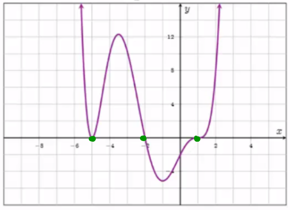
Figure 1



Warm up

1. Using figure 1, list the zeros & the multiplicity of each zero.

-5 mult. 2 (bounces)
-2 mult. 1 (crosses)
1 mult. 3 (flattens)
Te degree

2. Write a possible polynomial for the graph in figure 1 in FACTORED FORM.

$f(x) = (x+5)^2(x+2)(x-1)^3$

3. Given x = -4 and x = 2i, $+-2i$ write a possible polynomial with a lead coefficient of -3. Write in STANDARD FORM. (Expanded)

$y = -3(x+4)(x-2i)(x+2i)$

$x^2 - 4i^2$
$x^2 - 4(-1)$
$x + 4$

$y = (-3x-12)(x^2+4)$

$\boxed{y = -3x^3 - 12x^2 - 12x - 48}$

6. $x = 2 \quad x = 3i \quad x = -3i$

$y = (x-2)(x-3i)(x+3i)$
$y = (x-2)(x^2+9i^{\cancel{2}})$
$y = x^3 - 2x^2 + 9x - 18$

Know your VO....(vocabulary)

	$x=2$	$x=3i$	$x=-\sqrt{5}$
What is a **zero**?	2	$3i$	$-\sqrt{5}$
x-int.	$(2,0)$ $f(2)=0$		
What is a **root**?	2	$3i$	$-\sqrt{5}$
What is a **factor**?	$x-2$	$x-3i$	$x+\sqrt{5}$

The following statements are equivalent:
A real number r is a root of the polynomial equation $P(x) = 0$.
$P(r) = 0$
r is an x-intercept of the graph of $P(x)$.
$x - r$ is a factor of $P(x)$.
When you divide the rule for $P(x)$ by $x - r$, the remainder is 0.
r is a zero of $P(x)$.

Factor Theorem:

For any polynomial p(x), (x - a) is a factor of P(x) if and only if P(a) = 0

**** If a number is divided by any of its factors the remainder is 0. Likewise, if a polynomial is divided by any of its factors, the remainder is 0.**

x y (2,0)

If f(2) = 0 for some function f(x), then __2__ is a root and __x-2__ is a factor.

If f(3) = 5 for some functions f(x), then 3 is/is not a root.

(3,5) not on x-axis.

Determine whether the given binomial is a factor of p(x).

factor

$(x - 3)$; $p(x) = x^2 + 2x - 3$

zero = 3

$$\begin{array}{c|ccc} 3 & 1 & 2 & -3 \\ & \downarrow & 3 & 15 \\ \hline & 1 & 5 & \vdots\ 12 \leftarrow \text{remainder} \end{array}$$

No; x-3 isn't a factor bc the remainder isn't 0

Determine whether the given binomial is a factor of p(x).

$$(x + 4); \quad p(x) = 2x^4 + 8x^3 \overset{0x^2}{\swarrow} + 2x + 8$$

$$\begin{array}{r|rrrrr} -4 & 2 & 8 & 0 & 2 & 8 \\ & \downarrow & -8 & 0 & 0 & -8 \\ \hline & 2 & 0 & 0 & 2 & \vdots\, 0 \checkmark \end{array}$$

Yes, x+4 is a factor bc you have a remainder = 0

Remainder Theorem:

The remainder, r, of the quotient of a polynomial divided by a linear polymonial is equal to P(r).

Use the Remainder Theorem to evaluate:

a. $p(x) = x^3 - 7x - 6$ when $x = 4$

$$\begin{array}{r|rrrr} 4 & 1 & 0 & -7 & -6 \\ & \downarrow & 4 & 16 & 36 \\ \hline & 1 & 4 & 9 & :30 \end{array}$$

$p(4) = 30$
or
$(4, 30)$

b. $f(\overset{x}{3}) = 6x^3 - 5x^2 + 4x - 17$

$$\begin{array}{r|rrrr} 3 & 6 & -5 & 4 & -17 \\ & \downarrow & 18 & 39 & 129 \\ \hline & 6 & 13 & 43 & :112 \end{array}$$

