{1+t^2}\, dt$ and $g(y) = \int_3^y f(x)dx$, find $g''\left(\frac{\pi}{6}\right)$

$g(y) = \int_3^y f(x)dx$

$g'(y) = f(y)$

$g''(y) = f'(y)$

1st deriv $f'(x) = \cos x \sqrt{1+\sin^2 x}$

2nd deriv $g(y) = \cos y \sqrt{1+\sin^2 y}$

$g''\left(\frac{\pi}{6}\right) = \cos\frac{\pi}{6}\sqrt{1+\sin^2 \pi/6}$

$= \frac{\sqrt{3}}{2}\sqrt{1+\left(\frac{1}{2}\right)^2}$

$= \frac{\sqrt{3}}{2}\sqrt{\frac{5}{4}}$

$= \frac{\sqrt{15}}{4}$

19. If $f(1) = 12$, f' is continuous, and $\int_1^4 f'(x)dx = 17$, what is the value of $f(4)$?

$\int_1^4 f'(x)\,dx = f(x)\Big|_1^4$

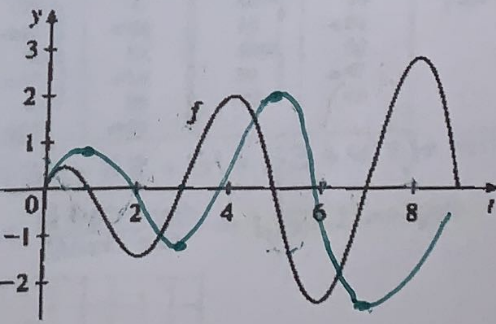
$f(4) - f(1) = 17$

$f(4) - 12 = 17$

$f(4) = 29$

20. Let $g(x) = \int_0^x f(t)dt$, where f is the function whose graph is shown.

$g'(x) = f(x)$



a. At what values of x does the local maximum and minimum of g occur? $f(x) = 0$

$+ \quad - \quad + \quad - \quad +$ (0, 1, 3, 5, 7, 9)

local max at $x = 1, 5$

local min at $x = 3, 7$

b. Where does g attain its absolute maximum value?

$x = 9$ since that is where you have the largest accumulated area

c. On what intervals is g concave downward? $f(x)$ is decr.

$(1/2, 2) \cup (4, 6) \cup (8, 9)$

d. Sketch the graph of g.

$f(x) = g'(x)$ $+ \; 0 \; - \; 0 \; + \; 0 \; - \; 0 \; + \; 0$ (0, 1, 3, 5, 7, 9)

$g(x)$ max min max min

Definite Integrals and Rate of Change

1. If $w'(t)$ is the rate of growth of a child in pounds per year, what does $\int_5^{10} w'(t)\, dt$ represent?

The # of lbs the child grew between 5 yrs old + 10 yrs old.

2. If oil leaks from a tank at a rate of $r(t)$ gallons per minute at time t, what does $\int_0^{120} r(t)dt$ represent?

The gallons of oil leaked from the tank in the 1st 2 hours

3. A honeybee population starts with 100 bees and increases at a rate of $n'(t)$ bees per week. What does $100 + \int_0^{15} n'(t)dt$ represent?

The bee population after 15 weeks

4. The linear density of a rod of length 4m is given by $p(x) = 9 + 2\sqrt{x}$ measured in kilograms per meter, where x is measured in meters from one end of the rod. Find the total mass of the rod.

$\int_0^4 9 + 2\sqrt{x}\, dx$

$9x + \frac{4}{3}x^{3/2}\Big|_0^4$

$9(4) + \frac{4}{3}\sqrt{4}^3 - 0 - 0$

$36 + \frac{32}{3} = \frac{140}{3}$ kg

5. Water flows from the bottom of a storage tank at a rate of $r(t) = 200 - 4t$ liters per minute, where $0 \le t \le 50$. Find the amount of water that flows from the tank during the first 10 minutes.

$\int_0^{10} 200 - 4t\, dt$

$200t - 2t^2\Big|_0^{10}$

$200(10) - 2(10)^2 - 0$

$2000 - 200 = 1800$ liters

6. The velocity of a car was read from its speedometer at 10-second intervals and recorded in the table. Use the Midpoint Rule to estimate the distance traveled by the car.

