Algebra 2 Unit 1: Quadratics Revisited

I CAN:

- Simplify, add and subtract complex numbers, including simplifying radicals with imaginary roots.
- Multiply and divide complex numbers, including rationalizing the denominator.
- Solve quadratic equations using the following methods:
  - Square Root Property
  - Factoring
  - Completing the Square
  - Quadratic Formula (and use Discriminant to determine # & type of solutions)

Monday	Tuesday	Wednesday	Thursday	Friday
3	4	5 DAY 1 Simplifying, Adding & Subtracting Complex Numbers	6 DAY 2 Multiplying & Dividing with Complex Numbers	7 DAY 3 Solving Quadratic Equations using Square Roots
10 DAY 4 Review and **Quiz 1: Operating and Solving with Complex Numbers**	11 DAY 5 Factoring GCF and trinomials with a=1	12 DAY 6 Factoring trinomials with a>1	13 DAY 7 Factoring with special patterns / Review	14 DAY 8 **Quiz 2: Factoring**
17 No Classes MLK Holiday	18 DAY 9 Solving Quadratic Equations by Factoring	19 DAY 10 Solving by Completing the Square	20 DAY 11 Solving by Quadratic Formula & Using Discriminant to Predict Types of Solutions	21 DAY 12 Solving: Choose the Best Method
24 DAY 13 Unit 1 Review	25 DAY 14 **Unit 1 Test**	26	27	28

THIS PLAN IS SUBJECT TO CHANGE. PLEASE REFER TO MY BLOG FOR UPDATES.

Complex Numbers

$$i = \sqrt{-1}$$

Examples:

1. $\sqrt{-16}$
2. $\sqrt{-81}$
3. $\sqrt{-45}$
4. $\sqrt{-200}$

"I one, I one"

Negatives in the middle.

- $i^1 =$ ______ , **remainder =** ______
- $i^2 =$ ______ , **remainder =** ______
- $i^3 =$ ______ , **remainder =** ______
- $i^4 =$ ______ , **remainder =** ______

Examples:

5. i^{13}
6. i^{27}
7. i^{54}
8. i^{72}

Add and Subtract Complex Numbers

◎ Add or subtract the real parts, and then, add or subtract the imaginary parts.

9. $(3+2i)+(7+6i)$
10. $(6-5i)-(1+2i)$
11. $(9-4i)-(-2+3i)$
12. $9-(10+2i)-5i$
13. $(11i^4+4i^3)-(2i^4-6i^3)$

Multiplying Complex Numbers

- Treat the i's like variables, then change any that are not to the first power.

Examples:

14. $-i(3+i)$

15. $(2+3i)(-6-2i)$

16. $(-3+i)(8+5i)$

17. $(4+3i)(4-3i)$

18. $-2i(1+4i)$

19. $(3-2i)(-5-9i)$

Conjugates

- Two complex numbers of the form a + b*i* and a – b*i* are complex conjugates.
- The product is always a real number.

Example $(2+4i)(2-4i)$

Dividing Complex Numbers

- Multiply the numerator and denominator by the conjugate of the denominator.
- Simplify completely.

Examples:

Write each expression as a complex number in standard form.

20. $\frac{5-2i}{3+8i}$

21. $\frac{3+11i}{-1-2i}$

22. $\frac{5}{1+i}$

23. $\frac{8+3i}{1-2i}$

24. $\frac{6-3i}{2i}$

25. $\frac{5+6i}{-3i}$

Complex Numbers – Practice

Write each expression in standard form.

1. i^2
2. i^{14}
3. i^7
4. i^{23}

5. $(4-2i)+(3-i)$
6. $(9+i^2)+i^2$
7. $(-2+i)+(-10-14i)$

8. $5-(15+i)-(3-5i)$
9. $7i+(12+2i)+4$
10. $(4i^2)+8i^2$

11. $(-2i)^2(3i)^3$
12. $(5-6i)(7-2i)$
13. $(2+3i)(2-3i)-4+i$

14. $\dfrac{2}{3-i}$
15. $\dfrac{-4i}{3-3i}$
16. $\dfrac{4+2i}{2+4i}$

15. Let r = (4 + i) and s = 1 – i. What is the value of $r^2 - s$?

a. 14 + i b. 15 + I c. 15 + 7i d. 14 + 9i

Simplifying Complex Radicals

Simplify.

