AP Calc AB - Derivative Rules – Fall 2020

EQ: What are some Short Cut Derivative Rules?

Day	Date	Topic	Assignment
1	Friday, September 11th	**4.1 The Power Rule** *EQ: How do you find the derivative of a polynomial using the power rule?*	Constant and Power Rule Practice (Packet p. 1 - 2)
2	Monday, September 14th	**4.2 The Product and Quotient Rules** *EQ: How do you find the derivative when functions are multiplied or divided?*	**Skills Check 4.1** Power and Quotient Rule (Packet p. 3 - 4) Mixed Derivatives (Packet p. 5 - 6)
3	Tuesday, September 15th	**4.3 Particle Motion** EQ: How are Derivatives used in the real world?	**Quick Derivatives Quiz 1** Particle Motion (Packet p. 7 – 8)
4	Wednesday, September 16th	**Optional Q&A Session at 10:00 AM**	**Get caught up on all Keeper Notes & HW**
5	Thursday, September 17th	**4.4 Derivative of Trigonometric Functions** *EQ: How can you take the derivative of trig functions?* **4.5 The Chain Rule** *EQ: How do you take the derivative of a composition of functions?*	**Quick Derivatives Quiz 2** Derivatives of Trig Functions (Packet p. 9) Chain Rule (Packet p. 10 – 12)
6	Friday, September 18th	**4.6 Derivatives of Logarithmic Functions and Exponential Functions** *EQ: How do you differentiate a logarithmic or exponential function?*	**Skills Check 4.1-4.5** Derivative of ln(x) (Packet p. 13) Derivative of e^x and a^x (Packet p. 14)
7	Monday, September 21st	**4.7 Implicit Differentiation** **4.8 Logarithmic Differentiation** *EQ: How can you take a derivative if y is in the problem more than once?*	**Quick Derivatives Quiz 3** Implicit Differentiation (Packet p. 15 – 16) Logarithmic Differentiation (Packet p. 17)
8	Tuesday, September 22nd	**4.9 Derivatives of Inverse Functions** *EQ: How do you find the derivative of an inverse function?*	**Skills Check 4.7/4.8** Derivatives of Inverse Functions (packet p. 20 – 21) Derivatives of Inverse Trig Functions (Packet p. 22 – 23)
9	Wednesday, September 23rd	**Optional Q&A Session at 10:00 AM**	Packet p. 18 - 19 **Get caught up on all Keeper Notes & HW**
10	Thursday, September 24th	**4.10 L'Hopital's Rule** *EQ: What is the quick way to evaluate limits in indeterminate form?* **Review**	**Quick Derivatives Quiz 4** Limits and L'Hopital's Rule (Packet p. 24 – 25) AP Calculus Multiple Choice and Free Response Practice (Packet p. 26 – 30)
11	Friday, September 25th	**Test**	Good Luck!

Constant and Power Rule Practice

Find the derivative of each function. Make sure your answers are factored completely. If a point is given, find the value of the derivative at that point.

1. $y = 3$

$y' = 0$

2. $f(x) = x + 1$

$f'(x) = 1$

3. $f(t) = -3t^2 + 2t - 4$

$f'(t) = -6t + 2$

4. $s(t) = t^3 - 2t + 4$

$s'(t) = 3t^2 - 2$

5. $y = 4t^{\frac{4}{3}}$

$y' = \frac{16}{3}t^{1/3}$ or $\frac{16\sqrt[3]{t}}{3}$

6. $f(x) = 4\sqrt{x} = 4x^{1/2}$

$f'(x) = 2x^{-1/2}$

$f'(x) = \frac{2}{x^{1/2}}$ or $\frac{2}{\sqrt{x}}$

7. $y = 4x^{-2} + 2x^2$

$y' = -8x^{-3} + 4x$

$y' = \frac{-8}{x^3} + 4$

8. $y = \frac{1}{4x^3} = \frac{1}{4}x^{-3}$

$y' = -\frac{3}{4}x^{-4}$

$y' = \frac{-3}{4x^4}$

9. $y = \frac{1}{(4x)^3} = \frac{1}{64x^3}$

$y = \frac{1}{64}x^{-3}$

$y' = \frac{-3}{64}x^{-4}$

$y' = \frac{-3}{64x^4}$

10. $y = \frac{\sqrt{x}}{x} = \frac{x^{1/2}}{x}$

$y = x^{-1/2}$

$y' = -\frac{1}{2}x^{-3/2}$

$y' = \frac{-1}{2x^{3/2}}$

11. $f(x) = x^2 - \frac{4}{x}$

$f(x) = x^2 - 4x^{-1}$

$f'(x) = 2x + 4x^{-2}$

$f'(x) = 2x + \frac{4}{x^2}$

12. $f(x) = x^2 - 2x - \frac{2}{x^4}$

$f(x) = x^2 - 2x - 2x^{-4}$

$f'(x) = 2x - 2 + 8x^{-5}$

$f'(x) = 2x - 2 + \frac{8}{x^5}$

13. $f(x) = \frac{2x^3 - 4x^2 + 3}{x^2}$

$f(x) = 2x - 4 + 3x^{-2}$

$f'(x) = 2 - 6x^{-3}$

$f'(x) = 2 - \frac{6}{x^3}$

14. $y = x(x^2 + 1)$

$y = x^3 + x$

$y' = 3x^2 + 1$

15. $f(x) = x^{4/5}$

$f'(x) = \frac{4}{5}x^{-1/5}$

$f'(x) = \frac{4}{5x^{1/5}}$ or $\frac{4}{5\sqrt[5]{x}}$

16. $f(x) = \sqrt[3]{x} + \sqrt[5]{x}$

$f(x) = x^{1/3} + x^{1/5}$

$f'(x) = \frac{1}{3}x^{-2/3} + \frac{1}{5}x^{-4/5}$

$f'(x) = \frac{1}{3x^{2/3}} + \frac{1}{5x^{4/5}}$

17. $f(x) = \frac{4}{x^{-3}}$

$f(x) = 4x^3$

$f'(x) = 12x^2$

18. $f(x) = \frac{\pi}{(3x)^2} = \frac{\pi}{9x^2}$

$f(x) = \frac{\pi}{9}x^{-2}$

$f'(x) = \frac{-2\pi}{9}x^{-3}$

$f'(x) = \frac{-2\pi}{9x^3}$

19. $f(x) = \frac{1}{\sqrt[9]{x^7}} = x^{-7/9}$

$f(x) = -\frac{7}{9}x^{-16/9}$

$f'(x) = \frac{-7}{9x^{16/9}}$

20. $f(x) = \frac{5x^7 + 9x^4 + 2x - 9}{10}$

$f(x) = \frac{1}{2}x^7 + \frac{9}{10}x^4 + \frac{1}{5}x - \frac{9}{10}$

$f'(x) = \frac{7}{2}x^6 + \frac{18}{5}x^3 + \frac{1}{5}$

Product and Quotient Rule

1. $f(x) = (1+\sqrt{x})(x^3)$

$f'(x) = (1+\sqrt{x})(3x^2) + (x^3)\left(\frac{1}{2\sqrt{x}}\right)$

$f'(x) = 3x^2 + 3x^{5/2} + \frac{1}{2}x^{5/2}$

$f'(x) = \frac{7}{2}x^{5/2} + 3x^2$

2. $g(t) = \left(\frac{2}{t} + t^5\right)(t^3+1)$

$g'(t) = (2t^{-1}+t^5)(3t^2) + (t^3+1)\left(\frac{-2}{t^2}+5t^4\right)$

$g'(t) = 6t + 3t^7 - 2t + 5t^7 - \frac{2}{t^2} + 5t^4$

$g'(t) = 8t^7 + 5t^4 + 4t - \frac{2}{t^2}$

3. $h(y) = \frac{1}{y^3+2y+1}$

$h'(y) = \frac{(y^3+2y+1)0 - 1(3y^2+2)}{(y^3+2y+1)^2}$

$h'(y) = \frac{-3y^2-2}{(y^3+2y+1)^2}$

4. $y = \frac{1}{x+\sqrt{x}}$

$y' = \frac{(x+\sqrt{x})0 - 1\left(1+\frac{1}{2\sqrt{x}}\right)}{(x+\sqrt{x})^2}$

