Find ALL Roots/Zeros

Step 1: List all possible rational roots (p/q)

Step 2: Test possible roots using synthetic division until you find a zero.

Step 3: Use synthetic division until you have a quotient that is quadratic. Solve the quadratic equation using any method to find the remaining zeros.
***Remember that your degree will match the number of roots you should have!

9. $f(x)=x^4+7x^3+9x^2-27x-54$

$x=-3$ $x=2$

x^4 x^3 x^2

	1	7	9	−27	−54
−3		−3	−12	9	54
	1	4	−3	−18	: 0
2		2	12	18	
	1	6	9	: 0	

$(x^3+4x^2-3x-18)$

$x^2(x+4)-3(x+6)$

$x^2+6x+9=0$

$(x+3)(x+3)=0$

$x=-3$

24. $1x^4-3x^3+3x^2-3x+2=0$ (Q = 1, P = 2)

P/Q: $\pm 1, \pm 2$

$(x^3-2x^2)+(x-2)=0$

$x^2(x-2)+1(x-2)=0$

$(x^2+1)(x-2)$

	1	-3	3	-3	2
1 ↓		1	-2	1	-2
2	1	-2	1	-2	: 0
↓		2	0	2	
	1	0	1	: 0	

$x^2+1=0$

$\sqrt{x^2}=\pm\sqrt{-1}$

$x=\pm i$

Find all zeros

1. $f(x)=1x^3-7x^2+14x-8$ (Q = 1, P = 8)

P/Q: $\pm 1, \pm 2, \pm 4, \pm 8$

	1	-7	14	-8
1 ↓		1	-6	8
	1	-6	8	: 0

$x^2-6x+8=0$

$(x-4)(x-2)=0$

$x=4 \quad x=2 \quad x=1$

Find all roots.

2. $f(x) = 2x^4 + 5x^3 - 5x^2 - 5x + 3$

3. Find all roots of $y = 1x^3 + 4x^2 + 5x + 20$ (Q = 1, P = 20)

P/q: $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

calc: $x = -4$, $x = \pm i\sqrt{5}$

$-4 \mid 1 \;\; 4 \;\; 5 \;\; 20$

$\downarrow \;\; -4 \;\; 0 \;\; -20$

$1 \;\; 0 \;\; 5 \;\; 0$

$x^2 + 5 = 0$

$\sqrt{x^2} = \sqrt{-5}$

$x = \pm i\sqrt{5}$

4. Find all solutions to $x^4 - 3x^3 + 6x^2 - 12x + 8 = 0$

P/Q : $\pm 1, \pm 2, \pm 4, \pm 8$

need 2 zeros from calc to do synth ÷ twice to get to x^2

calc: 1 & 2

x^4	2	1	-3	6	-12	8
		↓ 2	-2	8	-8
x^3	1	1	-1	4	-4 : 0
		↓ 1	0	4	
x^2		1	0	4 : 0	

$x^2 + 4 = 0$

$\sqrt{x^2} = \sqrt{-4}$

$x = \pm 2i, 1, 2$

5. Find all roots of the polynomial function

$f(x) = x^4 - 4x^3 + x^2 + 8x - 6$

6. Find all zeros $f(x) = x^5 + x^4 + 5x^3 + 5x^2$

$f(x) = x^2(x^3 + x^2 + 5x + 5)$

$= x^2[x^2(x+1) + 5(x+1)]$

$f(x) = x^2(x^2+5)(x+1)$

$x^2 = 0$

$x = 0$ mult. 2

$x^2 + 5 = 0$

$x^2 = -5$

$x = \pm i\sqrt{5}$

$x + 1 = 0$

$x = -1$

7. Find all zeros of the function by factoring.

$f(x) = x^4 + 34x^2 + 225$ → $(x+5i)(x-5i)$

$0 = (x^2+25)(x^2+9)$

$x^2 + 25 = 0$

$x^2 = -25$

$x = \pm 5i$

$x^2 + 9 = 0$

$x^2 = -9$

$x = \pm 3i$

Classwork/homework~

- I can apply the Rational Root and Remainder Theorem to find solutions to polynomials equations.

Quiz:

- Find zeros, multiplicity, end behavior, degree & sign of leading coefficient from equation or graph
- Write factored form from graph
- Factor & solve (grouping, cubes, quartic)
- Given a factor find other factors
- Write equations given roots
- List possible rational roots
- Find All roots

HW:
p.9 #20-21
p.11-12 all
p.13 # 15+18
but change #18 to $3i, \sqrt{2}$