$\Delta x = \frac{100}{5} = 20$

$\Delta x = 20$

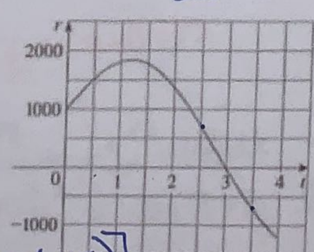
midpt

t(s)	v(mi/h)	t(s)	v(mi/h)
0	0	60	56
10	38	70	53
20	52	80	50
30	58	90	47
40	55	100	45
50	51		

$20(38 + 58 + 51 + 53 + 47) = 4940 \frac{mi}{hr}$

$d = \frac{4940 \text{ mph}}{3600 \text{ sec}} \approx 1.372$ miles

7. Water flows into and out of a storage tank. A graph of the rate of change $r(t)$ of the volume of water in the tank, in liters per day, is shown. If the amount of water in the tank at time $t = 0$ is 25,000 L, use the Midpoint Rule to estimate the amount in the tank 4 days later.



$25000 + (1)[r(.5) + r(1.5) + r(2.5) + r(3.5)]$

$25000 + 1500 + 1750 + 750 - 750$

28,250 L

Practice Test - Meaning of Integration

Calculator not allowed on this section.

1. What is the meaning of $\int_3^7 m(t)dt$, if $m(t)$ is the rate of traffic flow (number of cars per hour passing an observation point along a highway), and t is measured in hours from 8:00 am on December 20, 2005?

Total # of cars passing an observation point along a highway between 11am & 3pm on Dec. 20, 2005

3. A particle moves along a straight line with acceleration $a(t) = 5 + 4t - 6t^2$. The velocity at $t = 1$ second is 3 m/sec. Its position at time $t = 0$ is 10 meters. Find both the velocity function and the position function.

$v(t) = \int a(t)\,dt$

$v(t) = \int 5 + 4t - 6t^2\,dt$

$v(t) = 5t + 2t^2 - 2t^3 + C$

$3 = 5(1) + 2(1)^2 - 2(1)^3 + C$

$C = -2$

$v(t) = 5t + 2t^2 - 2t^3 - 2$

$s(t) = \int 5t + 2t^2 - 2t^3 - 2\,dt$

$s(t) = \frac{5t^2}{2} + \frac{2t^3}{3} - \frac{t^4}{2} - 2t + C$

$10 = 0 + 0 - 0 - 0 + C$

$C = 10$

$s(t) = \frac{5t^2}{2} + \frac{2t^3}{3} - \frac{t^4}{2} - 2t + 10$

4. If $\int_3^7 f(x)dx = 11$ and $\int_3^7 g(x) = -5$ and $\int_5^7 f(x)dx = 4$, find the following. (2 points each)

a) $\int_3^7 [2f(x) + 3g(x)] =$

$2\int_3^7 f(x) + 3\int_3^7 g(x)\,dx$

$2(11) + 3(-5) = 7$

b) $\int_7^3 4g(x)dx =$

$-4\int_3^7 g(x)\,dx$

$-4(-5) = 20$

c) $\int_5^7 (f(x) + 2)dx =$

$\int_5^7 f(x)dx + 2x\Big|_5^7$

$4 + 14 - 10 = 8$

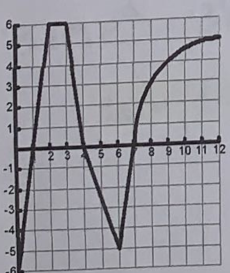
d) $\int_3^5 f(x)dx$

$\int_3^7 f(x)dx - \int_5^7 f(x)dx$

$11 - 4 = 7$

5. The graph of the velocity (in m/min) of a bicycle as a function of time t (in min) is graphed below. The graph consists of line segments and a quarter of a circle. Use the graph of velocity to determine:

v(t)



$\int v(t) = s(t)$

could've done

$\int_1^4 v(t)\,dt = \frac{1}{2} \cdot 6(3+1) = 12$

a. the displacement of the bicycle during the first 12 minutes.