$y' = \frac{-1 - \frac{1}{2\sqrt{x}}}{(x+\sqrt{x})^2}$

$y' = \frac{-2\sqrt{x}+1}{2\sqrt{x}(x+\sqrt{x})^2}$

5. $y = 2^x e^x$

$y' = 2^x e^x + e^x 2^x \ln 2$

6. $g(z) = \frac{z^2+1}{z^3-5}$

$g'(z) = \frac{(z^3-5)(2z) - (z^2+1)(3z^2)}{(z^3-5)^2}$

$g'(z) = \frac{2z^4 - 10z - 3z^4 - 3z^2}{(z^3-5)^2}$

$g'(z) = \frac{-z^4 - 3z^2 - 10z}{(z^3-5)^2}$

7. $y = \frac{\sqrt{x}}{x^3+1}$

$y' = \frac{(x^3+1)\left(\frac{1}{2x^{1/2}}\right) - x^{1/2}(3x^2)}{(x^3+1)^2}$

$y' = \frac{\frac{x^{5/2}}{2} + \frac{1}{2x^{1/2}} - 3x^{5/2}}{(x^3+1)^2}$

$y' = \frac{x^3 + 1 - 6x^3}{2x^{1/2}(x^3+1)^2}$

$y' = \frac{-5x^3+1}{2\sqrt{x}(x^3+1)^2}$

8. $z = \frac{t^2}{(t-4)(2-t^3)}$

$z = \frac{t^2}{2t - t^4 - 8 + 4t^3}$

$z' = \frac{(-t^4+4t^3+2t-8)(2t) - t^2(-4t^3+12t^2+2)}{(-t^4+4t^3+2t-8)^2}$

$z' = \frac{-2t^5 + 8t^4 + 4t^2 - 16t + 4t^5 - 12t^4 - 2t^2}{(-t^4+4t^3+2t-8)^2}$

$z' = \frac{2t^5 - 4t^4 + 2t^2 - 16t}{(-t^4+4t^3+2t-8)^2}$

9. $h(x) = \frac{\left((x^3+1)\sqrt{x}\right)}{x^2}$

$h(x) = \frac{x^{7/2} + x^{1/2}}{x^2}$

$h(x) = x^{3/2} + x^{-3/2}$

$h'(x) = \frac{3}{2}x^{1/2} - \frac{3}{2}x^{-5/2}$

$h'(x) = \frac{3}{2}x^{1/2} - \frac{3}{2x^{5/2}}$

10. $y(m) = \frac{(e^m)(\sqrt[3]{m})}{m^2+3}$ $\quad e^m m^{1/3}$

$y'(m) = \frac{(m^2+3)(e^m \cdot \frac{1}{3}m^{-2/3} + m^{1/3}e^m) - (e^m m^{1/3})2m}{(m^2+3)^2}$

$y'(m) = \frac{\frac{1}{3}m^{4/3}e^m + m^{7/3}e^m + m^{-2/3}e^m + 3m^{1/3}e^m - 2m^{4/3}e^m}{(m^2+3)^2}$

$y'(m) = \frac{e^m(\frac{5}{3}m^{4/3} + m^{7/3} + 3m^{1/3} + m^{-2/3})}{(m^2+3)^2}$

$y'(m) = \frac{e^m(5m^2 + 3m^3 + 9m + 3)}{3m^{2/3}(m^2+3)^2}$

11. $g(x) = (x + \sqrt{x})(3^x)$

$g'(x) = (x + x^{1/2})3^x \ln 3 + 3^x(1 + \frac{1}{2\sqrt{x}})$

$g'(x) = 3^x(x\ln 3 + \sqrt{x}\ln 3 + 1 + \frac{1}{2\sqrt{x}})$

12. Let $f(x) = g(x)h(x), g(10) = -4, h(10) = 560, g'(10) = 0$, and $h'(10) = 35$. find $f'(10)$.

$f(x) = g(x) \cdot h(x)$

$f'(x) = g(x)h'(x) + h(x)g'(x)$

$f'(10) = g(10)h'(10) + h(10)g'(10)$

$f'(10) = -4(35) + 560(0)$

$f'(10) = -140$

13. Let $y(x) = \frac{z(x)}{1+x^2}$, $z(-3) = 6$, and $z'(-3) = 15$. Find $y'(-3)$

$y'(x) = \frac{(1+x^2)z'(x) - z(x)(2x)}{(1+x^2)^2}$

$y'(-3) = \frac{(1+(-3)^2)z'(-3) - z(-3)(2\cdot -3)}{(1+(-3)^2)^2}$

$y'(-3) = \frac{10(15) - 6(-6)}{100}$

$y'(-3) = \frac{150+36}{100}$

$y(-3) = \frac{186}{100} = \frac{93}{50}$

Mixed Derivatives

Find the derivative using the power, product, or quotient rule. If necessary, rewrite first.

1. $y = 6x^3 + 4x^2 - 2x + 5$

$y' = 18x^2 + 8x - 2$

2. $y = \sqrt[4]{x^3}$

$y = x^{3/4}$

$y' = \frac{3}{4}x^{-1/4}$

$y' = \frac{3}{4x^{1/4}}$ or $\frac{3}{4\sqrt[4]{x}}$

3. $y = 3x^2 + \frac{12}{\sqrt{x}} - \frac{1}{x^2}$

$y = 3x^2 + 12x^{-1/2} - x^{-2}$

$y' = 6x - 6x^{-3/2} + 2x^{-3}$

$y' = 6x - \frac{6}{x^{3/2}} + \frac{2}{x^3}$

4. $y = 3 - 7x^3 + 3x^7$

$y' = -21x^2 + 21x^6$

5. $y = 3x^{-\frac{2}{3}} + x^{\frac{3}{4}}$

$y' = -2x^{-5/3} + \frac{3}{4}x^{-1/4}$

$y' = \frac{-2}{x^{5/3}} + \frac{3}{4x^{1/4}}$

6. $y = \frac{3x^3 - 5}{7}$

$y = \frac{3}{7}x^3 - \frac{5}{7}$

$y' = \frac{9}{7}x^2$

7. $y = \frac{4x^{\frac{3}{2}}}{x^1}$

$y = 4x^{1/2}$

$y' = 2x^{-1/2}$

$y' = \frac{2}{\sqrt{x}}$

8. $y = \frac{x^2 + 1}{x} = \frac{x^2}{x} + \frac{1}{x}$

$y = x + x^{-1}$

$y' = 1 - x^{-2}$

$y' = 1 - \frac{1}{x^2}$

9. $y = \dfrac{x^7 + 5x^6 - x^3}{x^2}$

$y = x^5 + 5x^4 - x$

$y' = 5x^4 + 20x^3 - 1$

10. $y = \dfrac{x+1}{\sqrt{x}} = \dfrac{x+1}{x^{1/2}}$

$y = x^{1/2} + x^{-1/2}$

$y' = \dfrac{1}{2x^{1/2}} - \dfrac{1}{2x^{3/2}}$

11. $y = (x^3 - 2)^2$

$y = (x^3-2)(x^3-2)$

$y = x^6 - 4x^3 + 4$

$y' = 6x^5 - 12x^2$

12. $y = \dfrac{x^2 - 4}{x+3}$

$y' = \dfrac{(x+3)(2x) - (x^2-4)(1)}{(x+3)^2}$

$y' = \dfrac{2x^2 + 6x - x^2 + 4}{(x+3)^2}$

$y' = \dfrac{x^2 + 6x + 4}{(x+3)^2}$

13. $y = \dfrac{2x+1}{2x-1}$

$y' = \dfrac{(2x-1)(2) - (2x+1)(2)}{(2x-1)^2}$

$y' = \dfrac{4x - 2 - 4x - 2}{(2x-1)^2}$

$y' = \dfrac{-4}{(2x-1)^2}$

14. $y = \dfrac{x^2+1}{x^2-1}$

$y' = \dfrac{(x^2-1)(2x) - (x^2+1)(2x)}{(x^2-1)^2}$

$y' = \dfrac{2x^3 - 2x - 2x^3 - 2x}{(x^2-1)^2}$

$y' = \dfrac{-4x}{(x^2-1)^2}$

15. $y = \dfrac{1}{1+\sqrt{x}}$

$y' = \dfrac{(1+\sqrt{x})(0) - 1\left(\frac{1}{2\sqrt{x}}\right)}{(1+\sqrt{x})^2}$