1. $\sqrt{-1}$	2. $\sqrt{-4}$
3. $\sqrt{-16}$	4. $\sqrt{-25}$
5. $\sqrt{-50}$	6. $\sqrt{-72}$
7. $\sqrt{-80}$	8. $\sqrt{-98}$
9. $\sqrt{-108}$	10. $\sqrt{-125}$
11. $\sqrt{-96}$	12. $\sqrt{-500}$
13. $\sqrt{-52}$	14. $\sqrt{-147}$

Solving Quadratic Equations Using Square Roots

Solving Quadratic Equations Using Square Roots

1. Get x^2 by itself.
2. Take the square root of both sides of the equation.
3. There will ALWAYS be a positive answer and a negative answer.
4. Check your answers!!!

Solve each equation.

1. $x^2 + 4 = 0$
2. $\frac{1}{2}x^2 + 3 = 12$
3. $2(x^2 - 5) = -x^2 - 1$
4. $\frac{1}{3}(x+4)^2 - 1 = 5$
5. $4(x+5)^2 = -64$
6. $2x^2 + 338 = 0$
7. $5(x-4)^2 = 125$
8. $\frac{1}{7}x^2 - 3 = 4$
9. $-\frac{3}{5}x^2 - 2 = -5$
10. $-9x^2 = 243$

Solving Quadratics by Using Square Roots – HW

Solve each quadratic equation.

1. $x^2 + 4 = 29$
2. $3x^2 - 7 = 47$
3. $x^2 + 11 = 16$
4. $(x+4)^2 = 121$
5. $(2x-3)^2 = 9$
6. $(x-7)^2 = 99$
7. $(x+3)^2 + 6 = 18$
8. $(2x+6)^2 - 8 = 24$
9. $x^2 + 21 = 5$
10. $3(x+4)^2 = -9$
11. $9(x^2 + 1) = 9$
12. $(x-5)^2 = -14$
13. $(r-1)^2 = -28$
14. $-3(x-8)^2 = 18$
15. $7(2x+3)^2 + 38 = 101$

Quiz 1 Review: Complex Numbers

Date_______________ Period____

Simplify.

1) $(2i) - (8 + 7i)$

2) $(1 - 5i) + (1 - i)$

3) $(5 + 4i) - (2i) - 5$

4) $(6i)(1 + 5i)$

5) $(1 + 5i)^2$

6) $(2 - 5i)(-6 + i)$

7) $\frac{3 + 6i}{9i}$

8) $\frac{-3 - 5i}{1 + 6i}$

9) $\frac{6 + 7i}{-2 - 8i}$

10) $\frac{4 + 6i}{7 - 6i}$

11) $\sqrt{-25}$

12) $\sqrt{-32}$

13) $\sqrt{-54}$

14) $\sqrt{-40}$

Solve each equation by taking square roots.

15) $r^2 = -63$

16) $b^2 - 6 = 3$

17) $3(v - 1)^2 = 15$

18) $9b^2 - 10 = -157$

Solving Quadratic Equations by Factoring

How to Factor

Warm up: Recall multiplying polynomials…DISTRIBUTE TO MULTIPLY.

a) $2x(4x-3)$

b) $(x+5)(x+2)$

The process of FACTORING is the reverse of the process of distributing.
The goal is to write an expression that is equivalent to the original.
To FACTOR, DIVIDE and "undistribute" any common factors.

GREATEST COMMON FACTOR

For every factoring problem, you should begin by looking for a ________________.

Ex 1: Factor each expression.

a. $2x^2+8x$

b. $15x^3-35x$

c. $12x^4-4x^2$

THREE TERMS – SUM & PRODUCT STRATEGIES

When $\boldsymbol{a}=\mathbf{1}$: GUESS & CHECK

If the trinomial is a quadratic expression in standard form, _________________________________, AND $\boldsymbol{a}=\mathbf{1}$, find two factors of ____ which have a sum equal to ____: then write the quadratic as the product of two binomial factors $(x+p)(x+q)$.