$\int_0^1 v(t)\,dt + \int_1^3 v(t)\,dt + \int_3^4 v(t)\,dt + \int_4^7 v(t)\,dt + \int_7^{12} v(t)\,dt$

$-\frac{1}{2}(1)(6) + \frac{1}{2} \cdot 6(2+1) + \frac{1}{2}(1)(6) - \frac{1}{2}(3)(5) + \frac{1}{4}\pi(5)^2 = -3 + 9 + 3 - \frac{15}{2} + \frac{25\pi}{4} = \frac{3}{2} + \frac{25\pi}{4}$ m

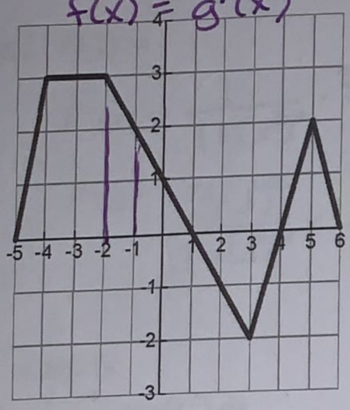
b. the total distance traveled by the bicycle during the first 12 minutes.

$|-3| + 12 + \left|-\frac{15}{2}\right| + \frac{25\pi}{4}$

$3 + 12 + \frac{15}{2} + \frac{25\pi}{4} = \frac{45}{2} + \frac{25\pi}{4}$ m

6. Let $g(x) = \int_{-1}^{x} f(t)dt$, where f is the function whose graph is shown below:

$f(x) = g'(x)$



$g(x) = \int_{-1}^{x} f(t)\,dt$

$g(0) = \int_{-1}^{0} f(t)\,dt = \frac{1}{2}\cdot 1(2+1) = \frac{3}{2}$

$g(-3) = \int_{-1}^{-3} f(t)\,dt = -1\int_{-3}^{-1} f(t)\,dt = -[(3\cdot 1) + \frac{1}{2}\cdot 1(3+2)]$

$= -(3+\frac{5}{2})$

$= -\frac{11}{2}$

a) Find g(0) and g(−3).

$g(0) = \frac{3}{2}$ $g(-3) = -\frac{11}{2}$

b) Find g' (0) and g' (−3) .

$g'(0) = 1$ $g'(-3) = 3$

c) Find g " (0) and g " (−3).

$g''(0) = f'(0) = -1$ $g''(-3) = f'(-3) = 0$

look at slope f(0) + f(-3)

c) At what value(s) of *x* does g attain a local max and/or local min?

$f(x) = g'(x)$

so look at zeros

$-5 \quad + \quad 1 \quad - \quad 4 \quad + \quad 6$

local max at $x = 1$

local min at $x = 4$

d) At what value(s) of *x* does *g* attain an absolute max and/or an absolute min?

$g(-5) = \int_{-1}^{-5} f(t)\,dt = -[\frac{1}{2}\cdot 3(3+2) + \frac{1}{2}\cdot 1(3+2)] = -10$ $g(4) = \int_{-1}^{4} f(t)\,dt = 2 - \frac{1}{2}(3)(2) = -1$

$g(1) = \int_{-1}^{1} f(t)\,dt = \frac{1}{2}(2)(2) = 2$ $g(6) = \int_{-1}^{6} f(t)\,dt = -1 + \frac{1}{2}(2)(2) = 1$

Abs. min: $x = -5$ Abs. max: $x = 1$

7. Evaluate the definite integrals.

a. $\int_{2}^{e^2} \frac{3}{t}\,dt$

$3\ln|t| \Big|_{2}^{e^2}$

$3\ln e^2 - 3\ln 2$

$6 - 3\ln 2$

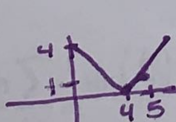
b. $\int_{0}^{\frac{\pi}{3}} \sec x \cdot \tan x\,dx$