$y = \dfrac{\frac{-1}{2\sqrt{x}}}{(1+\sqrt{x})^2} = \dfrac{-1}{2\sqrt{x}(1+\sqrt{x})^2}$

16. $y = \dfrac{(x+1)(2x-5)}{(x+2)}$

$y = \dfrac{2x^2 - 3x - 5}{x+2}$

$y' = \dfrac{(x+2)(4x-3) - (2x^2-3x-5)(1)}{(x+2)^2}$

$y' = \dfrac{4x^2 + 5x - 6 - 2x^2 + 3x + 5}{(x+2)^2}$

$y' = \dfrac{2x^2 + 8x - 1}{(x+2)^2}$

17. $y = (3x^3 + 4x)(x-5)(x+1)$

$y = (3x^3+4x)(x^2-4x-5)$

$y' = (3x^3+4x)(2x-4) + (x^2-4x-5)(9x^2+4)$

$y' = 6x^4 - 12x^2 + 8x^2 - 16x + 9x^4 + 4x^2 - 36x^3 + 16x - 45x^2 - 20$

$y' = 15x^4 - 36x^3 - 45x^2 - 32x - 20$

Particle Motion

Answer the following questions for each position function $s(t)$ in meters where t is in seconds if a particle is moving along the x-axis.

$s(t) = t^3 - 3t + 3 \quad [0,6]$

a. What is the velocity function?

$v(t) = 3t^2 - 3$

b. What is the velocity at $t = 3$ seconds?

$v(3) = 3(3)^2 - 3$

$v(3) = 24 \text{ m/s}$

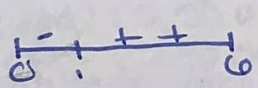
c. When is the particle at rest?

$0 = 3t^2 - 3$ $\quad t = 1 \text{ sec}$

$0 = t^2 - 1$

$0 = (t+1)(t-1)$

d. When is the particle moving right? Moving left?



right : $(0,6)$ sec

left : $(0,1)$ sec

e. What is the acceleration function?

$a(t) = 6t$

f. What is the acceleration at $t = 1$ second?

$a(1) = 6(1)$

$a(1) = 6 \text{ m/s}^2$

g. What is the displacement?

$s(0) = 3 \text{m} \quad s(6) = 201 \text{m}$

$201 - 3 = 198 \text{m}$

h. What is the total distance traveled?

$s(0) = 3 \text{ m}$ $\quad |s(1) - s(0)| = 2 \text{ m}$

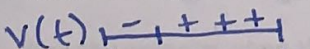
$s(1) = 1 \text{ m}$ $\quad |s(6) - s(1)| = 200 \text{ m}$

$s(6) = 201 \text{m}$ $\quad +$

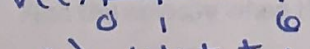
202 m

i. When is the particle speeding up? Slowing Down?

$v(t)$



$a(t)$



$6t = 0$

$t = 0$

speeding up: $(1,6)$

slowing down: $(0,1)$

j. Find the velocity when the acceleration is 0.

$6t = 0$ $\quad v(t) = 3t^2 - 3$

$t = 0 \text{m}$ $\quad v(0) = 3(0)^2 - 3$

$v(0) = -3 \text{m/s}$

$s(t) = t^3 - 6t^2 \quad [0,7]$

a. What is the velocity function?

$v(t) = 3t^2 - 12t$

b. What is the velocity at $t = 3$ seconds?

$v(3) = 3(3)^2 - 12(3)$

$v(3) = -9 \text{ m/s}$

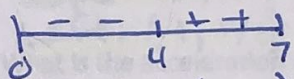
c. When is the particle at rest?

$0 = 3t^2 - 12t$

$0 = 3t(t-4)$

$t = 0 \text{sec} + 4 \text{sec}$

d. When is the particle moving right? Moving left?



right: $(4,7)$ sec

left: $(0,4)$ sec

e. What is the acceleration function?

$a(t) = 6t - 12$

f. What is the acceleration at $t = 1$ second?

$a(1) = 6(1) - 12$

$a(1) = -6 \text{m/s}^2$

g. What is the displacement?

$s(0) = 0 \text{m} \quad s(7) = 49 \text{m}$

$s(7) - s(0) = 49 \text{m}$

h. What is the total distance traveled?

$s(0) = 0 \quad s(4) = -32 \quad s(7) = 49$

$|s(4) - s(0)| = |-32 - 0| = 32 \text{m}$

$|s(7) - s(4)| = |49 - -32| = 81 \text{m}$

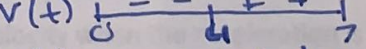
113m

i. When is the particle speeding up? Slowing Down?

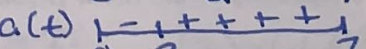
$6t - 12 = 0$

$t = 2$

$v(t)$



$a(t)$



speeding up: $(0,2) \cup (4,7)$ sec

slowing down: $(2,4)$ sec

j. Find the velocity when the acceleration is 0.

$6t - 12 = 0$ $\quad v(2) = 3(2)^2 - 12(2)$

$t = 2$ $\quad v(2) = -12 \text{ m/s}$

$s(t) = 2t^3 - 21t^2 + 60t + 3$ [0,8]

a. What is the velocity function?

$v(t) = 6t^2 - 42t + 60$

b. What is the velocity at $t = 3$ seconds?

$v(3) = 6(3)^2 - 42(3) + 60$

$v(3) = -12\ m/s$

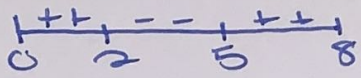
c. When is the particle at rest?

$0 = 6t^2 - 42t + 60$

$0 = t^2 - 7t + 10$

$0 = (t-2)(t-5)$ $t = 2, 5$ sec

d. When is the particle moving right? Moving left?



Right: $(0,2) \cup (5,8)$ sec

Left: $(2,5)$ sec

e. What is the acceleration function?

$a(t) = 12t - 42$

f. What is the acceleration at $t = 1$ second?

$a(1) = 12(1) - 42$

$a(1) = -30\ m/s^2$

g. What is the displacement?

$s(0) = 3m$ $s(8) = 163m$

$s(8) - s(0) = 160m$

h. What is the total distance traveled?

$s(0) = 3$, $s(2) = 55$, $s(5) = 28$, $s(8) = 163$

$|s(2) - s(0)| = |55 - 3| = 52m$

$|s(5) - s(2)| = |28 - 55| = 27m$

$|s(8) - s(5)| = |163 - 28| = 135m$

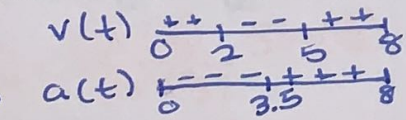
$+$ ____

$214m$

i. When is the particle speeding up? Slowing Down?

$0 = 12t - 42$

$t = \frac{42}{12} = 3.5$



Speed up: $(2, 3.5) \cup (5, 8)$ sec

Slow down: $(0,2) \cup (3.5, 5)$ sec

j. Find the velocity when the acceleration is 0.

$0 = 12t - 42$

$t = 3.5$

$v(3.5) = 6(3.5)^2 - 42(3.5) + 60$

$v(3.5) = -13.5\ m/s$

$s(t) = 2t^3 - 14t^2 + 22t - 5$ [0,6]

a. What is the velocity function?

$v(t) = 6t^2 - 28t + 22$

b. What is the velocity at $t = 3$ seconds?