Ex 2: Factor each trinomial

a. $x^2+7x+12$

product

sum

b. $x^2-10x+25$

c. $2x^2+4x-70$

d. $5x^2-20x-225$

When $a > 1$: Rewrite the middle term & use the grouping method.

1) Multiply $a \cdot c$
2) Find two factors of $a \cdot c$ that have a sum equal to b
3) Replace the middle term with the two terms you found in step 2 (don't forget x)
4) Group the first 2 terms & group the last 2 terms.
5) Take out the common factors from each group.
6) Since the remaining parentheses are now identical, factor it out so you have two binomial factors: $(x+p)(x+q)$

Ex 3: Factor each trinomial

a. $2x^2 - 9x - 18$

b. $8x^2 - 30x + 7$

c. $6x^2 - 5x - 4$

d. $3x^2 - 20x + 32$

Factoring with Special Patterns

Describe or explain the meaning of "perfect square."
Can variables be perfect squares?
Give some examples of perfect squares:

RECALL - Multiply each pair of binomials by double distributing: FOIL	
1. $(x+8)(x+8)$	2. $(x+8)(x-8)$
3. $(x-5)(x-5)$	4. $(x-5)(x+5)$
What patterns do you notice?	

Perfect Square Trinomial

$a^2 + 2ab + b^2 =$ ____________________

$=$ ____________________

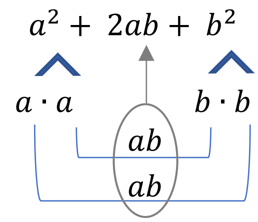
OR

$a^2 - 2ab + b^2 =$ ____________________

$=$ ____________________

When can I use this factoring pattern?

- The expression has ______ terms
- The first and last terms are ____________________________
- The middle term fits the pattern _________ or __________



Ex 3: Factor

a. $x^2 + 6x + 9$

b. $x^2 - 20x + 100$

c. $36x^2 - 12x + 1$

d. $4x^2 + 20x + 25$

Difference of Two Squares

$a^2 - b^2 =$ ____________________

When can I use this pattern to factor?

- The expression has _______ terms
- The terms are being __________________
- Each term is a ______________________

Ex 4: Factor

a. $x^2 - 4$

b. $x^2 - 121$

c. $25x^2 - 16$

d. $64x^2 - 1$

e. $36 - x^2$

f. $2x^2 - 162$

Always look for a ________ first!

Solving Quadratic Equations by Factoring

According to the Zero Product Property, if the product of two quantities is equal to zero, then one of the quantities must equal zero.

Step 1: Arrange terms in standard form
Step 2: Factor
Step 3: Set each factor = 0
Step 4: Solve each mini-equation

Recall: Factoring Strategies
- Look for a GCF first!
- 2 terms: Difference of Squares?
- 3 terms: Sum & Product or Grouping

Ex 6: Solve each equation by factoring.

a. $x^2 + 3x - 40 = 0$

b. $x^2 - 9x = 0$

c. $x^2 - 3x - 28 = 0$

d. $81x^2 - 100 = 0$

e. $2x^2 - 24x = -72$

f. $3x^2 - 8x + 4 = 0$

g. $6x + 16 = x^2 + 9$

h. $5x^2 + 20x + 20 = 0$

i. $15x^2 - 10x = 0$

j. $18x^2 + 25x - 3 = 0$

Factoring Practice

Date________________ Period____

Factor the common factor out of each expression.

1) $10r^3 + 10r^2$

2) $7x^5 - 10x^3$

3) $48m^5 + 48m^3 + 18m^2$

4) $90m + 18m^2 - 27m^6$

Factor each completely.