$f(3) = 112$
or
$(3, 112)$

Use the Remainder or Factor Theorem

1. Given that x + 7 is a factor of $3x^3 + 13x^2 - 52x + 28 = 0$, factor the polynomial completely.

$$\begin{array}{r|rrrr} -7 & 3x^3 & 13 & -52 & 28 \\ & \downarrow & -21 & 56 & -28 \\ \hline & 3 & -8 & 4 & :0 \end{array}$$

$(x+7)(3x^2 - 8x + 4)$

↑ factor

12
-6 -2
-8

$(3x^2 - 6x)(-2x + 4)$

$3x(x-2) - 2(x-2)$

$(x+7)(3x-2)(x-2)$

2. Given that x - 3 is a factor of $f(x) = x^3 + x^2 - 8x - 12$, factor the polynomial completely.

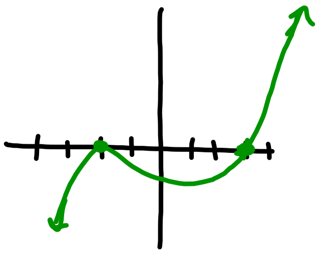
$x - 3 = 0$
$x = 3$

$$\begin{array}{r|rrrr} 3 & 1 & 1 & -8 & -12 \\ & \downarrow & 3 & 12 & 12 \\ \hline & 1 & 4 & 4 & :0 \end{array}$$

$(x-3)(x^2 + 4x + 4) = 0$

$(x-3)(x+2)(x+2)$

$x = 3$ mult 1
$x = -2$ mult 2



$x=2$

3. Given that 2 is a root of $x^3 - 4x^2 + 8 = 0$, find the other roots of the polynomial.

$$\begin{array}{r|rrrr} \boxed{2} & 1 & -4 & 0 & 8 \\ & \downarrow & 2 & -4 & -8 \\ \hline & 1 & -2 & -4 & \vdots\, 0 \end{array}$$

Once quadratic use quad. $x^2-2x-4=0$

formula, comp. the square, sq rt method, or factor.

Quad. Formula: $a=1$ $b=-2$ $c=-4$

$$x=\frac{2\pm\sqrt{(-2)^2-4(1)(-4)}}{2(1)}$$

$$x=\frac{2\pm\sqrt{20}}{2}=\frac{2\pm2\sqrt{5}}{2}$$

$$\boxed{1\pm\sqrt{5}}$$

4. Given that $f(-4/3)=0$ and $f(x)= 3x^3 + 4x^2 + 12x + 16$, find all of the zeros of the polynomial.

$$\begin{array}{r|rrrr} -\frac{4}{3} & 3x^3 & 4 & 12 & 16 \\ & \downarrow & -4 & 0 & -16 \\ \hline & 3x^2 & 0 & 12 & \vdots\, 0 ✓ \end{array}$$

$-\frac{4}{\cancel{3}}\cdot\overset{4}{\cancel{12}}$

$3x^2+12=0$

Square Rt method $b=0$

$3x^2=-12$

$\sqrt{x^2}=\pm\sqrt{-4}$

$$\boxed{x=\pm2i \;\&\; -4/3}$$

CHALLENGE: Given $-\sqrt{5}$ is a zero find the other zeros. $x^4+x^3+3x^2+5x-10=0$

$x=-\sqrt{5}$ $x=\sqrt{5}$

$(x+\sqrt{5})(x-\sqrt{5}) = x^2-5 \leftarrow$ factor

$$x^2+0x-5 \,\overline{\smash{)}\, x^4+x^3+3x^2+5x-10}$$

Rational Root Theorem

If $f(x) = a_nx^n + \ldots + a_1x + a_0$ has integer coefficients, then every rational zero of f has the following form:

$$\frac{p}{q} = \pm\frac{\text{Factors of constant term } a_0}{\text{Factors of leading coefficient } a_n}$$

List all possible rational zeros/roots

a. $f(x)=\overset{Q}{1}x^3 -9x^2 + 8x + \overset{P}{12}$

p: ±1, ±2, ±3, ±4, ±6, ±12

q: ±1

$P/q = \pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

b. $f(x) = \overset{Q}{2}x^4 - 9x^2 + x \overset{P}{-18}$

p: -18 ±1, ±2, ±3, ±6, ±9, ±18

q: 2 ±1, ±2

$P/q = \pm1, \pm2, \pm3, \pm6, \pm9, \pm18, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}$

c. $f(x) = \overset{q}{3}x^2 - 8x + 24$

p: ±1, ±2, ±3, ±4, ±6, ±8, ±12, ±24

q: ±1, ±3

$P/q = \pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$
$\pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}$