$v(3) = 6(3)^2 - 28(3) + 22$

$v(3) = -8\ m/s$

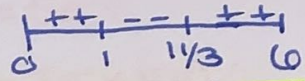
c. When is the particle at rest?

$0 = 6t^2 - 28t + 22$

$0 = 3t^2 - 14t + 11$

$0 = (3t - 11)(t - 1)$ $t = 1, \frac{11}{3}$ sec

d. When is the particle moving right? Moving left?



Right: $(0,1) \cup (11/3, 6)$ sec

Left: $(1, 11/3)$

e. What is the acceleration function?

$a(t) = 12t - 28$

f. What is the acceleration at $t = 1$ second?

$a(1) = 12(1) - 28$

$a(1) = -16 m/s^2$

g. What is the displacement?

$s(0) = -5m$ $s(6) = 55m$

$s(6) - s(0) = 55 - (-5) = 60m$

h. What is the total distance traveled?

$s(0) = -5$, $s(1) = 5$, $s(11/3) = -\frac{377}{27}$, $s(6) = 55$

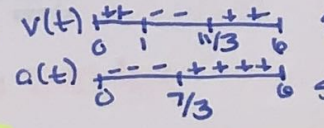
$|s(1) - s(0)| = |5 - -5| = 10m$

$|s(\frac{11}{3}) - s(1)| = |-\frac{377}{27} - 5| = \frac{512}{27} m$

$|s(6) - s(\frac{11}{3})| = |55 - \frac{-377}{27}| = \frac{1862}{27} m$

$2644/27 \approx 97.93\ m$

i. When is the particle speeding up? Slowing Down?



Speed up: $(1, 7/3) \cup (\frac{11}{3}, 6)$ sec

Slow down: $(0,1) \cup (\frac{7}{3}, \frac{11}{3})$ sec

j. Find the velocity when the acceleration is 0.

$0 = 12t - 28$

$t = \frac{28}{12} = \frac{7}{3}$

$v(\frac{7}{3}) = 6(\frac{7}{3})^2 - 28(\frac{7}{3}) + 22$

$v(\frac{7}{3}) = -\frac{32}{3}\ m/s$

Derivatives of Trigonometric Functions

1. $y = 4\sin^2 x + 5\cos^2 x$

$y = 4(\sin x)^2 + 5(\cos x)^2$

$y' = 8\sin x\cos x + 10\cos x \cdot -\sin x$

$y' = 8\sin x\cos x - 10\cos x\sin x$

$y' = -2\sin x\cos x$ or $y' = -\sin(2x)$

* Double angle identity!

2. $f(x) = \sin^3 x \cdot \cos x$

$f'(x) = \sin^3 x \cdot -\sin x + \cos x \cdot 3(\sin x)^2 \cdot \cos x$

$f'(x) = -\sin^4 x + 3\sin^2 x\cos^2 x$

3. $y = 2\sec x + \tan x$

$y' = 2\sec x\tan x + \sec^2 x$

4. $y = \frac{1+\tan^2 x}{\sec x} = \frac{\sec^2 x}{\sec x}$

$y = \sec x$

$y' = \sec x\tan x$

can factor + use trig identities 1st

5. $f(x) = \sin^4 3x - \cos^4 3x$

$f(x) = (\sin(3x))^4 - (\cos(3x))^4$

$f'(x) = 4\sin^3(3x)\cos(3x)(3) - 4\cos^3(3x)(-\sin(3x))(3)$

$f'(x) = 12\sin^3(3x)\cos(3x) + 12\cos^3(3x)\sin(3x)$

$f(x) = 12\sin(3x)\cos(3x)(\sin^2 3x + \cos^2 3x)$

$f'(x) = 12\sin(3x)\cos(3x) = 6 \cdot 2\sin(3x)\cos(3x)$

double angle

$f'(x) = 6\sin 6x$

6. $f(x) = \frac{\sec 4x}{\tan 4x}$ $\quad \frac{\sec x}{\tan x} = \frac{1}{\cos x} \cdot \frac{\cos x}{\sin x} = \frac{1}{\sin x}$