5) $v^2 + 2v - 24$

6) $b^2 - 5b - 24$

7) $n^2 + 9n + 20$

8) $x^2 - 11x + 28$

9) $3a^2 - 12a + 9$

10) $3x^2 - 3x - 60$

11) $5n^2 - 15n - 140$

12) $6n^2 - 72n + 120$

13) $7x^2 - 69x + 54$

14) $3r^2 + r - 10$

15) $7b^2 + 57b - 54$

16) $5x^2 - 47x + 56$

17) $10n^2 - 7n - 6$

18) $9k^2 + 21k - 8$

19) $4a^2 - 15a + 9$

20) $9a^2 + 34a - 8$

21) $16r^2 - 1$

22) $v^2 - 9$

23) $x^2 - 4$

24) $4a^2 - 9$

25) $n^2 - 10n + 25$

26) $9n^2 + 30n + 25$

Factoring Quadratics

1

look for a **gcf** first!

$15x^5 - 6x^2$

2 terms

Difference of Squares

$x^2 - 25$

$49x^2 - 1$

3 terms

Sum and Product

$x^2 + 10x + 16$

$2x^2 - 4x - 48$

Grouping

$3x^2 + 8x - 3$

$6x^2 - 11x + 3$

Quiz 2 Review: Factoring

Date________________ Period____

Factor completely.

1) $12x^4 - 15x^3$

2) $6x^7 + 24x^4$

Factor each completely.

3) $n^2 - 3n - 18$

4) $x^2 - 5x - 14$

5) $a^2 - 6a$

6) $x^2 - 8x - 9$

7) $5r^2 + 50r + 45$

8) $2b^2 - 7b + 3$

9) $5n^2 + 41n - 36$

10) $6k^2 - 29k + 30$

11) $x^2 - 1$

12) $x^2 - 16$

13) $9x^2 - 16$

14) $25r^2 - 9$

15) $4x^2 + 20x + 25$

16) $9x^2 + 6x + 1$

Solving Quadratic Equations with Factoring

Date________________

Solve each equation by factoring.

1) $(3a - 1)(a + 5) = 0$

2) $(m - 3)(m - 1) = 0$

3) $7n^2 - 448 = 0$

4) $6n^2 - 24n = 0$

5) $n^2 + 3n = 0$

6) $k^2 + 7k - 8 = 0$

7) $v^2 + 10v + 25 = 0$

8) $3a^2 - 21a + 30 = 0$

9) $b^2 - 11b + 24 = 0$

10) $3a^2 + 18a + 24 = 0$

11) $6v^2 + 9v - 27 = 0$

12) $2r^2 + 5r - 3 = 0$

13) $9x^2 + 21x - 18 = 0$

14) $2p^2 - 3p + 1 = 0$

Solving Quadratic Equations by Completing the Square

RECALL: Factor the following trinomials.

a. $x^2 + 4x + 4$	b. $x^2 - 8x + 16$	c. $x^2 - 20x + 100$
What pattern do you notice?		
Each of the quadratic expressions above is a ____.		

Ex 1: Find the value of c that makes each expression a perfect square trinomial.

a. $x^2 + 16x + c$ b. $x^2 - 12x + c$ c. $x^2 + 6x + c$

Ex 2: Write each perfect square trinomial in factored form, as a binomial squared.

a. $x^2 + 16x +$ ____ b. $x^2 - 12x +$ ____ c. $x^2 + 6x +$ ____

RECALL: We can solve some equations involving perfect square trinomials using the Square Root Property. $x^2 + 14x + 49 = 64$

What do we do when the left side is not a perfect square trinomial? **Solve by Completing the Square**	
1. Add/subtract the __________ to the other side. 2. Add $\left(\frac{b}{2}\right)^2$ to __________ sides of the equation. 3. Factor and write as a binomial _______________. 4. Take the ________________________ of both sides. ***Don't forget ______! 5. Isolate x. Then ADD & SUBTRACT to find BOTH solutions. 6. Check!	$x^2 + 4x - 12 = 0$

Ex 3: Solve each equation by completing the square.

a. $x^2 + 4x + 11 = 2$

b. $2x^2 - 4x - 10 = 20$

You Try: Solve by completing the square.	
1. $x^2 + 20x + 78 = 3$	2. $x^2 + 6x + 90 = 0$
3. $x^2 - 10x - 9 = 6$	4. $x^2 - 14x + 36 = -8$

Solving Quadratics by Completing the Square

Date________________ Period____

Solve each equation by completing the square.

