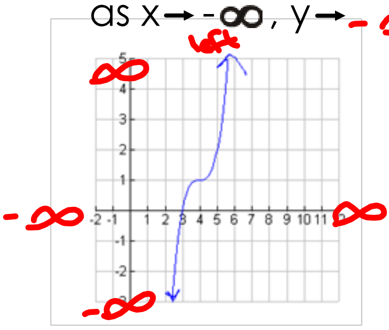
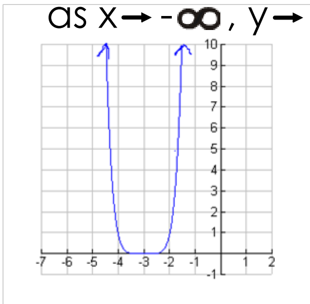
Warm up: State the end behavior of the graphs.

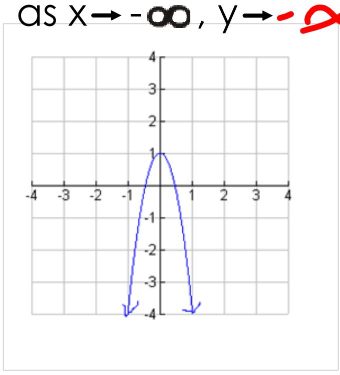
a. as x → ∞ (right), y → ∞
as x → -∞ (left), y → -∞



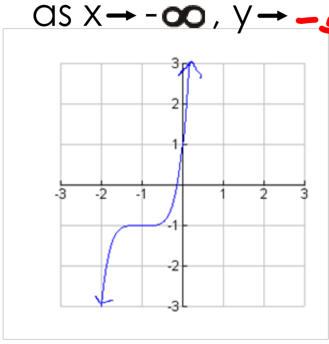
b. as x → ∞, y → ∞
as x → -∞, y → ∞



c. as x → ∞, y → -∞
as x → -∞, y → -∞



d. as x → ∞, y → ∞
as x → -∞, y → -∞



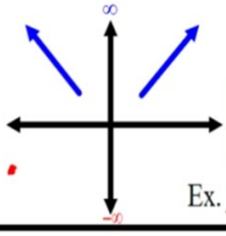
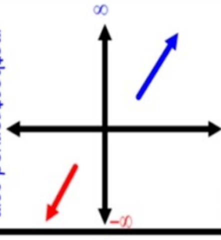
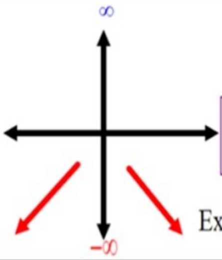
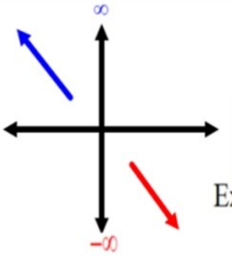
Graphs of polynomial functions of higher degrees

*Graphs are continuous and can be drawn without picking up your pencil. No jumps or holes.

*Graphs are smooth curves so there are no sharp turns (like absolute value)

*the y-intercept is the constant term

End Behavior of Polynomials

Degree / Sign of Leading Coefficient	Degree is Even Ex. $\begin{cases} y=-3x+2x^2+5 \\ y=-3x^6+3x^3+5 \end{cases}$	Degree is Odd Ex. $\begin{cases} y=2x^5-3x+4 \\ y=-2x^7-x+4 \end{cases}$
Leading Coefficient is Positive (+) Ex. $\begin{cases} y=-3x+2x^2+5 \\ y=2x^5-3x+4 \end{cases}$	 rises to left rises to right $y \to \infty$ as $x \to -\infty$ $y \to \infty$ as $x \to \infty$ Ex. $y=-3x+2x^2+5$	 falls to left rises to right $y \to -\infty$ as $x \to -\infty$ $y \to \infty$ as $x \to \infty$ Ex. $y=2x^5-3x+4$
Leading Coefficient is Negative (+) Ex. $\begin{cases} y=-3x^6+3x^3+5 \\ y=-2x^7-x+4 \end{cases}$	 falls to left falls to right $y \to -\infty$ as $x \to -\infty$ $y \to -\infty$ as $x \to \infty$ Ex. $y=-3x^6+3x^3+5$	 rises to left falls to right $y \to \infty$ as $x \to -\infty$ $y \to -\infty$ as $x \to \infty$ Ex. $y=-2x^7-x+4$

