Keeper 3.2 – The Definition of the Derivative

Virtual Problems

DEFINITION OF A DERIVATIVE

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

1. Find the slope of the tangent line at $x = 3$

$f(x) = \frac{5}{3x-4}$ $\quad f(3) = 1$

$\lim_{x\to 3} \frac{\frac{5}{3x-4} - 1}{x-3}$

$\lim_{x\to 3} \frac{5-3x+4}{3x-4} \cdot \frac{1}{x-3}$

$\lim_{x\to 3} \frac{-3}{3x-4} = -\frac{3}{5}$

2. Find the slope of the normal line at $x = 5$

$f(x) = \sqrt{2x-1}$ $\quad f(5) = 3$

$\lim_{x\to 5} \frac{\sqrt{2x-1}-3}{x-5}$

$\lim_{x\to 5} \frac{2x-1-9}{(x-5)(\sqrt{2x-1}+3)}$

$\lim_{x\to 5} \frac{2}{\sqrt{2x-1}+3} = \frac{2}{\sqrt{9}+3} = \frac{1}{3}$

$m_{nom} = -3$

3. Find the derivative:

$f(x) = x^2 + 2$

$f'(x) = \lim_{h\to 0} \frac{(x+h)^2+2-(x^2+2)}{h}$

$f'(x) = \lim_{h\to 0} \frac{x^2+2xh+h^2+2-x^2-2}{h}$

$f'(x) = \lim_{h\to 0} 2x+h = 2x+0$

$f'(x) = 2x$

4. Find the derivative:

$f(x) = \frac{1}{x}$

$f'(x) = \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$

$f'(x) = \lim_{h\to 0} \frac{x-x-h}{x(x+h)} \cdot \frac{1}{h}$

$f'(x) = \lim_{h\to 0} \frac{-1}{x(x+h)} = \frac{-1}{x(x+0)}$

$f'(x) = \frac{-1}{x^2}$

5. Find the derivative:

$f(x) = 2\sqrt{x+3}$

$f'(x) = \lim_{h\to 0} \frac{2\sqrt{x+h+3} - 2\sqrt{x+3}}{h}$

$f'(x) = \lim_{h\to 0}$

6. Find the derivative:

$f(x) = \frac{1}{\sqrt{x+1}}$

$f'(x) = \frac{-1}{2(x+1)^{3/2}}$

$f'(x) = \lim_{h\to 0} \frac{\frac{1}{\sqrt{x+h+1}} - \frac{1}{\sqrt{x+1}}}{h}$

$f'(x) = \lim_{h\to 0} \frac{\frac{\sqrt{x+1}-\sqrt{x+h+1}}{\sqrt{x+1}\sqrt{x+h+1}}}{h}$

$f'(x) = \lim_{h\to 0} \frac{x+1-x-h-1}{\sqrt{x+1}\sqrt{x+h+1}\cdot h(\sqrt{x+1}+\sqrt{x+h+1})}$

$f'(x) = \lim_{h\to 0} \frac{-1}{\sqrt{x+1}\sqrt{x+h+1}(\sqrt{x+1}+\sqrt{x+h+1})}$

$f'(x) = \frac{-1}{\sqrt{x+1}\sqrt{x+1}(\sqrt{x+1}+\sqrt{x+1})} = \frac{-1}{(x+1)2\sqrt{x+1}} = \frac{-1}{2(x+1)^{3/2}}$

Example: Derivative from a chart

7. The traffic speed S along a certain road (in mph) varies as a function of traffic density q (number of cars per mile on the road). Estimate the instantaneous rate of change at $q = 110$.

q (density) cars per mile	100	110	120	130	140
S (Speed) mph	45	42	39.5	37	35

$$m_{sec(100-110)} = \frac{42-45}{110-100} = \frac{-3}{10} = -.3$$

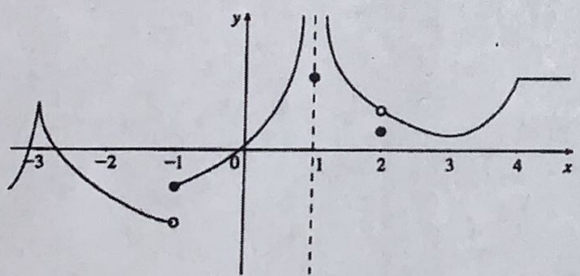
$$m_{sec(120-110)} = \frac{39.5-42}{120-110} = \frac{-2.5}{10} = -.25$$

$$m_{avg} = \frac{-.3 + -.25}{2} = \frac{-.55}{2} = -.275$$

$$m_{tan} \approx -.275 \text{ mph/car per mile}$$

Examples: State the x values where f is not differentiable and the reason.

8.



Reasons not differentiable
- Removable
- Infinite
- Jump

} Discontinuities

- Cusp
- Corner
- Vertical Tangents

x = -3 Cusp

x = -1 Jump

x = 1 Infinite discontinuity

x = 2 Removable discontinuity

x = 4 Corner