$f(x) = \csc(4x)$

$f'(x) = -\csc(4x)\cot(4x)\,4$

$f'(x) = -4\csc(4x)\cot(4x)$

7. $f(x) = \csc^4 x - 21\cot^2 x$

$f'(x) = 4\csc^3 x(-\csc x\cot x) - 42\cot x(-\csc^2 x)$

$f'(x) = -4\csc^4 x\cot x + 42\csc^2\cot x$

$f'(x) = -2\csc^2 x\cot x(2\csc^2 x - 21)$

8. $f(x) = (1 + \cos 3x)^2$

$f'(x) = 2(1+\cos 3x)(-\sin 3x)\,3$

$f'(x) = -6\sin 3x(1 + \cos 3x)$

9. $f(x) = \cot\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)$

$f(x) = \frac{\cos(\frac{x}{2})}{\sin(\frac{x}{2})} \cdot \sin(\frac{x}{2})$

$f(x) = \cos\left(\frac{x}{2}\right)$

$f'(x) = -\sin\left(\frac{x}{2}\right)\frac{1}{2}$

$f'(x) = -\frac{1}{2}\sin\left(\frac{x}{2}\right)$

10. $y = \frac{1}{\cos 6x}$

$y = \sec 6x$

$y' = 6\sec 6x\tan 6x$

Chain Rule

1. $y = (x^3 - 4)^4$

$y' = 4(x^3-4)^3(3x^2)$

$y' = 12x^2(x^3-4)^3$

2. $y = (2x^2 + 5)^7$

$y' = 7(2x^2+5)^6(4x)$

$y' = 28x(2x^2+5)^6$

3. $f(x) = (x^2 + 2x + 5)^6$

$f'(x) = 6(x^2+2x+5)^5(2x+2)$

$f'(x) = (12x+12)(x^2+2x+5)^5$

4. $f(x) = \sqrt[3]{x^2 + x} = (x^2+x)^{1/3}$

$f'(x) = \frac{1}{3}(x^2+x)^{-2/3}(2x+1)$

$f'(x) = \frac{2x+1}{3(x^2+x)^{2/3}}$

5. $y = \sqrt{(3x + 1)^3}$

$y = (3x+1)^{3/2}$

$y' = \frac{3}{2}(3x+1)^{1/2}(3)$

$y' = \frac{9\sqrt{3x+1}}{2}$

6. $y = (\sqrt{x} + 1)^2$

$y' = 2(\sqrt{x}+1)\left(\frac{1}{2\sqrt{x}}\right)$

$y' = \frac{\sqrt{x}+1}{\sqrt{x}}$

7. $f(x) = \frac{1}{\sqrt{2x^3-7x^2}}$

$f(x) = (2x^3-7x^2)^{-1/2}$

$f'(x) = -\frac{1}{2}(2x^3-7x^2)^{-3/2}(6x^2-14x)$

$f'(x) = \frac{-3x^2+7x}{(2x^3-7x^2)^{3/2}}$

8. $y = (5x^2 - 3x)^{-\frac{2}{3}}$

$y' = -\frac{2}{3}(5x^2-3x)^{-5/3}(10x-3)$

$y' = \frac{-20x+6}{3(5x^2-3x)^{5/3}}$

9. $f(x) = \sin^2 x$

$f'(x) = 2\sin x \cos x$

$f'(x) = \sin 2x$

* Double Angle Identity

10. $f(x) = \sin(x^2)$

$f'(x) = \cos(x^2) \cdot 2x$

$f'(x) = 2x\cos x^2$

11. $f(x) = \tan(3x)$

$f'(x) = 3\sec^2(3x)$

12. $f(x) = (3x - \cos x)^4$

$f'(x) = 4(3x-\cos x)^3(3+\sin x)$

13. $f(x) = e^{x^2+2x}$

$f'(x) = e^{x^2+2x}(2x+2)$

$f'(x) = (2x+2)e^{x^2+2x}$

14. $f(x) = \sec^2(4x)$

$f(x) = (\sec(4x))^2$

$f'(x) = 2\sec(4x)\cdot\sec(4x)\tan(4x)\cdot 4$

$f'(x) = 8\sec^2(4x)\tan(4x)$

15. $y = (1-x)(3x^2-5)^5$

$y' = (1-x)\cdot 5(3x^2-5)^4(6x) + (3x^2-5)^5(-1)$

$y' = (3x^2-5)^4(30x - 30x^2 - 3x^2 + 5)$

$y' = (3x^2-5)^4(-33x^2+30x+5)$

16. $y = (x^2-5x)^6(2x-5)^{-1} = \dfrac{(x^2-5x)^6}{2x-5}$

$y' = \dfrac{(2x-5)\cdot 6(x^2-5x)^5(2x-5) - (x^2-5x)^6(2)}{(2x-5)^2}$

$y' = \dfrac{2(x^2-5x)^5\left[3(4x^2-20x+25) - x^2+5x\right]}{(2x-5)^2}$

$y' = \dfrac{2(x^2-5x)^5(11x^2-55x+75)}{(2x-5)^2}$

17. $y = \left(\dfrac{7-2x^5}{5x^2-8}\right)^2$

$y' = 2\left(\dfrac{7-2x^5}{5x^2-8}\right)\left(\dfrac{(5x^2-8)(-10x^4) - (7-2x^5)(10x)}{(5x^2-8)^2}\right)$

$y' = \dfrac{2(7-2x^5)(-50x^6+80x^4-70x+20x^6)}{(5x^2-8)^3}$

$y' = \dfrac{2(7-2x^5)(-30x^6+80x^4-70x)}{(5x^2-8)^3} = \dfrac{-20x(7-2x^5)(3x^5-8x^3+7)}{(5x^2-8)^3}$

18. $f(x) = \sqrt{\dfrac{x^2+9}{x+3}}$

$f'(x) = \dfrac{1}{2}\left(\dfrac{x^2+9}{x+3}\right)^{-1/2}\left(\dfrac{(x+3)(2x) - (x^2+9)(1)}{(x+3)^2}\right)$

$f'(x) = \dfrac{1}{2}\dfrac{\sqrt{x+3}}{\sqrt{x^2+9}}\cdot\dfrac{x^2+6x+9}{(x+3)^2}$

$f'(x) = \dfrac{x^2+6x-9}{2(x^2+9)^{1/2}(x+3)^{3/2}}$

19. $f(x) = (x^2-3)^4(5x-1)^6$

$f'(x) = (x^2-3)^4\cdot 6(5x-1)^5\cdot 5 + (5x-1)^6\cdot 4(x^2-3)^3\cdot 2x$

$f'(x) = 2(x^2-3)^3(5x-1)^5(15(x^2-3) + 4x(5x-1))$

$f'(x) = 2(x^2-3)^3(5x-1)^5(15x^2+20x^2-4x-45)$

$f'(x) = 2(x^2-3)^3(5x-1)^5(35x^2-4x-45)$

20. $f(x) = \sqrt{4-\sqrt{x^2-5}}$

$f(x) = (4-(x^2-5)^{1/2})^{1/2}$

$f'(x) = \dfrac{1}{2}(4-(x^2-5)^{1/2})^{-1/2}\cdot\dfrac{-1}{2}(x^2-5)^{-1/2}\cdot 2x$

$f'(x) = \dfrac{-x}{2\sqrt{4-\sqrt{x^2-5}}\cdot\sqrt{x^2-5}}$

21. $f(x) = \cos^2(\sin 5x)$

$f(x) = [\cos(\sin(5x))]^2$

$f'(x) = 2[\cos(\sin(5x))] \cdot -\sin(\sin(5x)) \cdot \cos(5x) \cdot 5$

$= -10\cos(\sin(5x))\sin(\sin(5x))\cos(5x)$

22. $f(x) = \frac{(4x^2-6)^3}{(6x-7)^5}$

$f'(x) = \frac{(6x-7)^5 \cdot 3(4x^2-6)^2(8x) - (4x^2-6)^3 5(6x-7)^4 \cdot 6}{[(6x-7)^5]^2}$

$f'(x) = \frac{6(6x-7)^4(4x^2-6)^2[4x(6x-7) - 5(4x^2-6)]}{(6x-7)^{10}}$

$f'(x) = \frac{6(6x-7)^4(4x^2-6)^2(24x^2-28x-20x^2+30)}{(6x-7)^{10}}$

$f'(x) = \frac{6(4x^2-6)(4x^2-28x+30)}{(6x-7)^6}$

23. $f(x) = \frac{e^{3x}-5}{e^{2x}+7}$

$f'(x) = \frac{(e^{2x}+7)(3e^{3x}) - (e^{3x}-5)(2e^{2x})}{(e^{2x}+7)^2}$

$f'(x) = \frac{3e^{5x} + 21e^{3x} - 2e^{5x} + 10e^{2x}}{(e^{2x}+7)^2}$

$f'(x) = \frac{e^{5x} + 21e^{3x} + 10e^{2x}}{(e^{2x}+7)^2}$

24. $f(x) = \cot^3(e^{x^2})$

$f(x) = [\cot(e^{x^2})]^3$

$f'(x) = 3[\cot(e^{x^2})]^2 \cdot -\csc^2(e^{x^2}) \cdot e^{x^2} \cdot 2x$

$f'(x) = -6xe^{x^2}\csc^2(e^{x^2})\csc^2(e^{x^2})$

25. $f(x) = x^2 \sin\left(\frac{1}{x}\right)$

$f'(x) = x^2\cos\left(\frac{1}{x}\right) \cdot \left(\frac{-1}{x^2}\right) + \sin\left(\frac{1}{x}\right) 2x$

$f'(x) = -\cos\left(\frac{1}{x}\right) + 2x\sin\left(\frac{1}{x}\right)$

26. $f(x) = \sin^3(\sqrt{e^{3x} - 5x})$

$f'(x) = 3\sin^2(\sqrt{e^{3x}-5x}) \cdot \cos(\sqrt{e^{3x}-5x}) \cdot \frac{1}{2\sqrt{e^{3x}-5x}} \cdot (3e^{3x} - 5)$

$f'(x) = \frac{(9e^{3x} - 15)\sin^2(\sqrt{e^{3x}-5x})\cos(\sqrt{e^{3x}-5x})}{2\sqrt{e^{3x}-5x}}$