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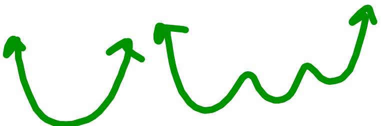
visual examples:

even degree
positive lead coeff $5x^4$

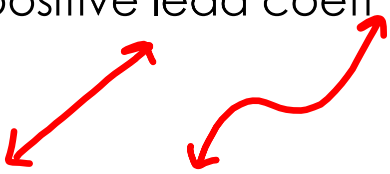


as $x \xrightarrow{\text{right}} \infty$, $f(x) \to \infty$
$x \to -\infty$ down, $f(x) \to \infty$ up

even degree $-5x^4$
negative lead coeff

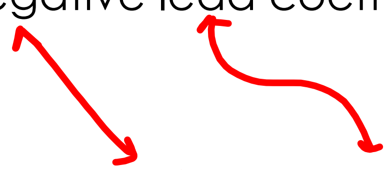
$x \xrightarrow{\text{right}} \infty$, $f(x) \to -\infty$
$x \to -\infty$ left, $f(x) \to -\infty$ down

odd degree
positive lead coeff $3x$



as $x \xrightarrow{\text{right}} \infty$, $f(x) \to \infty$ up
$x \to -\infty$ left, $f(x) \to -\infty$ down

odd degree $-3x$
negative lead coeff



as $x \xrightarrow{\text{right}} \infty$, $f(x) \to -\infty$ down
$x \to -\infty$ left, $f(x) \to \infty$ up

Give the end behavior for the polynomial

$y = 4x^3 + 5x^2 + 7x + 1$

odd positive

degree: 3 L.C: 4

left

$x \to -\infty, f(x) \to \underline{-\infty}$

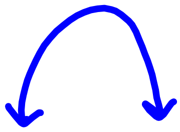
$x \to \infty, f(x) \to \underline{\infty}$

right

$y = -3x^2 + 2x - 1$

degree: 2 L.C: −3

even negative



$x \to -\infty, f(x) \to \underline{-\infty}$

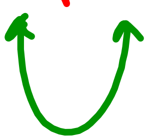
$x \to \infty, f(x) \to \underline{-\infty}$

Give the end behavior for the polynomial

$y = x^4 + 5x^2 + 3$

degree: 4 LC: 1

even positive



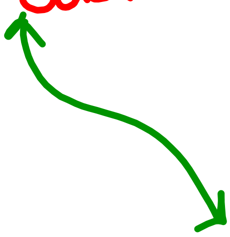
$x \to -\infty, f(x) \to \underline{\infty}$

$x \to \infty, f(x) \to \underline{\infty}$

$y = -x^5 + 4$

deg: 5 LC: −1

odd negative



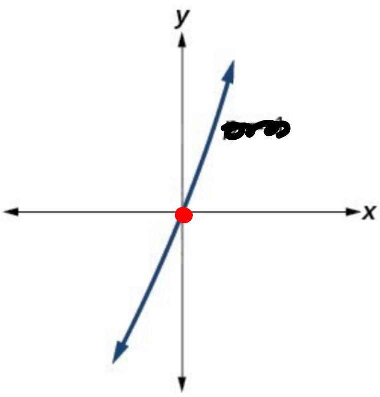
left

$x \to -\infty, f(x) \to \underline{\infty}$

$x \to \infty, f(x) \to \underline{-\infty}$

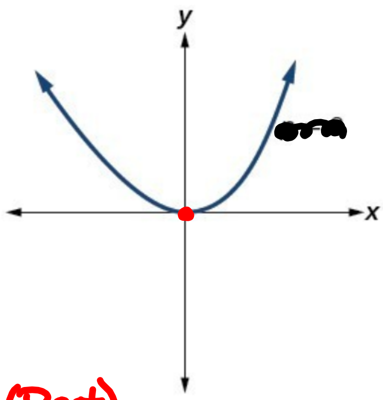
right

Multiplicity of Zeros (Roots)