1) $x^2 + 2x - 6 = 0$

2) $p^2 - 2p - 39 = 0$

3) $m^2 - 8m - 48 = 0$

4) $v^2 - 12v - 57 = 0$

5) $p^2 + 14p + 60 = -5$

6) $n^2 - 18n + 96 = 9$

7) $x^2 + 18x + 42 = -10$

8) $n^2 - 8n - 58 = 7$

9) $p^2 + 2p - 72 = -2$

10) $p^2 - 10p - 81 = -9$

11) $3x^2 + 18x + 15 = -6$

12) $10r^2 - 20r - 40 = -10$

The Quadratic Formula and Discriminant

For $ax^2 + bx + c = 0$, where $a \neq 0$,
the solutions are given by the Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Ex 1: Solve $\mathbf{x^2 - 10x = -21}$ using the specified method.	
a. Quadratic Formula	b. Factoring

Ex 2: Solve $\mathbf{x^2 + 6x + 9 = 0}$ using the specified method.	
a. Quadratic Formula	b. Square Root Property

Ex 3: Solve $\mathbf{2x^2 + 8x - 6 = 0}$ using the specified method.	
a. Quadratic Formula	b. Completing the Square

Ex 4: Solve $\boldsymbol{x^2 + 13 = 6x}$ using the Quadratic Formula.

Discriminant
The expression $b^2 - 4ac$ is called the ____________________________. The value of the discriminant can be used to determine the **number** and **type** of roots of a quadratic equation. • If $b^2 - 4ac$ is __________________, there are **2 real** roots. ➢If it is a <u>perfect square</u>, they are **<u>rational</u>**. ➢If it is <u>not a perfect square</u>, then they are **<u>irrational.</u>** • If $b^2 - 4ac$ is __________________, there is **1 real, rational** root. • If $b^2 - 4ac$ is __________________, there are **2 complex** roots.
Look back at Examples 1-4 and note the discriminant, number & type of solution(s). Ex 1: discriminant=______ # of solutions: _______ type of solutions: ________________ Ex 2: discriminant=______ # of solutions: _______ type of solutions: ________________ Ex 3: discriminant=______ # of solutions: _______ type of solutions: ________________ Ex 4: discriminant=______ # of solutions: _______ type of solutions: ________________

Ex 5: Find the value of the discriminant for each quadratic equation. Then describe the number and type of roots for the equation.

a. $x^2 + 3x + 5 = 0$ b. $x^2 - 11x + 10 = 0$ c. $-5x^2 + 8x - 1 = 0$

Quadratic Formula & Discriminant Practice

Date________________ Period____

Find the discriminant of each quadratic equation then state the number and type of solutions.

1) $-3p^2 - 10p - 3 = 0$

2) $4v^2 - v + 1 = 0$

3) $x^2 + 2x + 8 = 7$

4) $-4p^2 - 5p - 4 = 2$

Solve each equation with the quadratic formula.

5) $5x^2 - 12x + 9 = 0$

6) $4v^2 + 9v + 1 = 0$

7) $3k^2 + 10k + 12 = 4$

8) $3x^2 - 9x + 5 = -6$

9) $r^2 - 2r - 27 = -12$

10) $8a^2 - 8a = -5$

Solving Quadratic Equations Using All Four Methods

Solve each equation by completing the square.

Ex. 1 $2r^2 - 8r - 6 = 0$ $r^2 - 4r - 3 = 0$ $+3 \quad +3$ $r^2 - 4r + [\,4\,] = 3 + [\,4\,]$ $(r-2)^2 = 7$ $r - 2 = \pm\sqrt{7}$ $r = 2 \pm\sqrt{7}$ $\{2-\sqrt{7},\ 2+\sqrt{7}\}$	1. $b^2 + 2b + 11 = 0$ 2. $b^2 - 10b + 21 = 0$ 3. $x^2 + 2x + 15 = 0$ 4. $n^2 - 8n + 20 = 0$ 5. $3x^2 - 6x - 9 = 0$

Solve each equation by factoring.