Derivatives of $\ln x$

Find each Derivative

1. $y = \ln(x^3 + 1)$

$y' = \frac{1}{x^3+1} \cdot 3x^2$

$y' = \frac{3x^2}{x^3+1}$

2. $y = \ln\sqrt{x}$

$y' = \frac{1}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$

$y' = \frac{1}{2x}$

3. $y = \sqrt{\ln(x)} = (\ln(x))^{1/2}$

$y' = \frac{1}{2}(\ln(x))^{-1/2}\frac{1}{x} \cdot 1$

$y' = \frac{1}{2x\sqrt{\ln x}}$

4. $y = \ln|\sin x|$

$y' = \frac{1}{\sin x} \cdot \cos x$

$y' = \cot x$

5. $y = \ln(\sec x)$

$y' = \frac{1}{\sec x} \cdot \sec x \tan x$

$y' = \tan x$

6. $y = x \cdot \ln x$

$y' = x \cdot \frac{1}{x} + \ln x \cdot 1$

$y' = 1 + \ln x$

7. $y = \frac{\ln x}{x^2}$

$y' = \frac{x^2 \cdot \frac{1}{x} - \ln x \cdot 2x}{(x^2)^2}$

$y' = \frac{x - 2x\ln x}{x^4} = \frac{1 - 2\ln x}{x^3}$

8. $y = \ln(\ln x)$

$y' = \frac{1}{\ln x} \cdot \frac{1}{x}$

$y' = \frac{1}{x \ln x}$

9. $y = (\sin x)(\ln x)$

$y' = (\sin x)\left(\frac{1}{x}\right) + (\ln x)(\cos x)$

$y' = \frac{\sin x}{x} + \ln x \cdot \cos x$

10. $y = \frac{x^2}{\ln x}$

$y' = \frac{\ln x \cdot 2x - x^2 \cdot \frac{1}{x}}{(\ln x)^2}$

$y' = \frac{2x\ln x - x}{\ln^2 x}$

11. $y = \ln\left(\frac{5}{5-x}\right)$

$y = \ln 5 - \ln(5-x)$

$y' = 0 - \frac{1}{5-x} \cdot -1$

$y' = \frac{1}{5-x}$

12. $y = \ln\sqrt{x^2+4} = \ln(x^2+4)^{1/2}$

$y = \frac{1}{2}\ln(x^2+4)$

$y' = \frac{1}{2} \cdot \frac{1}{x^2+4} \cdot 2x$

$y' = \frac{x}{x^2+4}$

13. $y = \ln(2 - \cos x)$

$y' = \frac{1}{2-\cos x} \cdot \sin x$

$y' = \frac{\sin x}{2-\cos x}$

14. $y = \ln(5-x)^6$

$y = 6\ln(5-x)$

$y' = 6 \cdot \frac{1}{5-x} \cdot -1$

$y' = \frac{-6}{5-x}$

15. $y = e^{\ln x^2}$

$y = x^2$

$y' = 2x$

16. $y = \ln(3x^2+2)^3$

$y = 3\ln(3x^2+2)$

$y' = 3 \cdot \frac{1}{3x^2+2} \cdot 6x$

$y' = \frac{18x}{3x^2+2}$

17. $y = \ln x^3 + (\ln x)^3$

$y' = \frac{1}{x^3} \cdot 3x^2 + 3(\ln x)^2 \cdot \frac{1}{x}$

$y' = \frac{3}{x} + \frac{3\ln^2 x}{x}$

$y' = \frac{3 + 3\ln^2 x}{x}$

18. $y = \ln\sqrt{\ln(x)}$

$y = \ln(\ln x)^{1/2}$

$y = \frac{1}{2}\ln(\ln x)$

$y' = \frac{1}{2} \cdot \frac{1}{\ln x} \cdot \frac{1}{x}$

$y' = \frac{1}{2x\ln x}$

Derivatives of e^x and a^x

Find the derivative of each.

1. $y = e^{2x}$

$y' = e^{2x} \cdot 2$

$y' = 2e^{2x}$

2. $y = e^{5x^2}$

$y' = e^{5x^2} \cdot 10x$

$y' = 10x e^{5x^2}$

3. $y = e^{\sin x}$

$y' = e^{\sin x} \cdot \cos x$

$y' = \cos x \, e^{\sin x}$

4. $y = e^{\tan x}$

$y' = \sec^2 x \, e^{\tan x}$

5. $y = e^{x^2+2x}$

$y' = e^{x^2+2x}(2x+2)$

6. $y = e^{\sqrt{x}}$

$y' = e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$

$y' = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$

7. $y = 5^x$

$y' = 5^x \ln 5$

8. $y = e^{e^x}$

$y' = e^{e^x} \cdot e^x$

9. $y = 7^{x^2+2x^3}$

$y' = 7^{x^2+2x^3} \ln 7 \, (2x+6x^2)$

10. $y = \sin e^{3x}$

$y' = \cos e^{3x} \cdot e^{3x} \cdot 3$

$y' = 3e^{3x} \cos e^{3x}$

11. $y = xe^x$

$y' = xe^x + e^x$

12. $y = (\sin x)e^x$

$y' = (\sin x)e^x + e^x \cos x$

13. $y = x^2 e^x$

$y = x^2 e^x + 2xe^x$

14. $y = \frac{e^x}{x^2}$

$y' = \frac{x^2 e^x - 2xe^x}{x^4}$

$y' = \frac{xe^x - 2e^x}{x^3}$

15. $y = 2^x(x^2+1)$

$y' = 2^x \cdot 2x + (x^2+1)2^x \ln 2$

16. $y = 3^{\ln x}$

$y' = 3^{\ln x} \ln 3 \cdot \frac{1}{x}$

$y' = \frac{3^{\ln x} \ln 3}{x}$

17. $y = x^2 + 4^x$

$y' = 2x + 4^x \ln 4$

18. $y = \ln e^{x^2}$

$y = x^2$

$y' = 2x$

19. $y = e^{\ln x^3}$

$y = x^3$

$y' = 3x^2$

20. $y = e^{3x} \cdot 4^{5x}$

$y' = e^{3x} \cdot 4^{5x} \cdot \ln 4 \cdot 5 + 4^{5x} \cdot e^{3x} \cdot 3$

$y' = e^{3x} 4^{5x} (5 \ln 4 + 3)$

21. $y = e^{\csc x}$

$y' = e^{\csc x} \cdot -\csc x \cot x$

$y' = -e^{\csc x} \csc x \cot x$

22. $y = 10^{\sin x}$

$y' = 10^{\sin x} \ln 10 \cos x$

23. $y = x^2 e^x - xe^x$

$y = e^x(x^2 - x)$

$y' = e^x(2x-1) + (x^2-x)e^x$

$y' = e^x(x^2+x-1)$

24. $y = xe^2 - e^x$

$y' = e^2 - e^x$

Implicit Differentiation

Find the derivative:

1. $(3x+7)^2 = 2y^3$

$2(3x+7)3 = 6y^2\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{6(3x+7)}{6y^2}$

$\frac{dy}{dx} = \frac{3x+7}{y^2}$

2. $x^2 = \frac{x-y}{x+y}$

$2x = \frac{(x+y)(1-\frac{dy}{dx}) - (x-y)(1+\frac{dy}{dx})}{(x+y)^2}$

$2x(x+y)^2 = x - x\frac{dy}{dx} + y - y\frac{dy}{dx} - x - x\frac{dy}{dx} + y + y\frac{dy}{dx}$

$2x(x+y)^2 = 2y - 2x\frac{dy}{dx}$

$2x(x+y)^2 - 2y = -2x\frac{dy}{dx} \rightarrow \frac{dy}{dx} = \frac{2x(x+y)^2 - 2y}{-2x}$

$\frac{dy}{dx} = \frac{x(x+y)^2 - y}{-x}$

3. $y^2 = \frac{x-1}{x+1}$

$2y\frac{dy}{dx} = \frac{(x+1)(1) - (x-1)(1)}{(x+1)^2}$

$\frac{dy}{dx} = \frac{2}{2y(x+1)^2}$

$\frac{dy}{dx} = \frac{1}{y(x+1)^2}$

4. $x^3 - xy + y^3 = 1$

$3x^2 - (x\frac{dy}{dx} + y) + 3y^2\frac{dy}{dx} = 0$

$3x^2 - x\frac{dy}{dx} - y + 3y^2\frac{dy}{dx} = 0$

$\frac{dy}{dx}(-x + 3y^2) = y - 3x^2$

$\frac{dy}{dx} = \frac{y - 3x^2}{3y^2 - x}$

5. $x = \tan y$

$1 = \sec^2 y\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{\sec^2 y}$

$\frac{dy}{dx} = \cos^2 y$

6. $x + \sin y = xy$

$1 + \cos y\frac{dy}{dx} = x\frac{dy}{dx} + y$

$\cos y\frac{dy}{dx} - x\frac{dy}{dx} = y - 1$

$\frac{dy}{dx}(\cos y - x) = y - 1$

$\frac{dy}{dx} = \frac{y-1}{\cos y - x}$

7. $y^2\cos\left(\frac{1}{y}\right) = 2x + 2y$

$y^2 \cdot -\sin(\frac{1}{y})(\frac{-1}{y^2})\frac{dy}{dx} + \cos(\frac{1}{y})2y\frac{dy}{dx} = 2 + 2\frac{dy}{dx}$

$\sin(\frac{1}{y})\frac{dy}{dx} + 2y\cos(\frac{1}{y})\frac{dy}{dx} - 2\frac{dy}{dx} = 2$

$(\sin\frac{1}{y} + 2y\cos(\frac{1}{y}) - 2)\frac{dy}{dx} = 2$

$\frac{dy}{dx} = \frac{2}{\sin(\frac{1}{y}) + 2y\cos(\frac{1}{y}) - 2}$

8. $e^{xy} + \ln(y) = 2x$

$e^{xy}(x\frac{dy}{dx} + y) + \frac{1}{y}\frac{dy}{dx} = 2$

$e^{xy}x\frac{dy}{dx} + ye^{xy} + \frac{1}{y}\frac{dy}{dx} = 2$

$(e^{xy}x + \frac{1}{y})\frac{dy}{dx} = 2 - ye^{xy}$

$\frac{dy}{dx} = \frac{2 - ye^{xy}}{e^{xy}x + \frac{1}{y}}$

$\frac{dy}{dx} = \frac{2y - y^2e^{xy}}{xye^{xy} + 1}$

Find y' and y''.