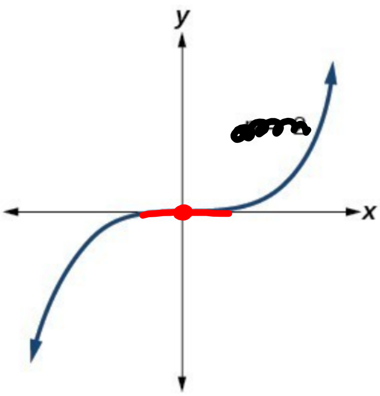
Single zero

Cross through x-axis at that zero (root)



(Root) Zero with multiplicity 2

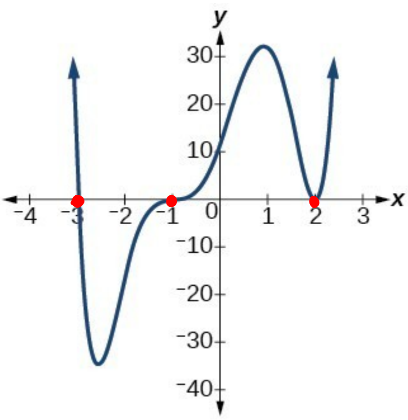
Bounces off x-axis



Zero with multiplicity 3

Flattens (horizontal tangent)

Identify whether the function graphed has an odd or even degree and a positive or negative leading coefficient. Define multiplicities.



Degree even

Lead coeff +

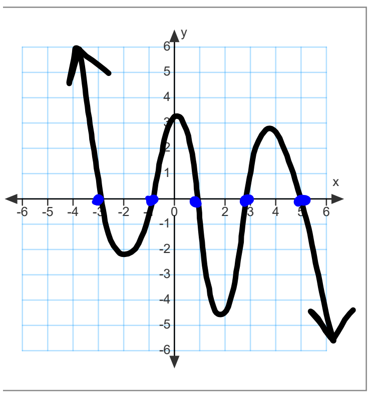
roots and mult:

$x=-3$ mult. 1

$x=-1$ mult. 3

$x=2$ mult. 2

$1+3+2=6$ least degree x^6

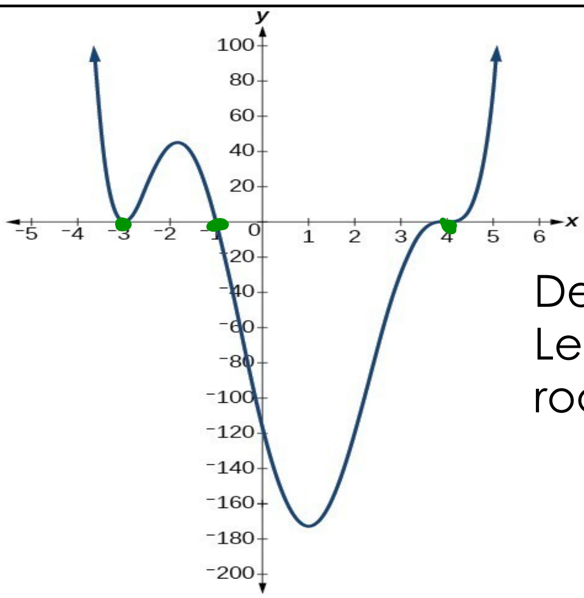


Degree odd

Lead coeff −

roots and mult:

$x=-3$, $x=-1$, $x=1$, $x=3$, $x=5$ all w/ mult. 1



Degree even

Lead coeff +

roots and mult:

$x=-3$ mult. 2

$x=-1$ mult. 1

$x=4$ mult. 3

The Fundamental Theorem of Algebra

- If P(x) is a polynomial of degree n, there are exactly n complex roots. x^7 means 7 roots
- This means that some of the roots might be imaginary and some might be real.

imaginary solutions always come in pairs $\pm 3i$

Zeros of a Polynomial Function:

An $\mathbf{n}^{th}$ degree polynomial function in one variable has at most **n** real zeros. There will be exactly **n** real or complex zeros.