Ex. 2 $3x^2 + x - 2 = 0$ $3x^2 - 2x + 3x - 2 = 0$ $(3x^2 - 2x) + (3x - 2) = 0$ $x(3x-2) + 1(3x-2) = 0$ $(3x-2)(x+1) = 0$ (-6 | 1; 1 · 6 | 5; -2 · 3 | 1) $3x - 2 = 0$ $\quad$ $x + 1 = 0$ $3x = 2$ $\quad$ $x = -1$ $x = 2/3$ $\{2/3, -1\}$	6. $x^2 - 6x = 0$ 7. $a^2 - 2a - 35 = 0$ 8. $2n^2 + 5n - 3 = 0$ 9. $2x^2 = -3x + 20$ 10. $21a^2 + 42a = -21$

Solve each equation by taking square roots.

Ex. 3 $6x^2 - 3 = -27$ $6x^2 = -24$ $x^2 = -4$ $x = \sqrt{-4}$ $x = \pm 2i$ $\{-2i, 2i\}$	11. $r^2 = -63$
	12. $x^2 + 3 = 56$
	13. $4(n-1)^2 = 8$
	14. $6x^2 + 1 = 55$
	15. $9(b+7)^2 = -36$

Solve each equation with the Quadratic Formula.

Ex. 4 $4n^2 - 8n = -1$ $4n^2 - 8n + 1 = 0$ $x = \frac{8 \pm \sqrt{(-8) - 4(4)(1)}}{2(4)}$ $x = \frac{8 \pm \sqrt{48}}{8} = \frac{8 \pm 4\sqrt{3}}{8}$ $x = \frac{2 \pm \sqrt{3}}{2}$	16. $x^2 - 3x - 4 = 0$
	17. $3r^2 + 2r + 2 = 0$
	18. $a^2 - 9 = 0$
	19. $2m^2 + 5m + 2 = 0$
	20. $3m^2 + 4m = 1$

Solve. Choose the best method for each, using each method at least once.

Which method should I choose? **Square Root Method** is best when there is **no b-term**. ex: $5x^2 - 15 = 0$ **Factoring** is best when you can easily find the correct sum/product pair or special pattern. *If b^2-4ac is a perfect square, the expression is **factorable**!* ex: $x^2 + 7x + 12 = 0$ **Completing the Square** is best when **a = 1** and **b is an even number**. ex: $x^2 + 6x + 2 = 4$ **Quadratic Formula** solves **any** quadratic equation!	21. $2x^2 + 32 = 0$ 22. $5v^2 + 2v - 2 = 0$ 23. $x^2 - 17x + 60 = 0$ 24. $r^2 - 12r + 52 = 0$ 25. $3x^2 - 48 = 0$

Unit 1 Review

Simplify.

1. $\sqrt{-72}$

2. $\sqrt{-99}$

3. $(3 + 2i) - (4 - 6i)$

4. $(2 - i)(3 + 4i)$

5. $(-4i + 2) + (6 - 5i)$

6. $\frac{3-2i}{4+i}$

Factor.

7. $x^2 + 19x + 90$

8. $3x^2 - 13x + 12$

9. $15x^2 - x - 2$

10. $x^2 - 121$

11. $4x^2 - 25$

12. $5x^2 + 20x + 15$

13. $x^2 - 20x + 100$

Solve by using square root.

14. $2x^2 - 162 = 0$

15. $x^2 + 1 = 3x^2 - 13$

16. $(x + 1)^2 = 7$

Solve by factoring.

17. $x^2 + x - 30 = 0$

18. $x^2 = 12x - 32$

19. $3x^2 + 5x = x^2 - 2x + 4$

Solve by using the quadratic formula.

20. $5x^2 - 20x + 20 = 0$

21. $2x^2 - 98 = 0$

22. $3x^2 - x = -5$

Solve by completing the square.

23. $x^2 - 4x + 12 = 0$

24. $2x^2 - 4x - 4 = 0$

25. $x^2 + 6x + 48 = 0$