9. $x^2 + y^2 = 1$

$2x + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2x}{2y}$

$\frac{dy}{dx} = \frac{-x}{y}$

$\frac{d^2y}{dx^2} = \frac{y(-1) - (-x)\frac{dy}{dx}}{y^2}$

$\frac{d^2y}{dx^2} = \frac{-y + x\left(\frac{-x}{y}\right)}{y^2}$

$\frac{d^2y}{dx^2} = \frac{-y - \frac{x^2}{y}}{y^2} = \frac{-y^2 - x^2}{y^3}$

10. $y^2 = x^2 + 2x$

$2y\frac{dy}{dx} = 2x + 2$

$\frac{dy}{dx} = \frac{x+1}{y}$

$\frac{d^2y}{dx^2} = \frac{y(1) - (x+1)\frac{dy}{dx}}{y^2}$

$\frac{d^2y}{dx^2} = \frac{y - (x+1)\left(\frac{x+1}{y}\right)}{y^2}$

$\frac{d^2y}{dx^2} = \frac{y - \frac{(x+1)^2}{y}}{y^2}$

$\frac{d^2y}{dx^2} = \frac{y^2 - (x+1)^2}{y^3}$

11. $x^{2/3} + y^{2/3} = 1$

$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$

$\frac{2}{3\sqrt[3]{y}}\frac{dy}{dx} = \frac{-2}{3\sqrt[3]{x}}$

$\frac{dy}{dx} = \frac{-2}{3\sqrt[3]{x}} \cdot \frac{3\sqrt[3]{y}}{2}$

$\frac{dy}{dx} = \frac{-\sqrt[3]{y}}{\sqrt[3]{x}}$ or $-\left(\frac{y}{x}\right)^{1/3}$

$\frac{d^2y}{dx^2} = -\frac{1}{3}\left(\frac{y}{x}\right)^{-2/3}\left(\frac{x\frac{dy}{dx} - y}{x^2}\right)$

$\frac{d^2y}{dx^2} = -\frac{1}{3}\left(\frac{x}{y}\right)^{2/3}\left(\frac{x\left(\frac{-y}{x}\right)^{1/3} - y}{x^2}\right)$

12. $xy + y^2 = 1$

$x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$

$(x+2y)\frac{dy}{dx} = -y$

$\frac{dy}{dx} = \frac{-y}{x+2y}$

$\frac{d^2y}{dx^2} = \frac{(x+2y)\left(-\frac{dy}{dx}\right) - (-y)\left(1 + 2\frac{dy}{dx}\right)}{(x+2y)^2}$

$\frac{d^2y}{dx^2} = \frac{(x+2y)\left(\frac{y}{x+2y}\right) + y\left(1 - \frac{2y}{x+2y}\right)}{(x+2y)^2}$

$\frac{d^2y}{dx^2} = \frac{y + y - \frac{2y^2}{x+2y}}{(x+2y)^2} = \frac{2y(x+2y) - 2y^2}{(x+2y)^3}$

$\frac{d^2y}{dx^2} = \frac{2xy + 2y^2}{(x+2y)^3}$

Find the equation of the lines that are tangent and normal to the curve at the given point.

13. $x^2y^2 = 9$ at $(-1, 3)$

$x^2 2y\frac{dy}{dx} + y^2(2x) = 0$

$\frac{dy}{dx} = \frac{-2xy^2}{2x^2y}$

$\frac{dy}{dx} = \frac{-y}{x}$

$\left.\frac{dy}{dx}\right|_{(-1,3)} = \frac{-3}{-1} = 3$

Tangent: $y - 3 = 3(x+1)$

Normal: $y - 3 = \frac{-1}{3}(x+1)$

14. $2xy + \pi\sin y = 2\pi$ at $\left(1, \pi/2\right)$

$2x\frac{dy}{dx} + 2y + \pi\cos y\frac{dy}{dx} = 0$

$(2x + \pi\cos y)\frac{dy}{dx} = -2y$

$\frac{dy}{dx} = \frac{-2y}{2x + \pi\cos y}$

$\left.\frac{dy}{dx}\right|_{(1,\pi/2)} = \frac{-2(\pi/2)}{2(1) + \pi\cos(\pi/2)} = \frac{-\pi}{2}$

Tangent: $y - \frac{\pi}{2} = -\frac{\pi}{2}(x-1)$

Normal: $y - \frac{\pi}{2} = \frac{2}{\pi}(x-1)$

Logarithmic Differentiation

1. $y = x^x$

$\ln y = \ln x^x$

$\ln y = x \ln x$

$\frac{1}{y}\frac{dy}{dx} = x \cdot \frac{1}{x} + \ln x$

$\frac{dy}{dx} = y(1 + \ln x)$

$\frac{dy}{dx} = x^x(1 + \ln x)$

2. $y = x^{2x+1}$

$\ln y = \ln x^{2x+1}$

$\ln y = (2x+1)\ln x$

$\frac{1}{y}\frac{dy}{dx} = (2x+1)\cdot\frac{1}{x} + \ln x(2)$

$\frac{dy}{dx} = y\left(\frac{2x+1}{x} + 2\ln x\right)$

$\frac{dy}{dx} = x^{2x+1}\left(\frac{2x+1}{x} + 2\ln x\right)$

3. $y = x^{\sin x}$

$\ln y = \ln x^{\sin x}$

$\ln y = \sin x \cdot \ln x$

$\frac{1}{y}\frac{dy}{dx} = \sin x \cdot \frac{1}{x} + \ln x \cdot \cos x$

$\frac{dy}{dx} = y\left(\frac{\sin x}{x} + \ln x \cos x\right)$

$\frac{dy}{dx} = x^{\sin x}\left(\frac{\sin x}{x} + \ln x \cos x\right)$

4. $y = x^{\frac{1}{x}}$

$\ln y = \frac{1}{x}\ln x$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x}\cdot\frac{1}{x} + \ln x \cdot -\frac{1}{x^2}$

$\frac{dy}{dx} = y\left(\frac{1-\ln x}{x^2}\right)$

$\frac{dy}{dx} = x^{1/x}\left(\frac{1-\ln x}{x^2}\right)$ or $x^{\frac{1}{x}-2}(1-\ln x)$

5. $y = (4x+3)^{x+2}$

$\ln y = (x+2)\ln(4x+3)$

$\frac{1}{y}\frac{dy}{dx} = (x+2)\cdot\frac{1}{4x+3}\cdot 4 + \ln(4x+3)$

$\frac{dy}{dx} = y\left(\frac{4x+8}{4x+3} + \ln(4x+3)\right)$

$\frac{dy}{dx} = (4x+3)^{x+2}\left(\frac{4x+8}{4x+3} + \ln(4x+3)\right)$

6. $y = (\ln x)^x$

$\ln y = x\ln(\ln x)$

$\frac{1}{y}\frac{dy}{dx} = x\cdot\frac{1}{\ln x}\cdot\frac{1}{x} + \ln(\ln x)$

$\frac{dy}{dx} = y\left(\frac{1}{\ln x} + \ln(\ln x)\right)$

$\frac{dy}{dx} = (\ln x)^x\left(\frac{1}{\ln x} + \ln(\ln x)\right)$

7. $y = (x^2+5x+1)^{x+2}$

$\ln y = (x+2)\ln(x^2+5x+1)$

$\frac{1}{y}\frac{dy}{dx} = (x+2)\cdot\frac{1}{x^2+5x+1}\cdot(2x+5) + \ln(x^2+5x+1)$