An $\mathbf{n}^{th}$ degree polynomial will have **at most n-1** "turns" or "relative max or min".

$x^6 \rightarrow$ 6th deg. $\rightarrow 6-1=5$ turns (extrema or max min)

x^2 $\quad 2-1=1$ turn

x^3 $\quad 3-1=2$ turns

---

* x = a is a *zero* or *root* of the function f(x).

$x=4$

* x = a is a *solution* of the equation f(x) = 0.

$x=4$ $\qquad f(4)=0$

* (x - a) is a *factor* of the function f(x).

$x-4$

* (a,0) is an *x-intercept* of the graph of f(x).

$(4,0)$

1. Consider the polynomial function g(x) that has **zeros** at $x = 3$, $x = -\sqrt{2}$, and $x = \sqrt{2}$.

change to factors for part

a. what is the minimum degree of the polynomial function **g(x)**? 3 bc there are 3 real zeros

b. Assuming all of the coefficients of the polynomial are real and the leading coefficient is 2, create the polynomial function in factored form that should describe **g(x)**.

$g(x)=2(x-3)(x+\sqrt{2})(x-\sqrt{2})$

mult. these 1st

c. Rewrite the polynomial, **g(x)**, in expanded form.

$g(x)=2(x-3)(x+\sqrt{2})(x-\sqrt{2})$

$=(2x-6)(x^2\cancel{-\sqrt{2}x+\sqrt{2}x}-2)$

$g(x)=(2x-6)(x^2-2)$

$g(x)=2x^3-6x^2-4x+12$

2. Consider the polynomial function **p(x)** that has **zeros** at **x = 2**, **x =-2**, and **x = 4**.

a. what is the minimum degree of the polynomial function **p(x)**? 3

b. write the polynomial function, **p(x)**, in expanded form.

$p(x)=(x-2)(x+2)(x-4)$

$(x^2-4)(x-4)$

$\boxed{p(x)=x^3-4x^2-4x+16}$

3. Consider the polynomial function **q(x)** that has zeros at **x = 1** and **x = 3i**, + $-3i$

+ imaginary sol. + radicals come in opposite pairs

Assuming the leading coefficient is 1, create a polynomial function in factored form that should describe **q(x)**.

multiply i ( ) 1st

$i^2 = -1$

$$q(x) = (x-1)(x-3i)(x+3i)$$

$$= (x-1)(x^2 \cancel{-3ix+3ix} - 9i^2)$$

$$(x-1)(x^2 - 9(-1))$$

$$(x-1)(x^2+9)$$

q(x) = $x^3 - x^2 + 9x - 9$

c. Rewrite the polynomial function, **q(x)**, in expanded form.

q(x) =

$i + -i \rightarrow (x-i)(x+i) = x^2 - i^2 = x^2 + 1$

$2i + -2i \rightarrow (x-2i)(x+2i) = x^2 - 4i^2 = x^2 + 4$

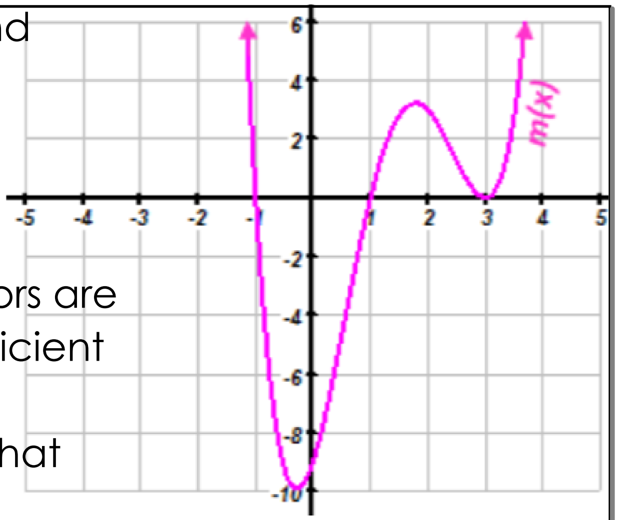
$3i + -3i \qquad x^2 + 9$

$4i + -4i \qquad x^2 + 16$

$5i + -5i \qquad x^2 + 25$

$6i + -6i \qquad x^2 + 36$

4. a. List all of the zeros and multiplicities.