$\sec x \Big|_{0}^{\pi/3}$

$\sec\frac{\pi}{3} - \sec 0$

$2 - 1 = 1$

c. $\int_{0}^{5} |4 - x|\,dx$



$\int_{0}^{4} |4-x|\,dx + \int_{4}^{5} |4-x|\,dx$

$\frac{1}{2}(4)(4) + \frac{1}{2}(1)(1)$

$\frac{17}{2}$

d. $\int_{8}^{27} \frac{4}{\sqrt[3]{x}}\,dx$

$\int_{8}^{27} 4x^{-1/3}\,dx$

$4x^{2/3}\cdot\frac{3}{2}\Big|_{8}^{27} = 6\sqrt[3]{x^2}\Big|_{8}^{27}$

$6\sqrt[3]{27}^2 - 6\sqrt[3]{8}^2$

$54 - 24 = 30$

8. Use the Fundamental Theorem of Calculus to simplify:

a. if $f(x) = \int_1^{x^3} \frac{1}{1+t^2}\,dt$

Find $F(1) =$

$F'(x) = \frac{1}{1+(x^3)^2} \cdot 3x^2 = \frac{3x^2}{1+x^6}$

$F'(1) = \frac{3(1)^2}{1+(1)^6} = \frac{3}{2}$

9. Simplify.

a. $\frac{d}{dx}\int_{3x^2}^{4} \sin(t^2)\,dt$

$-\sin((3x^2)^2) \cdot 6x$

$-6x\sin(9x^4)$

b. $F(x) = \int_{x^2}^{\sec x} \sqrt{1+t}\,dt$, find $F'(x)$

$\sqrt{1+\sec x} \cdot \sec x\tan x - \sqrt{1+x^2} \cdot 2x$

$\sec x\tan x\sqrt{1+\sec x} - 2x\sqrt{1+x^2}$

c. $\int_1^{x} \frac{1}{1+t^2}\,dt$

$\tan^{-1} t\Big|_1^x$

$\tan^{-1} x - \tan^{-1} 1$

$\tan^{-1} x - \frac{\pi}{4}$

10. Express the limit as a definite integral on the given interval and <u>evaluate</u>.

$$\lim_{n\to\infty}\sum_{i=1}^{n}[6(x_i^*)^2 - 3x_i^*]\Delta x \qquad [1, 2]$$

Integral: $\int_1^2 6x^2 - 3x\,dx$

Value of Integral: $2x^3 - \frac{3}{2}x^2\Big|_1^2 = 2(2)^3 - \frac{3}{2}(2)^2 - 2(1)^3 + \frac{3}{2}(1)^2$

$= 16 - 6 - 2 + \frac{3}{2} = \frac{19}{2}$

11. Express the limit $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}\sin\left(\frac{k}{n}\right)$ as an integral.

$\Delta x = \frac{1}{n}$ $a = 0$ $f(x) = \sin x$

$b = 1$

$\int_0^1 \sin x\,dx$

Practice Test - Meaning of Integration
Calculator allowed

12. The penguin population on an island is modeled by a differentiable function P of time t, where P(t) is the number of penguins and t is measured in years, for $0 \le t \le 40$. There are 100,000 penguins on the island at time $t = 0$. The birth rate for the penguins on the island is modeled by $B(t) = 1000e^{0.06t}$ penguins per year. To the nearest whole number, find how many penguins are on the island after 40 years.

initial

$100{,}000 + \int_0^{40} 1000e^{0.06t}\,dt$

$100{,}000 + 167{,}053 = 267{,}053$ penguins

13. Use your calculator to find the value of the integral.

$$\int_4^{10} (\ln(x) + 5\sin(x) - 1)dx$$

≈ 6.408

14. The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a differentiable and strictly increasing function R of time t. A table of selected values of R(t), for the time interval $0 \le t \le 90$ minutes, are shown below. Approximate the value of $\int_0^{90} R(t)dt$ using a trapezoidal approximation with the five subintervals indicated by the data in the table.

t (minutes)	R(t) (gal per min)
0	15
10	25
40	50
50	55
70	65
90	70

[0,10] [10,40] [40,50] [50,70] [70,90]

$\frac{1}{2}[(10)(15+25)+(30)(25+50)+(10)(50+55)+(20)(55+65)+20(65+70)]$

$\frac{1}{2}(8800) = 4400$ gallons