$\frac{dy}{dx} = y\left[\frac{(x+2)(2x+5)}{x^2+5x+1} + \ln(x^2+5x+1)\right]$

$\frac{dy}{dx} = (x^2+5x+1)^{x+2}\left[\frac{(x+2)(2x+5)}{x^2+5x+1} + \ln(x^2+5x+1)\right]$

8. $y = x^{\ln x}$

$\ln y = \ln x \ln x$

$\frac{1}{y}\frac{dy}{dx} = \ln x\cdot\frac{1}{x} + \ln x\cdot\frac{1}{x}$

$\frac{dy}{dx} = y\left(\frac{2\ln x}{x}\right)$

$\frac{dy}{dx} = x^{\ln x}\left(\frac{2\ln x}{x}\right)$

9. $y = (3x-7)^4(8x^2-1)^3$

$\ln y = \ln\left((3x-7)^4(8x^2-1)^3\right)$

$\ln y = 4\ln(3x-7) + 3\ln(8x^2-1)$

$\frac{1}{y}\frac{dy}{dx} = 4\cdot\frac{1}{3x-7}\cdot 3 + 3\cdot\frac{1}{8x^2-1}\cdot 16x$

$\frac{dy}{dx} = y\left(\frac{12}{3x-7} + \frac{48x}{8x^2-1}\right)$

$\frac{dy}{dx} = (3x-7)^4(8x^2-1)^3\left(\frac{12}{3x-7} + \frac{48x}{8x^2-1}\right)$

10. $y = (2x-1)^3(4x^2+5)^5$

$\ln y = \ln(2x-1)^3(4x^2+5)^5$

$\ln y = 3\ln(2x-1) + 5\ln(4x^2+5)$

$\frac{1}{y}\frac{dy}{dx} = 3\cdot\frac{1}{2x-1}\cdot 2 + 5\cdot\frac{1}{4x^2+5}\cdot 8x$

$\frac{dy}{dx} = y\left(\frac{6}{2x-1} + \frac{40x}{4x^2+5}\right)$

$\frac{dy}{dx} = (2x-1)^3(4x^2+5)^5\left(\frac{6}{2x-1} + \frac{40x}{4x^2+5}\right)$

Derivatives from Charts and Graphs

1. If $f(3)=4,\ g(3)=2,\ f'(3)=6$ and $g'(3)=5$ find the following.

a) $(f+g)'(3)$ $= f'(3)+g'(3)$
$= 6+5$
$= 11$

b) $-5g'(3)$ $= -5(5)$
$= -25$

c) $(f\cdot g)'(3) = f(3)g'(3)+g(3)f'(3)$
$= 4(5)+2(6)$
$= 32$

d) $\left(\frac{f}{g}\right)'(3) = \frac{g(3)f'(3)-f(3)g'(3)}{[g(3)]^2}$
$= \frac{2(6)-4(5)}{(2)^2} = \frac{12-20}{4} = \frac{-8}{4} = -2$

2 – 9 Given the following chart, find the indicated derivatives.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-2	3	1	-5	8
-1	-9	7	4	1
0	5	9	9	-3
1	3	-3	2	6
2	-5	3	8	-4

2. If $h(x)=f(x)+g(x)$, find $h'(-1)$

$h'(x) = f'(x)+g'(x)$
$h'(-1) = f'(-1)+g'(-1)$
$h'(-1) = 7+1$
$h'(-1) = 8$

3. If $h(x)=7g(x)$, find $h'(0)$

$h'(0) = 7g'(0)$
$h'(0) = 7(-3)$
$h'(0) = -21$

4. If $h(x)=g(x)\cdot f(x)$, find $h'(0)$

$h'(0) = g(0)\cdot f'(0)+f(0)\cdot g'(0)$
$h'(0) = 9\cdot 9+5\cdot -3$
$h'(0) = 81-15$
$h'(0) = 66$

5. If $h(x)=\frac{f(x)}{g(x)}$, find $h'(0)$

$h'(0) = \frac{g(0)f'(0)-f(0)g'(0)}{(g(0))^2}$
$h'(0) = \frac{9(9)-5(-3)}{9^2} = \frac{81+15}{81} = \frac{96}{81} = \frac{32}{27}$

6. If $h(x)=-4f(x)\cdot g(x)$, find $h'(-2)$

$h'(-2) = -4f(-2)g'(-2)+g(-2)(-4)f'(-2)$
$= -4(3)(8)+(-5)(-4)(1)$
$= -96+20$
$= -76$

7. If $h(x)=f(g(x))$, find $h'(1)$

$h'(1) = f'(g(1))\cdot g'(1)$
$= f'(2)\cdot g'(1)$
$= 3\cdot 6$
$= 18$

8. If $j(x)=g(f(x))$. Find j'(2)?

$j'(2) = g'(f(2))\cdot f'(2)$
$= g'(-5)\cdot f'(2)$
$= 3g'(-5)$

9. If $h(x)=x^2g(x)$, find $h'(-2)$

$h'(x) = x^2g'(x)+g(x)\cdot 2x$
$h'(-2) = 4g'(-2)+g(-2)\cdot -4$
$= 4(8)+(-5)(-4)$
$= 52$

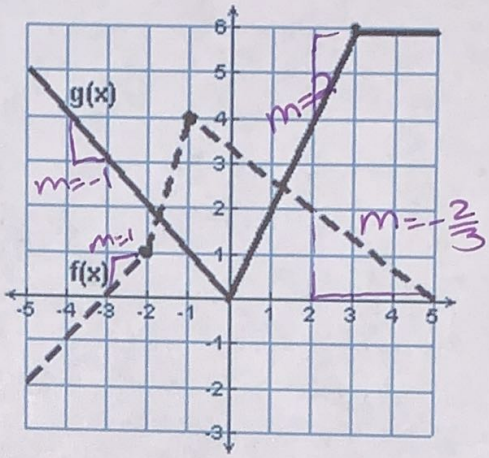
10– 15 Use the graph to find the derivative.

10. If $K(x) = f(x) + g(x)$, find $K'(-3)$

$K'(-3) = f'(-3) + g'(-3)$

$K'(-3) = 1 + -1$

$K'(-3) = 0$



11. If $K(x) = f(x) \cdot g(x)$, find $K'(2)$

$K'(2) = f(2) \cdot g'(2) + g(2) \cdot f'(2)$

$K'(2) = 2 \cdot 2 + 4 \cdot -\frac{2}{3}$

$K'(2) = 4 - \frac{8}{3}$

$K'(2) = \frac{4}{3}$

12. If $K(x) = \dfrac{f(x)}{g(x)}$, find $K'(2)$

$K'(2) = \dfrac{g(2)f'(2) - f(2)g'(2)}{[g(2)]^2}$

$K'(2) = \dfrac{4(-2/3) - (2)(2)}{4^2}$

$K'(2) = -\frac{20}{3} \cdot \frac{1}{16}$

$K'(2) = \frac{-5}{12}$

13. If $K(x) = f(g(x))$, find $K'(1)$

$K'(1) = f'(g(1)) \cdot g'(1)$

$K'(1) = f'(2) \cdot g'(1)$

$K'(1) = -\frac{2}{3} \cdot 2$

$K'(1) = -\frac{4}{3}$

14. If $K(x) = (g \circ f)(x)$, find $K'(-4)$

$K(x) = g(f(x))$

$K'(-4) = g'(f(-4)) \cdot f'(-4)$

$K'(-4) = g'(-1) \cdot f'(-4)$

$K'(-4) = -1 \cdot 1$

$K'(-4) = -1$

15. If $K(x) = f(x) - g(x)$, find $K'(0)$

$K'(0) = f'(0) - g'(0)$

$K'(0) = -2/3 - \text{DNE}$

DNE