$x=-1$ mult 1
$x=1$ mult 1
$x=3$ mult 2

b. Assuming all of the factors are real and the leading coefficient is 1, create a polynomial function in factored form that should describe **m(x)**.

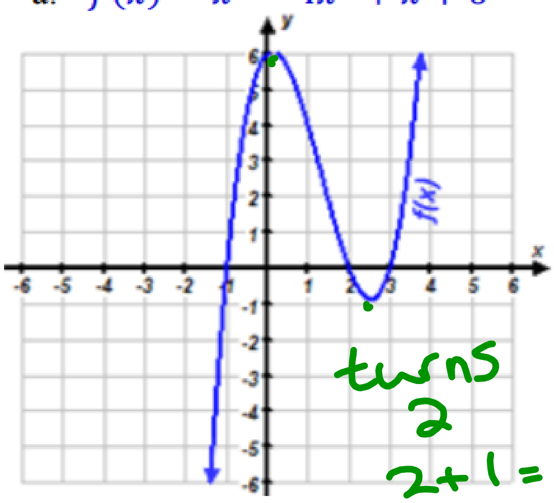
$(x+1)(x-1)(x-3)(x-3)$
$(x^2-1)(x^2-6x+9)$
$x^4-6x^3+9x^2$
$-1x^2+6x-9$

$\boxed{m(x) = (x+1)(x-1)(x-3)^2}$

c. rewrite the polynomial function, **m(x)**, in expanded form.

$\boxed{m(x) = x^4-6x^3+8x^2+6x-9}$

5. a. $f(x) = x^3 - 4x^2 + x + 6$



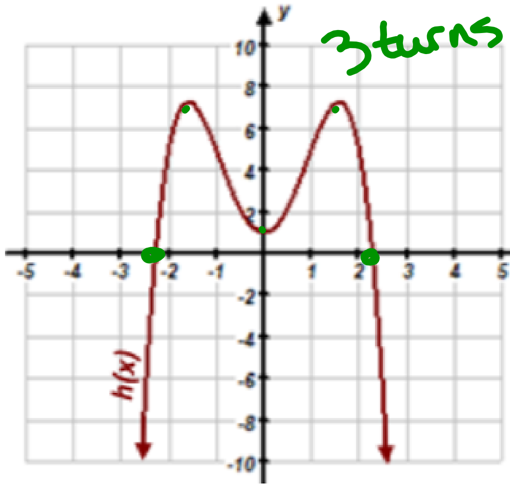
turns
2
$2+1=$ degree 3

number of real zeros:
(indicate any with higher multiplicity)
3

number of imaginary zeros:
0

b. $h(x) = -x^4 + 5x^2 + 1$

3 turns + 1 = degree 4



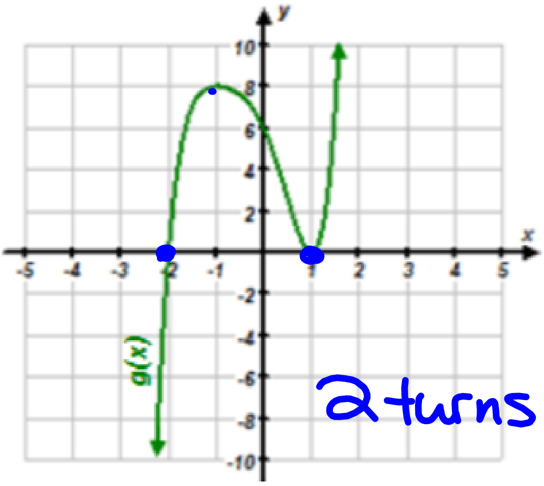
number of real zeros:

2

number of imaginary zeros:

2

c. $g(x) = x^5 + 2x^4 - 4x^2 - 5x + 6$



number of real zeros:

3

number of imaginary zeros:

0

2 turns + 1 = deg 3